

Chapter 1

Introduction

In this introductory chapter, we first give some motivations for the use of nonlinear systems in engineering, discussed in Section 1.1. In Section 1.2 we sketch the broad picture of this book. A chapter by chapter overview is given in Section 1.3. In Section 1.4 a list of new contributions related to this book is given.

1.1 Nonlinear systems in engineering

Linear systems theory has been extensively used in engineering applications *e.g.* for modelling and controlling the behavior of dynamical systems. Hence, what would be the reason for studying nonlinear systems instead of linear systems? One important reason is that the dynamics of linear systems are often not sufficiently rich to describe many commonly observed phenomena, as also motivated in [226] in reply to the question “Why do we need a nonlinear theory?” A few examples of such phenomena are

- a. Multiple equilibria or multiple operating points,
- b. Periodic behaviour of state variables or limit cycles,
- c. Bifurcation,
- d. Chaos.

The study of nonlinear dynamics has become an interdisciplinary field of science including physics, mathematics and others. From an engineering point of view, a main objective is trying to exploit these phenomena towards novel applications, beyond the fact that one is also driven by curiosity in order to understand these phenomena. In many engineering disciplines, nonlinearities were in the past often regarded as something which has to be

avoided, *e.g.* in control systems and electrical circuits design the validity of linear theories is either completely destroyed by nonlinearities or otherwise restricted to local regions in state space or to specific operating points. In more recent years, one has also recognized positive aspects with nonlinear systems that enable to develop novel information processing systems. At this point new mathematical foundations are needed in order to ensure reliable designs for systems based on nonlinear phenomena.

The Hopfield network [117] which is used for associative memories, transiently chaotic neural networks [38], chaos communication [1] and active wave computing [211] are some of the well-known examples which use multiple equilibria, bifurcation phenomena, synchronization and complex dynamical behavior in their related applications. For associative memories, equilibria of the Hopfield network correspond to several patterns which one wants to store and memorize. Transiently chaotic neural networks use bifurcation phenomena such that the network can search for the global optimum for a given optimization problem. In chaos communications, a chaotic signal is used as a carrier for transmitting information and synchronization methods are used to recover the information from the carrier. Active wave computing techniques that make use of spatial-temporal waves are used in image processing applications.

When the above mentioned phenomena are considered as methods for solving engineering problems, there is a need for platforms that can deal with the computational complexity needed to handle these phenomena. Cellular neural/nonlinear networks (CNN) [50] is such a framework, followed by the CNN universal machine (CNN-UM) [215] which is the first algorithmically programmable analog array computer. The CNN-UM has created a platform to exploit the above-mentioned phenomenas. Today we witness physical implementations of CNN-UMs [163, 164, 26] which have a profound impact in the engineering community such that it brings these phenomena from mathematical abstraction into the core of electrical engineering.

Figure 1.1 shows two different perspectives on computing for engineering applications. On the one hand one has the classical approach based on classical digital computers and digital signal processing. On the other hand one has computing based on principles of nonlinear dynamics. *Analogic* (from the contraction of *analog* and *logic*) computers [215] exploit both perspectives with their dual computing capability.

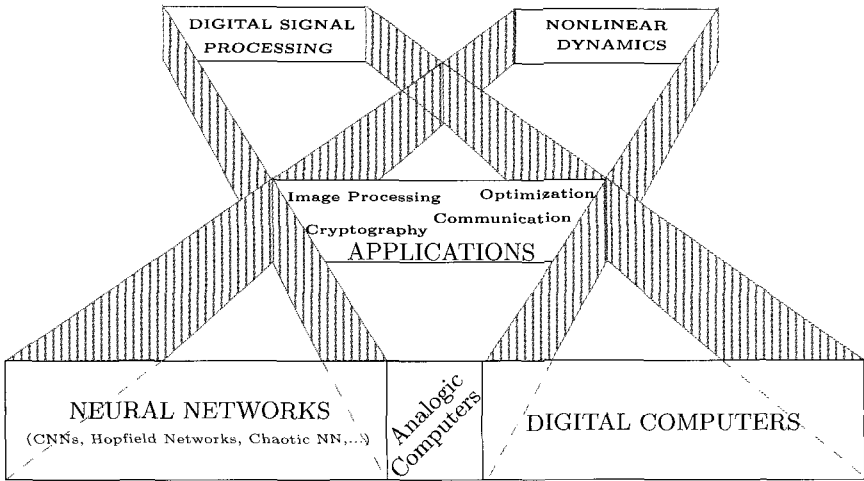


Fig. 1.1 Two different perspectives on computing for solving engineering problems. Classical digital computers handle the problems based on digital signal processing techniques. A new perspective is given from the viewpoint of nonlinear dynamics studies that addresses the same problem via neural networks. Analogic computers based on CNN-UMs serve as a platform at the intersection of these two different perspectives.

1.2 This book

This book is basically centered around three cornerstones:

- Cellular neural networks
- Chaotic systems
- Synchronization methods.

Related to these subjects, we investigate applications that aim at exploiting the dynamics of nonlinear systems (see Figure 1.1). Figure 1.2 shows the cornerstones in relation to the different topics of the book. Cellular neural networks serve at this point as architectural models to obtain programmable networks. These networks are composed of nonlinear dynamical cells. An important class of nonlinear systems studied in this book are *Lur'e systems*. This class of systems can be represented as a linear system interconnected by feedback to a nonlinearity that satisfies a sector condition. The subject of Lur'e systems touches upon the three cornerstones CNNs, chaotic systems and synchronization methods. The studied chaotic systems are generalizations of Chua's circuit, which can all be represented in Lur'e form. Chua's circuit can exhibit multiple equilibria, limit cycle behaviour,

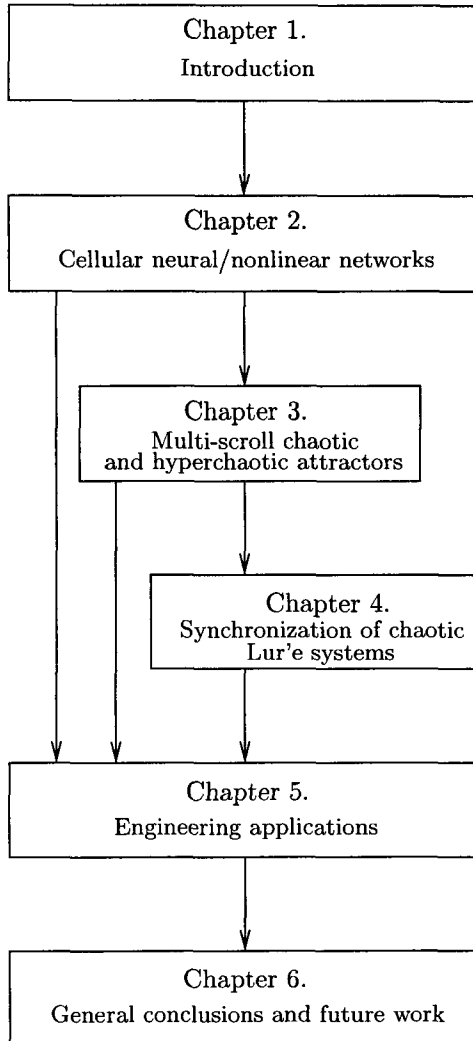


Fig. 1.3 Flowchart for this book. Chapter 5 discusses applications. The applications and the chapters that are used in these applications are shown in Table 1.1.

1.3 Chapter by chapter overview

Chapter 2. Cellular neural/nonlinear networks

This Chapter is devoted to Cellular Neural/Nonlinear Networks which are locally coupled nonlinear dynamical systems. We start with defining the

CNN acronym which is independent from its interpretation either as Cellular Neural Networks or Cellular Nonlinear Networks. We also present its mathematical definition by four specifications. The technical and special terms which are used in the CNN area research are explained. Then we discuss a number of well-known CNN models. We also review two generalized CNN models. While the result of one of these indicates that most of the known neural network architectures are covered by the generalized CNN model, the other leads to two new models, *i.e.* delay type and nonlinear CNNs. Then the CNN universal machine (CNN-UM) is discussed together with recent technological developments. We aim at reflecting the results of the VLSI hardware realizations of CNN-UMs (and also CNNs) without going into many details. Next background material is surveyed and a number of new research directions are given. An important new research direction is wave computing being driven by the VLSI implementation of CNN-UMs. We present a number of basic concepts of wave computing. Another research direction proposed in this Chapter is Coupled Local Minimizers (CLM). CLMs are presented here under the CNN paradigm and and its link to wave computing is explained.

Chapter 3. Multi-scroll chaotic and hyperchaotic attractors

We give an exhaustive overview of multi-scroll chaotic and hyperchaotic attractors. First we present multi-scroll chaotic attractors. Our outline for this Chapter is illustrated in Table 1.2 which chronologically shows authors and their contributions to the area of multi-scroll attractors. Historically, the double scroll attractor as generated from Chua's circuit is probably the first reported multi-scroll attractor. In this context theoretical and the heuristic approaches are discussed that prove the existence of chaotic behavior from Chua's circuit. We provide a short overview on circuit realizations of Chua's circuit. A realization methodology that allows to adjust independently the slopes and breakpoints of the nonlinearity is exploited to realize generalized Chua's circuits. Then we discuss several realizations and present experimental confirmations of 1-, 3- 5- and 6-scroll attractors. We also introduce a scaling of the piecewise linear (PWL) nonlinearity of the generalized Chua's circuit such that n -scroll attractors for large n values become feasible for implementations. We collect alternative realizations of n -scroll attractors based on Chua's circuit. Then a new family of scroll grid attractors is presented. These families are classified into three classes called 1-D, 2-D and 3-D grid scroll attractors, depending on the location

Table 1.2 Chronology of multi-scroll chaotic attractors, milestones and contributing authors.

| Date | Contribution | Reference |
|------|---|----------------------------------|
| 1986 | Double scroll family | Chua <i>et al.</i> [45] |
| 1991 | n -double scroll attractors | Suykens <i>et al.</i> [249, 250] |
| 1996 | 2-double scroll attractor implementation | Arena <i>et al.</i> [7] |
| 1997 | n -scroll attractors | Suykens <i>et al.</i> [248] |
| 1999 | 6-scroll attractor implementation | Yalçın <i>et al.</i> [309] |
| 2000 | 3- and 5-scroll attractors implementation | Yalçın <i>et al.</i> [310] |
| 2001 | n -scroll attractors via sine function | Tang <i>et al.</i> [265] |
| | Families of scroll grid attractors | Yalçın <i>et al.</i> [302] |
| | 1-D grid scroll attractors | |
| | 2-D grid scroll attractors | |
| | 3-D grid scroll attractors | |
| 2002 | 10-scroll attractor implementation | Zhong <i>et al.</i> [330] |
| 2003 | Prototype chip for a 3-scroll attractor | Fujiwara <i>et al.</i> [88] |
| | n - and $n \times m$ -grid scroll attractors from a second-order linear system with hysteresis series switchings. | Han <i>et al.</i> [106] |
| 2004 | n -, $n \times m$ - and $n \times m \times l$ -grid scroll attractors from a third-order linear system with hysteresis series switchings. | Lu <i>et al.</i> [167] |

of the equilibrium points in state space. The scrolls generated from 1-D, 2-D and 3-D grid scroll attractors are located around the equilibrium points either on a line, within a plane or in 3-D, respectively. Due to the generalization of the nonlinear characteristics, it is possible to increase the number of scrolls in all state variable directions. A number of strange attractors from the scroll grid attractor families are presented. They have been experimentally verified using current feedback opamps. We collect the recent research directions on the family of scroll grid attractors and their realizations. Then we focus on hyperchaotic attractors and introduce multi-scroll hyperchaotic attractors. While hyperchaotic n -scroll attractors are obtained from a generalized model like the generalized Chua's circuit, n -scroll hypercubes attractors are observed on a 1-D CNN consisting of n -scroll cells with weak unidirectional or diffusive coupling between the cells. Then we introduce scroll maps which generate imitated behavior of a two-sided Poincaré map of the n -scroll attractors. Finally we present Lur'e representations for the multi-scroll chaotic and hyperchaotic circuits which are studied in this Chapter. Hence, many results concerning stability and synchronization are applicable to it.

Chapter 4. Synchronization of chaotic Lur'e systems

In this Chapter we give a systematic overview of methods for master-slave synchronization of Lur'e systems. First we introduce different coupling configurations which help to distinguish between several synchronization schemes. Then we focus on the master-slave coupling configuration which implies that the master system evolves freely and drives the dynamics of the slave system. We basically divide the synchronization schemes into autonomous and non-autonomous schemes. For autonomous schemes the link is made between synchronization and absolute stability theory of Lur'e systems. First, the master-slave synchronization schemes and their synchronization criteria are given for both the full static state feedback and dynamic output feedback cases (when the full state vector cannot be measured). The proofs of the synchronization criteria are also given. A second scheme for the autonomous case is the robust synchronization scheme which deals with two non-identical Lur'e systems taking into account the influence of parameter mismatch between the master and the slave systems. The synchronization criteria and their proofs for the robust synchronization are given for both full static state feedback and dynamic output feedback cases. Then we deal with propagation delay in the master-slave synchronization scheme. A criterion for master-slave synchronization of Lur'e systems is presented when a time-delay exists in the master and slave systems. Synchronization criteria that are either time-dependent or time-independent are studied based on a Lyapunov-Krasovskii function for global asymptotic stability of the error system. A delay-dependent synchronization criterion is given based on a new Lyapunov function. Then we introduce non-autonomous schemes which are basically motivated by chaotic systems applications to communications. For non-autonomous schemes the link is made between synchronization and nonlinear H_∞ theory. Using a full state error or dynamic output feedback mechanism and formulating the synchronization problem with the standard plant framework of modern control theory, it is shown how the message signal can be recovered for a continuous time reference input (or message signal) which drives the master system. Synchronization criteria and their proofs for the nonlinear H_∞ synchronization are given for both full static state feedback and dynamic output feedback cases. We also study the influence of parameter mismatch between the master and slave systems with respect to the method of nonlinear H_∞ synchronization. As a last synchronization scheme, impulsive synchronization is discussed for state and dynamic measurement feedback. It has to

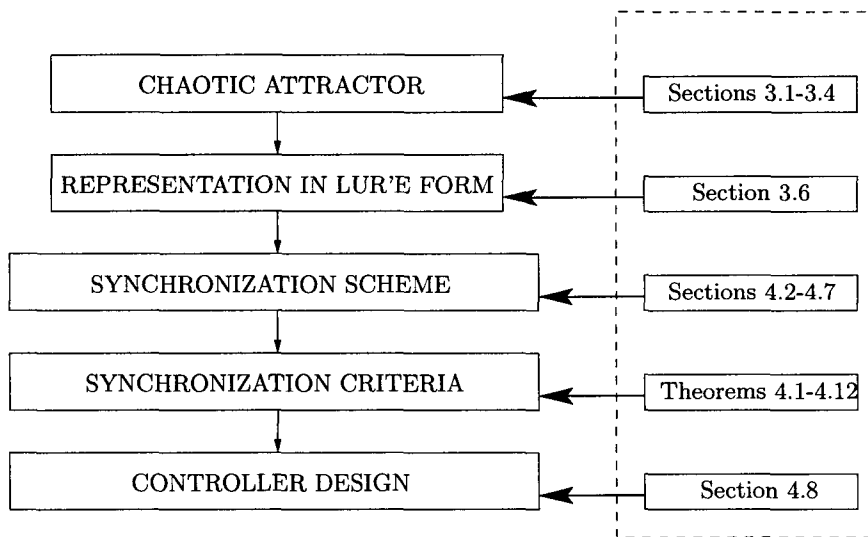


Fig. 1.4 Chaotic systems, representations in Lur'e form, synchronization methods and control design.

be pointed out that all given criteria are global but sufficient conditions. Furthermore the conditions are expressed in terms of matrix inequalities which lead to optimization based design of the controllers for achieving synchronization. Hence all of the given synchronization criteria correspond to related optimization problems. This may help the reader who wishes to skip theoretical derivations and design a controller for his/her application at hand. We provide an algorithmic procedure to employ a synchronization scheme (Figure 1.4) which also shows how Chapters 3 and 4 provide solutions for each block. At the end of this Chapter, we illustrate a number of examples which experimentally confirm the theoretical results. We use the circuits that are designed in the previous Chapter in order to implement a master-slave and a nonlinear H_∞ synchronization schemes.

Chapter 5. Engineering applications

In this Chapter, we face the question of how the methods and ideas from the previous Chapters can be used towards applications. Table 1.1 shows several applications which are considered in this book together with the Chapters that relate to these applications. We first present chaos communications. Although chaos may have potential applications in different

functional blocks of a communication system, we focus on applications for modulation and demodulation blocks. First we provide the reader with an overview on schemes which have exploited chaos for these blocks. Then we discuss and highlight the role of synchronization schemes from Chapter 4 for these blocks. Our discussions take into account the recent results of studies in chaos communications. A new application for chaos is also found in optimization. First, we give an overview of methods that have been presented in the literature. Then we introduce two new methods: chaotic annealing and coupled chaotic annealing. Chaotic annealing is similar to the use of noise in continuous simulated annealing. However, the role of the noise is played by chaos. Coupled chaotic annealing integrates chaotic annealing methods within the coupled local minimizers method. This method is interpreted within the CNN framework such that two 1-D CNNs work cooperatively towards finding good local minima. The method can also be considered as a wave computing algorithm that is controlled by state synchronization. A next application of chaotic signals in engineering is its use as a source for random number generators. We present a random bit generator that uses a double scroll attractor from a simple circuit model. The proposed TRBG is subjected to statistical tests using the well-known tests suites FIPS-140-1 and Diehard in cryptography. The proposed TRBG successfully passes all the statistical tests of the FIPS-140-1 test suite and a strong and more complete test suite of Diehard. Furthermore, Cellular Automata are applied for pseudorandom pattern generation thereby extending the related algorithms under the CNN framework. The discussed algorithms were tested on a CNN chip. The last application exploits the CNN-UMs for a watermarking-based image and video authentication and the pseudorandom pattern generators are used as a watermark generator. On-chip experimental results are reported, confirming the suitability of CNN-UMs to successfully act in real-time watermarking. Although VLSI implementations of the CNN-UMs (and the CNNs) have led to new directions in CNNs research, we discuss a side effect of the VLSI implementation. Analog implementations are advantageous in terms of speed but on the other hand are less robust. Novel approach towards chip-specific robustness are discussed for this purpose.

1.4 Main contributions

In this section we state a number of main contributions and new results, as this book is based on the PhD thesis of the first author:

- *Spatial-temporal pattern formation:*

It has been observed on the ACE16k CNN chip how pattern formation can take place. The CNN chip can be programmed with cloning templates in order to generate spiral waves and autowaves. The waves diffract from internal sources that can not be relocated on the network. However, by using initial and/or input images, an (external) source can be located at any place on the network. Furthermore, a competition between autowaves generated by external and internal sources can be observed. Propagation of autowaves on the inhomogeneous CNN array formed by the fixed-state map is discussed. This opens new directions for CNN applications based on spatial-temporal pattern formation. This contribution can be found in Section 2.4.3. The related reference is [315].

- *Implementation of n -scroll attractors from generalized Chua's circuits:*

We present experimental confirmations of 3- and 5-scroll attractors from a generalized Chua's circuit. The piecewise linear characteristic of a generalized Chua's circuit for generating n -scroll attractor is synthesized by a nonlinear VCVS which is realized by operational amplifiers in such a way that this realization allows that the slopes and break points of the nonlinearity can be independently adjusted. We discuss scaling properties of the nonlinearity in a generalized Chua's circuit. After scaling, n -scroll attractors for larger n values become feasible for implementation. This contribution can be found in Sections 3.2.1.1 and 3.2.1.2. References are [309] and [310].

- *Families of scroll grid attractors:*

A new family of scroll grid attractors is presented. These families are classified into three classes called 1-D, 2-D and 3-D grid scroll attractors depending on the location of the equilibrium points, related to the location of the equilibrium points in state space. The scrolls generated from 1-D, 2-D and 3-D grid scroll attractors are

located around the equilibrium points either on a line, within a plane or in 3-D, respectively. Thanks to the generalization of the nonlinear characteristics, it is possible to increase the number of scrolls in all state variable directions. This has been experimentally verified using current feedback opamps. Furthermore Lur's representations are given for the scroll grid attractor families. This contribution can be found in Section 3.3. References are [301], [300], [299], [302] and [258].

- *Hyperchaotic n -scroll attractors and scroll maps:*

Hyperchaotic n -scroll attractors that are generated from a generalized Matsumoto-Chua-Kobayashi circuit, are presented. Furthermore, n -scroll maps are constructed via a Poincaré cross section in the n -scroll attractor. Explicit equations for the 1- and 2-scroll maps are given. Also a 1-scroll map has been verified with a simple circuit realization. This contribution can be found in Sections 3.4.1 and 3.5. References are [311], [303] and [258].

- *Time-delay synchronization scheme:*

Time-delay effects on master-slave synchronization schemes are investigated. Sufficient conditions for master-slave synchronization of Lur's systems are presented for a known time-delay in the master and slave systems. A delay-dependent synchronization criterion is given based upon a new Lyapunov-Krasovskii function. The derived criterion is a sufficient condition for global asymptotic stability of the error system, expressed by means of a matrix inequality. The feedback matrix follows from solving a nonlinear optimization problem. The method is illustrated for the synchronization of Chua's circuits, 5-scroll attractors and hyperchaotic attractors. This contribution can be found in Sections 4.4.1, 4.4.2 and 4.9.2. References are [312], [313] and [259].

- *Experimental confirmations of synchronization schemes:*

An experimental confirmation of a master-slave synchronization scheme with double scroll attractors is given with a circuit realization. Furthermore experimental confirmations of a nonlinear H_∞ synchronization scheme with double and 5-scroll attractors are presented together with a circuit realization. For both schemes the controllers are designed based on the synchronization criteria for

Lur'e systems. This contribution can be found in Sections 4.9.1 and 4.9.3. References are [307], [308] and [259].

- *Chaotic annealing and coupled chaotic annealing:*

Methods of chaotic simulated annealing within the context of coupled local minimizers are formulated. Interpreted within the cellular nonlinear networks context, coupled local minimizers consider local optimization algorithms as cells with local connections between the cells. As a result, information exchange is taking place between the minimizers. Instead of taking local optimization methods as individual cells, we explore the use of chaotic signals as an additional driving force. This is similar to continuous simulated annealing, but deterministic chaos now plays the role of noise. On a number of examples, improved results are obtained with coupled chaotic annealing. In general, the coupling of the minimizers also leads to a variance reduction on the optimal cost function values simulated for many different runs. This contribution can be found in Sections 5.2.1 and 5.2.2. Reference is [257].

- *True random bit generation from a double scroll attractor:*

A novel true random bit generator (TRBG) based on a double scroll attractor is proposed. The double scroll attractor is obtained from a simple model which is qualitatively similar to Chua's Circuit. In order to face the challenge of using the proposed TRBG in cryptography, the proposed TRBG is subjected to statistical tests which are the well-known FIPS-140-1 and Diehard test suits in the area of cryptography. The proposed TRBG successfully passes all these tests and can be implemented in integrated circuits. This contribution can be found in Section 5.3.1. References are [314] and [317].

- *Image and Video authentication on the CNN-UM:*

A new approach to the fragile watermarking technique is introduced. This problem is particularly interesting in the field of modern multimedia applications, when image and video authentication are required. The approach exploits the suitability of Cellular Automata to work as Pseudorandom Pattern Generators and extends the related algorithms under the framework of the cellular nonlinear networks. The result is a novel way to perform watermarking generation in real time, using the currently available CNN-UM

chip prototypes. The CNN algorithms for fragile watermarking as well as on-chip experimental results are reported, confirming that CNNs can successfully act as real-time watermarking generators. The availability of CNN-based visual microprocessors allows to have powerful algorithms for real-time watermarking of images or videos for efficient smart camera applications. This contribution can be found in Sections 5.3.2, 5.4.2 and 5.4.3. References are [318], [10] and [319].