PREFACE TO THE EXPANDED EDITION

Quantum mechanics is a difficult subject for students to learn after years of rigorous training in classical physics, which eliminated their sloppy thinking and unscientific notions. Then in quantum mechanics they have to abandon what they have laboriously learned and adopt a new system of thinking. Yet most amazing is the fact that after much trial and tribulation, the good old Newton's law came out intact unscathed.

In the old edition of this book published four decades ago the author reformulated Newtonian mechanics in a new form of wave motion in a classical theory with an undetermined constant H. When H approaches zero the theory reduces to Newton's exactly. When H is set equal to the Planck constant h, the theory reduces to the Schrödinger version of quantum mechanics. Thus the new theory, at least its mathematical format, can be learned without tortuous brain washing. The ideological changes on the fundamentals of physics have to be taken care of but can be done cleanly without ramification and complexity. Dr. Ralph Ciceroni, the Chancellor of UC, Irvine, has commented that it is more than a textbook. Over the years the book has shepherded the growth of a generation of physicists.

In this expanded edition the same trick is applied to introduce matrix mechanics. In the new formulation based on matrix, H and h are replaced by matrix $\mathbf{H}$ and $h\mathbf{I}$. We can expect the same benefit of introducing something alien under the security blanket of something familiar. One novel result is the derivation of the Sommerfeld quantum condition from the matrix commutation relation of $\mathbf{p}$ and $\mathbf{x}$. This wraps up the last loose end in the fabric of quantum mechanics, which was not known before, and is a gratifying concluding act to close the theoretical structure of the quantum theory.

It also clarifies the physical meaning of the enigmatic noncommutative algebra of $\mathbf{p}$ and $\mathbf{x}$ in quantum theory — it is an esoteric mathematical representation of the fundamentally inexplicable fact of the quantization of the action variable to the units of h. We can only marvel but not explain. The historical wonder is that the limiting case of h approaching zero was discovered earlier in the same algebra system but reduced to commutative when h is zero. This is the more direct connection of the two theories than obtainable from the Schrödinger theory. The matrix formulation also allows the quantum theory to be generalized to new physical systems such as the electron spin, which cannot be done by the Schrödinger approach.

P. F.

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PREFACE TO THE FIRST EDITION

Quantum mechanics is usually a frustrating course for many physics students. Sometimes it is the stumbling block on their way to completing an education in physics. Many hard-working students may finish the course with an understanding of the mathematical manipulations but without a grasp of the underlying physical principles. They have seen that assumptions disconnected from reality and concepts alien to experience, when put together, may yield such familiar results as the Balmer formula and the Rutherford scattering equation. To them these results are amazing, but the physical principles that make the results possible seem beyond comprehension.

To simplify the presentation of a complicated subject, quantum mechanics may be developed by the deductive method, starting from a few axioms. However, the axioms are so highly mathematical in nature (e. g., observables are represented by noncommutative operators) that the student feels he is learning a branch of mathematics which, by some magic, predicts correctly the experimental results in physics. Physical meaning becomes obscure, in spite of the addition of piecemeal interpretations. To overcome these difficulties, it seems best to keep the development of quantum mechanics as close to classical mechanics as possible and to emphasize the fact that quantum mechanics is a natural extension of classical mechanics. The generalization of a well-understood theory is usually easier to accept than a completely new one; the physical picture is clearer and the logical relation simpler. It is not by good luck that quantum mechanics, starting with assumptions very different from classical mechanics, ends with results very similar. It is possible to trace out the intimate relations of the two at the beginning and at the different stages of development so that the similarity of their end results appears to be a matter of course rather than a surprise. To make clear the relations among all elements involved, one looks for a logical structure, not merely a few independent threads of thought but an interwoven texture embracing all logical connections through which we can foresee some of the results of quantum mechanics without having to await the laborious solution of a differential equation which seems so remote from the physical world.

In this introductory volume we try to develop as thoroughly as possible a few, but not all, of the new concepts introduced in quantum mechanics. Essentially, we concern ourselves with the many aspects of the Schrödinger equation (without spin) and its applications. In presenting our material we shall make rather extensive use of the concept of the wave packet. The wave packet is by no means of basic importance, but it serves as a useful scaffolding by means of which quantum mechanics may be built. The kinematics and dynamics of a wave packet are closely related to the classical mechanics of a particle; thus the mechanics of a wave packet may be considered as a formal extension of classical mechanics. On the other hand, a wave packet not only resembles a classical particle but also has an additional property, the uncertainty relation. Therefore, the results of the Schrödinger equation can either be reduced to the results of classical mechanics or be traced to the uncertainty relation (quantum phenomena). Take the linear harmonic oscillator, for example. It can be shown that the results of classical mechanics can be derived from a wave packet formed by superposition of eigenfunctions of large quantum numbers, and that a quantum effect, namely, the existence of the so-called zero-point motion, may be traced back to the uncertainty relation. The consequences of the extension from classical mechanics to quantum mechanics are thus made clear. Often a student, after learning quantum mechanics for several months, finds himself in possession of a large body of information concerning energy quantization, but fails to appreciate quantum mechanics as a dynamical system, i.e., a theoretical system capable of predicting the future development of physical systems from their initial conditions. The use of the wave packet helps bring out the dynamical aspect of the new mechanics. After quantum mechanics is thus introduced and made familiar to the student, a more general, abstract formulation is then presented in the later part of the book.

Following a historical introduction in Chapter 1, the mathematical formalism of quantum mechanics in a limited form will be presented in Chapter 2. The Schrödinger equation will be introduced. The meaning of its solutions will be discussed according to the two assumptions of Born. Schrödinger's equation and Born's assumptions form a self-consistent theoretical system. The similarity of and difference between this system and classical mechanics will be demonstrated by the derivation of Newton's second law and of the uncertainty relation.

In three following chapters this system of mechanics will be applied to three special cases: the free particle, the linear harmonic oscillator, and the potential barrier problems. To emphasize the similarity to classical mechanics, wave packet solutions will be obtained which reproduce the properties of the classical solutions. To emphasize the difference from classical mechanics, applications will be made to three quantum phenomena, i.e., the wave property of matter, the quantization of energy, and the penetration of potential barrier.

After having demonstrated the usefulness of this mathematical theory, we shall turn our attention in Chapter 6 to the physical meaning of quantum mechanics. Heisenberg's uncertainty principle will be discussed after a critical examination of the physical concepts used in classical mechanics. The causality law will be examined in connection with the quantum-mechanical description of physical processes. The inquiry into the physical meaning of quantum mechanics is usually expressed by students in the following questions: Is an electron a particle or a wave? Why is the energy quantized? How can a particle penetrate through a potential barrier? In anticipation of such inquiries, an explanation of the quantum phenomena will be given (in Section 6–3) in terms of the uncertainty principle and of the quantum potential of Eq. (2–65).

Chapters 7–11 will contain straightforward applications of quantum mechanics to various problems; they will be solved either exactly or approximately by the perturbation methods. When a problem may be solved in several ways, it is usually instructive to look into the relations among the different methods. Thus, the three-dimensional harmonic oscillator problem will be solved by using rectangular, cylindrical, and spherical coordinates respectively, and the relations among the solutions will be discussed. The Rutherford scattering problem will be treated by methods developed in the time-independent, as well as in the time-dependent, perturbation theory.

Before the book closes, the wave-packet scaffolding will be torn down and quantum mechanics will be reformulated in the operator form in Chapter 12. The operator formulation is the only one by which quantum mechanics may be adequately presented. But its abstractness makes it preferable, if not necessary, to delay its introduction until after the concepts and results of quantum mechanics have been made reasonably familiar. This chapter also serves the purpose of preparing students to read more advanced treatises.

In writing this book the author was inspired by the writings of Heisenberg, de Broglie, Schrödinger, Bohr, and Dirac. Nevertheless, he is solely responsible for the logical organization of the presentation. The subjects treated here, except for the derivation of a number of results, are found in published literature, although a few gaps must be filled to make possible a systematic presentation on an elementary level. Suggestions from users of the book will be appreciated. It is fitting to acknowledge the help the author has received in preparing this book. Dr. Melba Phillips made valuable suggestions in the earlier chapters. Two friends, Y. F. Bow and Robert Sanders, have read the manuscript and checked the equations carefully; they also made many important suggestions. The late Mrs. Ruth Hoople edited the manuscript. Mrs. Dorothy Sickels provided editorial advice and helped in proofreading. To them and to many others the author takes this opportunity to express his deep gratitude.

If the reader finds the assumptions involved in this presentation not like repulsive strangers, if the reader finds in the logical texture no loose thread, the author has achieved his aim.

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