

## PREFACE

This book arose from a friendship formed when we were both faculty members of the Department of Physics, Universidad Autonoma Metropolitana, Iztapalapa Campus, in Mexico City. Plaschko was teaching an intermediate to advanced course in mathematical physics. He had written, with Klaus Brod, a book entitled, “Hoehere Mathematische Methoden fuer Ingenieure und Physiker”, that Henderson admired and suggested that be translated into English and be updated and perhaps expanded somewhat.

However, we both prefer new projects and this suggested instead that a book on Stochastic Differential Equations be written and this project was born. This is an important emerging field. From its inception with Newton, physical science was dominated by the idea of determinism. Everything was thought to be determined by a set of second order differential equations, Newton’s equations, from which everything could be determined, at least in principle, if the initial conditions were known. To be sure, an actual analytic solution would not be possible for a complex system since the number of dynamical equations would be enormous; even so, determinism prevailed. This idea took hold even to the point that some philosophers began to speculate that humans had no free will; our lives were determined entirely by some set of initial conditions. In this view, even before the authors started to write, the contents of this book were determined by a set of initial conditions in the distant past. Dogmatic Marxism endorsed such ideas, although perhaps not so extremely.

Deterministic Newtonian mechanics yielded brilliant successes. Most astronomical events could be predicted with great accuracy.

Even in case of a few difficulties, such as the orbit of Mercury, Newtonian mechanics could be replaced satisfactorily by equally deterministic general relativity. A little more than a century ago, the case for determinism was challenged. The seemingly random motion of the Brownian motion of suspended particles was observed as was the sudden transition of the flow of a fluid past an object or obstacle from laminar flow to chaotic turbulence. Recent studies have shown that some seemingly chaotic motion is not necessarily inconsistent with determinism (we can call this quasi-chaos). Even so, such problems are best studied using probabilistic notions. Quantum theory has shown that the motion of particles at the atomic level is fundamentally nondeterministic. Heisenberg showed that there were limits to the precision with which physical properties could be determined. One can only assign a probability for the value of a physical quantity. The consequence of this idea can be manifest even on a macroscopic scale. The third law of thermodynamics is an example.

Stochastic differential equations, the subject of this monograph, is an interesting extension of the deterministic differential equations that can be applied to Brownian motion as well as other problems. It arose from the work of Einstein and Smoluchowski among others. Recent years have seen rapid advances due to the development of the calculi of Ito and Stratonovich.

We were both trained as mathematicians and scientists and our goal is to present the ideas of stochastic differential equations in a short monograph in a manner that is useful for scientists and engineers, rather than mathematicians and without overpowering mathematical rigor. We presume that the reader has some, but not extensive, knowledge of probability theory. Chapter 1 provides a reminder and introduction to and definition of some fundamental ideas and quantities, including the ideas of Ito and Stratonovich. Stochastic differential equations and the Fokker–Planck equation are presented in Chapters 2 and 3. More advanced applications follow in Chapter 4. The book concludes with a presentation of some numerical routines for the solution of ordinary stochastic differential equations. Each chapter contains a set of exercises whose purpose is to aid the reader in understanding the material. A CD-ROM that provides

MATHEMATICA and FORTRAN programs to assist the reader with the exercises, numerical routines and generating figures accompanies the text.

*Douglas Henderson*

*Peter Plaschko*

Provo Utah, USA

Mexico City DF, Mexico

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