Preface

Twenty Years of Białowieża: An Anthology

This volume marks the twentieth anniversary (which in fact passed three years ago) of the Białowieża series of meetings on Differential Geometric Methods in Physics. What once started out as a summer rendezvous for a few interested physicists and mathematicians from within Poland and some neighbouring countries, the Białowieża meetings have now grown into an annual pilgrimage for a devoted and growing international group, sharing scientific and professional experiences. Additionally, these meetings, held each year at the beginning of July, in the sylvan setting of the ageless Białowieża forests, have been the rite of passage for scores of graduate students and young researchers, starting out on their careers. The world of mathematical and theoretical physics knows no dearth of meetings, workshops, conferences, symposia ... each with its own series of proceedings volumes. What sets the Białowieża meetings apart is perhaps the "purity of expression", in mathematical terms and the close interaction between physicists and mathematicians that takes place during these week-long shared sojourns. More than at most mathematical physics meetings, the focus of attention here is rather on the mathematical structures than on the phenomenological aspects of the physical problems of interest. Added to that are the stimulating social ambience and camaraderie that have developed over the years.

The present collection is not a proceedings volume. Rather, it was conceived as a means to replicate the spirit of the Białowieża Workshops and to reflect its scientific tenor, as a tribute to the completion of two decades of a shared scientific experience. While the focus of the Białowieża workshops has naturally been on a small subset of areas within the broad gamut of mathematical physics – prominent among them being quantization techniques, coherent states, symplectic geometry, Poisson structures, infinite dimensional systems and new trends in the application of geometric methods to physics – yet even this narrow spectrum could not have been adequately represented in a single volume. Consequently, only a few topics could be chosen for inclusion in this collection. With this constraint in mind, a number of former participants and invited speakers were approached to contribute to this commemorative anthology, as it were, with papers that would represent a cross-section of the themes and topics that were, or could have been, discussed in these Workshops. Unfortunately, not everyone approached was able to contribute at this time. Nevertheless, in the opinion of the editors, what is presented here, does in many ways crystallize the Białowieża approach to mathematical physics.

In the course of its twenty-year history, the Workshop has offered a substantial core of mathematical lectures presented by mainstream mathematicians. Yet, as a result of the subject of the lectures and also of the personality of the lecturers, the audience has evolved to attract a number of mathematical physicists. The participation of the latter contributed different motivations and favored the raising of questions of intent. We all advertise to our students – and to our administrations – that mathematical techniques are demanded for the solution of problems in the physical sciences and in engineering; we know that this is no accident in the obvious sense that these techniques were often developed in direct response to questions from outside the ethereal realm of pure mathematics; and we all have our favorite examples. Nevertheless, what remains mysterious, as it did even to such creative practitioners as Wigner, is the "unreasonable effectiveness of mathematics". Mathematics that was developed as pure mathematics suddenly gets applied in fields ignored by the mathematical community. Flows running in these opposite directions have naturally surfaced in Białowieża also. While no external relevance is recognized as a precondition, the question of such relevance has been put forward as often as it has been brushed aside. These opposite pulls have contributed to the vitality of the Białowieża Workshops. We hope that some of the readers of the present collection will want to examine the various contributions from that angle, either demanding physical interpretation and motivation or insisting on mathematical purity and ingenuity. In this connection, we are reminded of the aphorism of the Polish mathematician Mark Kac who, upon considering the courtiers in attendance to the Queen of the Sciences, was moved to fear for her virtue. While Kac meant it as a barb against the purists nestled in the mathematical establishment, it seems to us that the aphorism could very well be double-edged. In this spirit, several of the contributions to this volume were solicited without concern for actual applications, but we believe none should be immune to this type of scrutiny.

A few words of introduction to the papers presented in this volume are in order. There is a cycle of four papers on different aspects of the theory of quantization – one of the most prominently represented areas of research at these workshops. The paper by N.P. Landsman entitled, "Functorial Quantization and the Guillemin-Sternberg Conjecture", addresses the issue of quantization in the presence of constraints; specifically, to find definitions for the arrows in following diagram that are reasonable and sharp enough to make sense of the question as to whether the diagram is commutative.

$$\begin{array}{ccc} \textit{Unconstrained} & Q & \textit{Unconstrained} \\ \textit{Classical system} & \longrightarrow & \textit{Quantum system} \\ & & \\ R \downarrow & & \downarrow R \\ & Q & \\ \textit{Classical system} & \longrightarrow & \textit{Quantum system} \\ \textit{with Constraints} & & \textit{with Constraints} \end{array}$$

Building up on the quantization approach now associated to the name of Raoul Bott, Landsman had recently proposed to view quantization as a functor between two categories, namely isomorphism classes of symplectic dual pairs and homotopy classes of Kasparov bimodules. To provide the definition of this functor between these categories is the main purpose of the present paper.

The two papers entitled, "Diffeomorphism Groups and Quantum Configurations as Mathematical Objects", by G.A. Goldin and "The Group of Volume Preserving Diffeomorphisms and the Lie Algebra of Unimodular Vector Fields: Survey of some Classical and not so Classical Results", by C. Roger, have a unifying aspect in that they both deal with the use of diffeomorphism groups as "receptacles" for several classical and quantum theories, notable among them being geometric and deformation quantization, local current algebras and their relation to quantized fields, vortex quantization in hydrodynamics, and anyons. The paper by Goldin has more of a physical flavour, with the mathematical structures introduced and explained in relatively non-technical terms. The author manages to survey an impressive spectrum of problems for which the diffeomorphism group could be used as an analyzing tool. The paper by C. Roger, bears distinctly a mathematician's stamp. It is a rigorous description of certain aspects of Lie algebras and "groups" of unimodular vector fields. The latter are vector fields that exponentiate to volume preserving diffeomorphisms. A number of mathematical results on the cohomology of the Lie algebra of vector fields is presented, one of the most important being the rigidity theorem, demonstrating that the Lie algebra of unimodular vector fields in dimensions greater or equal 3 admits no non-trivial formal deformations (Theorem 3.2). The theory is then extended to supermanifolds.

The fourth paper on the subject of quantization is, "Coherent State Method in Geometric Quantization" by A. Odzijewicz. Coherent states have been a constant theme running through the Białowieża meetings. The discussion in this paper is based on the construction of a *coherent state map*, between the classical phase space and a family of vectors on the quantized Hilbert space, which under conditions that are spelled out in the paper, yield a quantization of the original classical system. The situations where such a map yields the same results as the geometric quantization of Kostant-Souriau are worked out in some detail. A number of examples illustrate the theory.

There are two papers on the subject of symplectic and Poisson geometry. In the paper entitled, "Moduli Space of Germs of Symplectic Connections of Ricci type", by M. Cahen, the prototype for the structures studied arise, in the particular case of symplectic manifolds that are compact and simply connected, from complex projective spaces $\mathsf{P}_n(\mathsf{C})$, equipped with their standard symplectic form and the Levi-Cevita connection associated to the Fubini-Study metric. The last section of the paper is devoted to the study of the moduli space of Ricci-type connections of $\mathsf{P}_n(\mathsf{C})$ and it makes explicit the sense in which the Levi-Cevita connection for the Fubini-Study metric is an isolated point in this space. The general motivation for the paper is provided by an analogy with the moduli space of Einstein metrics, i.e. the space of Einstein metrics on a manifold M *modulo* the action of the diffeomorphism group of M. The other paper entitled, "Banach Lie-Poisson Spaces", by A. Odzijewicz and T.S. Ratiu, is a summary of the authors' work in this field spanning several years. The paper sets out the category of Banach Lie-Poisson manifolds, which are Poisson manifolds defined over Banach spaces, such that the duals of these spaces form Banach Lie-algebras under the Poisson bracket operation. The morphisms for the category of such spaces are the *linear Poisson maps*. The construction is illustrated with a number of examples of both mathematical and physical interest.

The paper by R. Picken, "A Cohomological Description of Abelian Bundles and Gerbes", stands by itself in that the geometric structures discussed there have potential applications to non-abelian gauge theories – a subject which has not featured in the mainstream of the Białowieża Workshops. Nevertheless, the paper adds a new flavour to the volume, even more so because of the easy pedagogical style of presentation. Gerbes with connection are higher order generalizations of abelian bundles with connection. The aim of this paper is to achieve a cohomological description of both gerbes and abelian bundles in which this generalization is seen to emerge in a natural way.

The paper by R.P. Langlands, entitled "The Renormalization Fixed Point as a Mathematical Object", also opens a new direction in the Białowieża repertoire, namely the delineation of universality classes in the critical phenomena of classical statistical mechanical systems exhibiting phase transitions. There is nevertheless a link to the main paths traditionally trod at the Workshop, namely the emergence of classical, or macroscopic, behaviour that are revealed in the course of asymptotic developments. Here, this process is provided by the successive iterations of rescaling known as renormalization (semi-)group techniques. Some of the immediacy of the present paper stems from its character as a review based mostly on numerical results obtained for well chosen models, specifically percolation and 2-D Ising.

In the paper by J.Hilgert entitled, "An Ergodic Arnold-Liouville Theorem for Locally Symmetric spaces", the main result is stated as the last theorem on the second last page of the paper, and it may be a good idea for the reader, before studying the paper in detail, to have a quick glance at this theorem. The author proposes to view it as an "ergodic Arnold-Liouville theorem" in analogy with the standard Arnold-Liouville theorem on integrable Hamiltonian systems. While the analysis is resolutely carried out within the realm of classical dynamical systems, the author also alludes to possible relevance to the quantization of some of the structures considered.

The paper entitled, "Spectra of Operators Associated with Dynamical Systems: from Ergodicity to the Duality Principle", by Antonevitch, *et al*, is of a functional analytic nature. The authors extend from reversible to irreversible shifts several aspects of the study of classical dynamical systems. Their main results for irreversible shifts is stated in Theorem 4.6 which establishes a duality – realized explicitly as a Legendre transform – between two quantities they define separately for the abstract version, *viz*, the spectral exponent of weighted shifts and a new dynamical entropy. To illustrate the theory, the last section of the paper is devoted to a discussion of the Perron-Frobenius systems and some of their properties.

In the paper by W. Pusz and S.L. Woronowicz entitled, "On Quantum Group of Unitary Operators: Quantum '$az + b$' Group", the authors discuss a theory whose inception and growth has in fact been coeval with the Białowieża meetings. The paper briefly surveys the present status of quantum group theory and introduces the concept of a quantum group of unitary operators, relevant to a study of non-compact, locally compact quantum groups. The theory is then illustrated by constructing a quantum "$az + b$" group.

As our commentaries should have amply demonstrated, we are happy to present this volume to our readers as a representative of the scientific variety of the Białowieża experience.

There remains only the pleasant task of expressing our debt of gratitude to our friend, colleague and co-sharer of the Białowieża experience, Mikhail Shubin, for his generous help with the editing of this volume and bringing it to fruition.

The Editors
(S.T. Ali, G.G. Emch,
A. Odzijewicz, M. Schlichenmaier
and S.L. Wronowicz)