Chapter 1

Introduction

1.1 A brief historical overview

The nearshore coastal region is the region between the shoreline and a fictive offshore limit which usually is defined as the limit where the depth becomes so large that it no longer influences the waves. This depth depends on the wave motion itself and in simple terms it can be identified as a depth of approximately half the wave length. Thus in storms with larger and longer waves the offshore limit moves further out to sea. This definition is practical because the influence of the bottom on the waves is one of the most important mechanisms in nearshore hydrodynamics.

Nearshore hydrodynamics could probably be said to have been founded by G. G. Stokes, who in 1847 developed the first linear and nonlinear wave theory. Today this theory is often referred to as *Stokes waves* (see also Stokes, 1880). Over the following century various wave phenomena were analysed and a great number of results, remarkable from a mathematical point of view, were obtained. Of particular importance from todays perspective was the development by Boussinesq (1872) of the consistent approximation for nonlinear waves in shallow water, a situation for which Stokes himself recognized that his theory was failing. Korteweg and DeVries (1895) added to this result by finding analytical solutions to the Boussinesq equations. These solutions are known as *cnoidal* and *solitary waves.* Interestingly the infinitely long solitary waves had already been observed in real channels by Russell (1844). Finally even this ultra brief historical review would be incomplete without mentioning the pioneering discovery of the wave radiation stress by Longuet-Higgins and Stewart (1962). This established the insight that forms an essential element in all later research related to currents and long wave generation in the nearshore.

The advent of computers has radically changed the perspective of what is relevant hydrodynamics in todays world. Equations or theories that, when developed before the computer age, were merely of theoretical interest have become central to modern engineering applications while many of the remarkable mathematical results that helped the understanding of how waves behave have become mainly of academical interest. The content of this book partially reflects that in the choice of which subjects and results are pursued in detail.

1.2 Summary of content

As an introduction Fig. 1.2.1 from Svendsen and Jonsson (1976) shows a schematic of most of the major wave phenomena that occur in the nearshore. These, and some more that are not visible in such a picture, are the phenomena that are analysed further in the following chapters.

Chapter 2

The first chapter (Chapter 2) is meant as a reference chapter that essentially presents the main hydrodynamical results used later in the book. For most sections there are no derivations in this chapter. If the reader needs further explanation reference is made to the textbooks quoted in the list of references at the end of the chapter. Exceptions are the sections on boundary conditions, turbulence and energy flux which contain material not so easily found in standard books.

Chapter 3

Remaining central to the understanding of nearshore wave and current motion is the Stokes theory, which in its simplest linear form represents the most important theoretical background for nearshore hydrodynamics. Chapter 3 therefore gives a thorough analysis, not only of the linear wave theory itself but also of the most important of the results that have been derived on the basis of that theory.

The main objective of the linear theory is to establish a first approximation for all the flow details of small amplitude waves on a constant depth. This is done in Section 3.2.

The characteristic surface profile of such waves is described by the sine function, whence they are also called **sinusoidal** waves. It turns out that even though the average over a wave period of such wave profiles is zero

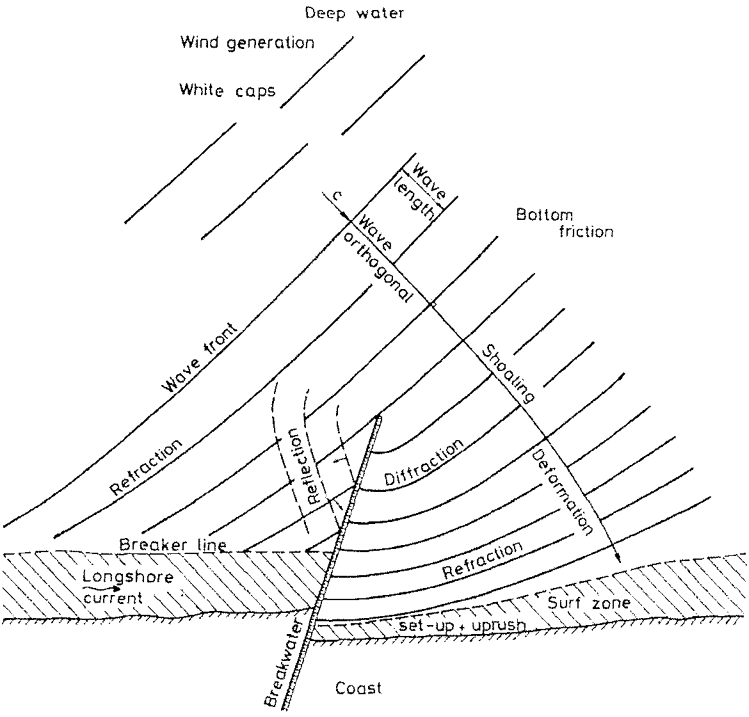

Fig. 1.2.1 Nearshore wave processes (Svendsen and Jonsson, 1976).

they still have properties that in average over a period are non-zero (Section 3.3). Linear waves transport energy (the socalled **energy flux**) which is the mechanism that causes waves generated in an area to spread forward in the direction of wave propagation. Waves also represent both a **mass (or volume) flux** of fluid and they exert a net force on the surroundings that is called the **radiation stress**.

Section 3.4 explores what happens when we utilize the freedom of linear theory to form new wave solutions by adding solutions of waves. Thus two waves added can form **standing waves** or **wave groups**. And in particular the results of adding arbitrarily many waves leads to the concept of **wave spectra** which can be used in the analysis and description of random seas.

A particularly important element in the hydrodynamics of waves on a coast is the variation of the water depth. The effect of wave propagation over a varying depth is analysed in Section 3.5. The major effect causing changes of the waves is the depth dependence of the propagation velocity for the wave forms. This induces **wave refraction** which is shown to follow laws similar to the laws controling the propagation of light and sound. The depth variations also cause change in wave heights. This becomes particularly important as the waves approach the shore, because the decreasing depth increases the waves heights so that they eventually break. The process is termed **shoaling** and the concepts of energy flux generated by the waves and of energy conservation controls the development of the wave height.

In the nearshore the currents also play an important role in changing the waves. Section 3.6 gives a brief introduction to the main mechanisms of combined **waves and currents**, including the doppler effect which is also known from optics and from the propagation of sound.

The refraction theories described in Section 3.5 makes assumptions about the wave motion theatre not satified when there are rapid changes along wave fronts such as when waves propagate around the tip of breakwaters and also out in a general wave field the when wave height changes over short distances along a wave front. This influences the propagation pattern even for linear waves, a phenomenon called **diffraction**. In Section 3.7 we develop a theoretical approach to the combination of depth refraction and diffraction. This leads to the socalled **Mild Slope Wave Equation (MSE)** which describes the variation over a domain of the wave height and wave pattern. The derivation and properties of this equation is discussed in detail.

Chapter 4

Chapter 4 is dedicated to a closer look at the energy balance in waves both before and after breaking. This expands the analysis in Section 3.5 and involves discussion of the various types of energy present in the nearshore and derivation of the **energy equation** which is an equation that describes the transformation and propagation of energy in areas with varying depth and currents.

Chapter 5

One of the most improtant physical processes in the nearshore region is the **wave breaking** that occurs close to the shore of beaches. As mentioned this is caused by the (gradual) decrease in depth closest to the shore. On sufficiently gently sloping beaches such as most littoral beaches the breaking process destroys or dissipates almost all the incoming wave energy in the nearshore region called the **surfzone**. This causes rapid changes in the waves with violent particle velocities that highly contribute to the movement of sediment material and beach erosion. The rapid changes in wave height also imply rapid changes in radiation stresses for the waves. This create the most important forcing mechanism for nearshore currents. Our knowledge about the wave breaking is still limited and in Chapter 5 we use both measured data and theoretical analysis in an attempt to describe and understand the details of the wave motion.

Chapter 6

In Chapter 6 a brief overview is given of the *types* of wave models based on of the results described in the previous chapters that are frequently used today.

Chapter 7

The following three chapters are focusing on the effect of finite wave heights. In the linear wave theory discussed and utilized in Chapter 3 we assumed the wave height was infinitely small (which is the reason the equations become linear). Real waves, however, can be quite steep and the finite wave height has profound effects on the motion. In Chapter 7 we first derive the different forms of the model equations that properly describe the flow in different parameter ranges for the waves. The key parameters here are the water depth, the wave length (or wave period) and the wave height. It is shown how that this leads to the governing equations for the classical theories of **Stokes waves, Boussinesq waves** and **nonlinear shallow water waves** to mention the most important.

Chapter 8

Chapter 8 is then presenting a detailed derivation of the **second order stokes theory** and briefly outlines third and fifth order versions of the theory. The chapter also includes a description of the special computer version of the stokes wave theory called the **stream function theory**, which makes it possible to calculate the properties of stokes waves to very high order. This theory has been proven a very effective numerical tool.

Chapter 9

In recent years the **Boussinesq wave theory** has become one of the most effective ways of analysing nearshore wave motion computationally. Chapter 9 derives the basic equations. It also gives a relatively detailed account of the constant form solution for Boussinesq waves called **cnoidal waves** which is the Boussinesq wave equivalent to the sinusoidal waves of linear wave theory, and the infinitely long version of those waves waves called **solitary waves**.

A strength of the Boussinesq equations is that when solved computationally they provide the development in time and space of the entire wave motion in a coastal domain, which makes it possible also to analyze irregular waves such as wind generated storm waves. One of the weak points of Boussinesq wave theory is its limitation to relatively shallow water. However, numerous recent results have modified the equations to forms that extend the validity of the theory almost to the limt of what we have defined as the nearshore region. From the perspective of practical applications this has tremendous importance by making the method viable. These developments are also presented in the chapter. Another problem is that solving the Boussinesq equations in a realistically large domain over a sufficiently long time period of time for practical applications still requires very substantial computational efforts.

Chapter 10

When waves propagate over a domain with depth small enough that the depth influences the waves (as in the nearshore region) a **boundary layer** develops at the bottom. In the traditional approach to wave motion analysis (such as described in the chapters above) the effect of this boundary layer is disregarded: the motion is considered irrotational described by a velocity potential and at the bottom we essentially have a **slip velocity**. However, the boundary layer is real and it does produce both a local disturbance of the flow near the bottom and a shear stress (or **bottom friction**) acting on the fluid above. This stress dissipates energy and when waves propagate over longer distances the accumulative effect of the energy dissipation due to the bottom friction causes the wave height to decrease slowly but significantly. Chapter 10 presents the classical theory of viscous wave boundary layers for stokes waves to first and second order in the wave amplitude. It also derives and discusses the expressions for the socalled **steady streaming** in the boundary layer which is a net current generated by nonlinear mechanisms active inside the boundary layer. The chapter then proceeds with analysis of turbulent boundary layers giving results based on the empirical concept of a fricion coefficient. The general case of combined wave-current motion is presented in detail.

Chapter 11

The wave averaged properties are important parts of the mechanisms resposible for the wave generated currents, such as longshore and cross-shore currents, socalled **nearshore circulation**. These currents are important in the nearhsore environments where they contribute significantly to the morphodynamic changes of beaches. Because the currents are esentially wave averaged flows those currents are governed by wave averaged equations which are also depth integrated. In Chapter 11 we describe the derivation of those equations which also reveals the exact definitions of the wave mass (or rather volume) flux and the radiation stress. Those concepts are analyzed in detail for linear waves which is the form most frequently used in applications and also put into context of waves in two horizontal dimensions.

The chapter then goes through two important special ("canonical") cases of nearshore circulation: the cross-shore momentum balance on a long straight coast with shorenormal wave incidence, and the wave generated longshore current on such a coast with oblique wave incidence. Because the equations are wave averaged they require as input information about the volume flux and the radiation stresses at all points of the domain which correspnds to demanding the wave motion known. This information is usually provided by wave models of the type described in earlier chapters, particularly linear models.

Finally the use of boundary conditions along the free boundaries of nearshore models is described. Such boundaries are artificial in the sense that they only exist because we limit the computations to a section of a coast. The demand along such boundaries is that they form no obstacle to the wave motion. In particular the waves that would want to propagate out of the domain - either because they were generated inside or were reflected from the beach or engineering structues inside the computational domain - should be able to do so freely.

Chapter 12

The nearshore currents covered by the equations discussed in the previous chapter are essentially depth averaged and therefore no resolution is obtained for the vertical variation of those currents. In the case of a long straight beach with shorenormal wave incidence it is clear, however, that since the waves have shoreward net volume flux then there must also be a seaward going current. This is called the **undertow** and the mechanisms governing this flow are analyzed in Section 12.2.

Chapter 13

The results for the undertow can be supplemented with a similar analysis of the longshore currents and this reveals that the nearshore currents in general vary over depth both in magnitude and direction. It turns out that this feature is important for the way in which the currents interact and change in the horizontal direction (Chapter 13). This chapter analyses general 3-dimensional currents by expanding the depth integrated and time averaged equations for nearshore circulation derived in Chapter 11 to depth varying currents and gives analytical results for the vertical profiles of the currents. The resulting equations are called **quasi-3D equations** because the depth varying currents is represented in the modified depth integrated equations as coefficients that account for the horizontal effects of the depth variations, much like the momentum correction factor in engineering hydraulics equations account for the depth variation of the flow in a river.

Chapter 14

Finally the variation in height and period of the irregular wind waves approacing a beach leads to variation in the radiation stresses which ends up generating new, much longer waves. These **infragravity or IG** waves become particular important in the inner part of the nearhsore region where the wind waves are breaking while the IG-waves usually are not (Section 14.1). Canonical examples of IG waves are the socalled **edge waves** which anre waves propagating along the shore with their strongest motion closest to the shoreline and decreasing seaward.

The chapter also analyses the fact that the simple models of nearshore currents, developed under the assumption of steady flow, turn out to be unstable - socalled **shear instabilities**. A consequence is that many (or most) longshore currents show fluctuations in time and space that again have profound influence also on the mean currents. The initial linear instability theory is developed and numerical computations of what happens as the instabilites grow into complex longshore flows are discussed.

The advanced present day nearshore models are now opening such situations from natural beaches to realistic computational analysis. A food for thought discussion is offered at the end of the Chapter 14 about these and other complex flow situations found on natural beaches and how models can help improve our understanding.

1.3 References - Chapter 1

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