

# Chapter 1

## BASIC CONCEPTS

### 1.1 Introduction

Conventional iron dominated electromagnets are components of low/medium energy accelerator systems and also used for low/medium energy charged beam transport. Low/medium energy particle accelerators are those whose beam stiffness,  $B\rho$ , is limited to a few tens of Tesla-meters. The beam stiffness is given in many physics texts including Livingood[1] by

$$B\rho = \frac{1}{qc} \sqrt{T^2 + 2TE_0}, \quad (1.1)$$

where  $q$  = charge in Coulombs,  $c$  = the speed of light in  $\frac{m}{sec}$ ,  $T$  = beam energy and  $E_0$  = the particle rest mass energy. Written in conventional accelerator units

$$B\rho \approx \frac{1}{299.8Z} \sqrt{T^2 + 2TE_0}, \quad \text{where} \quad (1.2)$$

$Z$  is the number of charge units,  
 $B\rho$  (Tesla - meters),  
 $T$  (MeV),  
 $E_0 = \left( \begin{array}{l} 0.51MeV \text{ for electrons} \\ 938MeV \text{ for protons} \end{array} \right)$ .

High energy accelerators and beam transport lines for accelerators whose beam stiffness is greater than a few tens of Tesla-meters require higher fields not achievable with iron dominated magnets and must rely on superconducting technology.

The conventional magnets described herein are those whose fields are shaped by iron poles, where the maximum field level in the yoke is less than the iron saturation level and whose excitation is provided by current carrying coils. Understanding the function of these magnets requires understanding the forces and the force directions on charged particle beams with conventionally defined magnet polarities. This chapter introduces the different magnet types, describes the forces and defines the polarities for different magnets used for particle beams. Means of electrically connecting separate coils in different type magnets to achieve the desired polarities are also described. The physics accelerators using different magnets can be found in other Physics texts[2][3].

Since it is difficult to explain all concepts of magnetic fields without some basic background, some expressions, developed more fully in later chapters, are used to describe relationships of the electrical current flowing in a single conductor, the magnetic field it generates and the forces the field exerts on moving charged particles. Other conventions are used to identify the polarities of magnet poles and the flux directions determined by these polarities.

## 1.2 Magnet Types

Many of the magnet types and their functions are described in a chapter written by Dr. Neil Marks of Daresbury in Great Britain in a collection of articles covering various aspects of synchrotron radiation accelerators edited by Dr. Herman Winick[4].

Conventional electromagnets can be divided among several different types.

- Dipole Magnets
  - Gradient Magnet
- Quadrupoles
- Sextupoles
- Correctors
  - Vertical and Horizontal Steering
  - Skew Quadrupole
- Specialized Magnets
  - Current Sheet Septum (Horizontal Bend)
  - Lambertson Septum (Vertical Bend)
  - Bump and Kicker Magnets

## 1.3 Pictures

The photographs reproduced in this chapter are those of magnets manufactured for the PEP-II and SPEAR-3 projects. The PEP-II accelerator at SLAC is an electron-positron asymmetric collider, constructed to investigate the fundamental nature of subatomic particles. The SPEAR-3 project is an upgrade of an existing 3 GeV electron synchrotron at SLAC, constructed to exploit the synchrotron radiation emitted by electrons bent through magnetic fields.

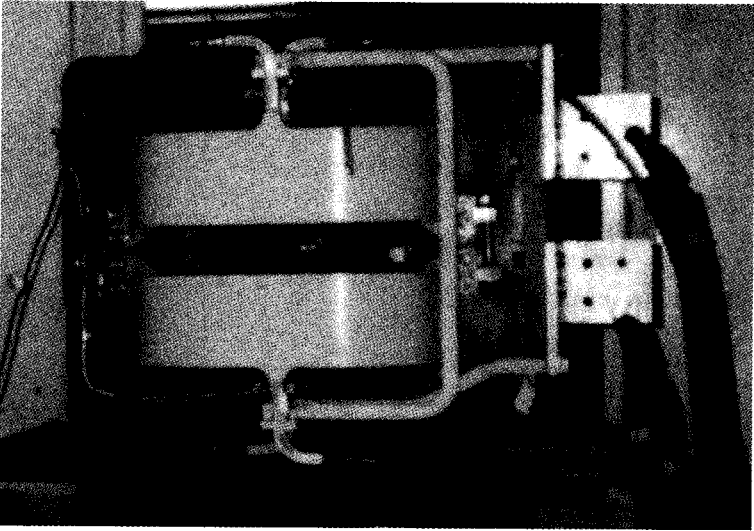


Figure 1 PEPII Low Energy Ring Dipole

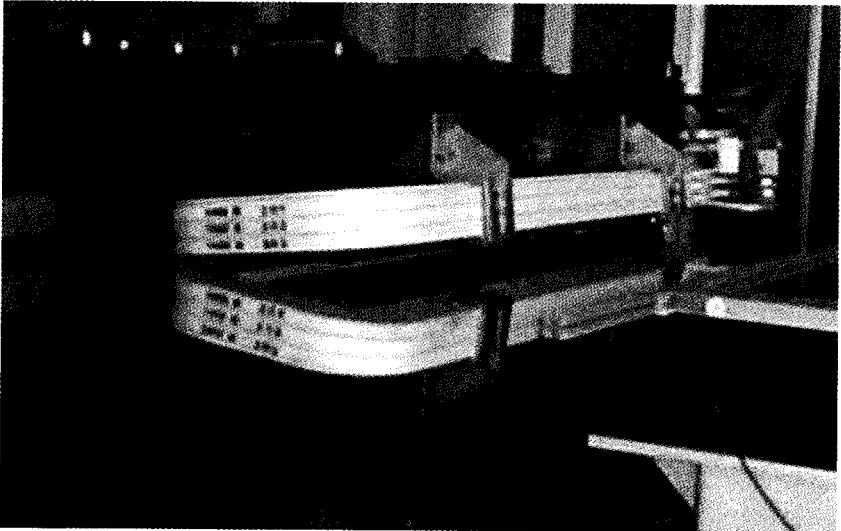


Figure 2 SPEAR3 Gradient Dipole Magnet

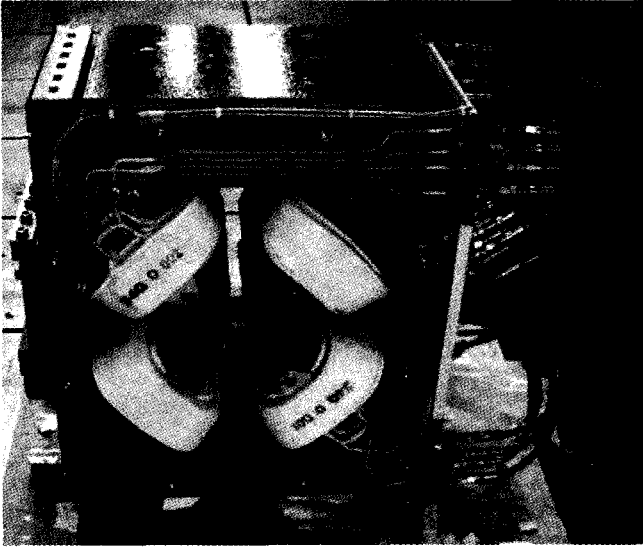


Figure 3 SPEAR3 Quadrupole Magnet

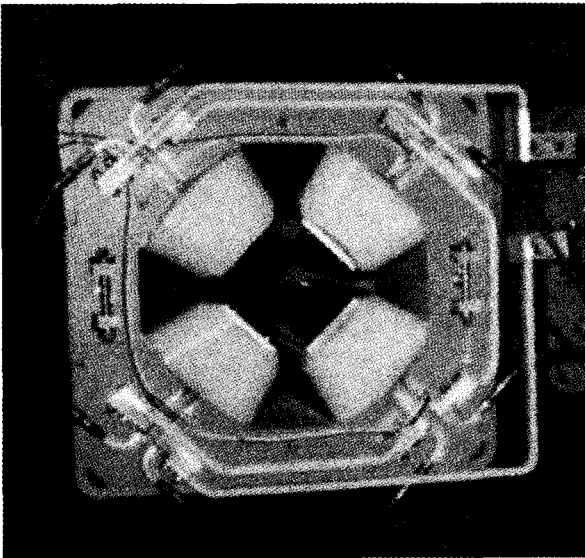


Figure 4 PEP-II Quadrupole

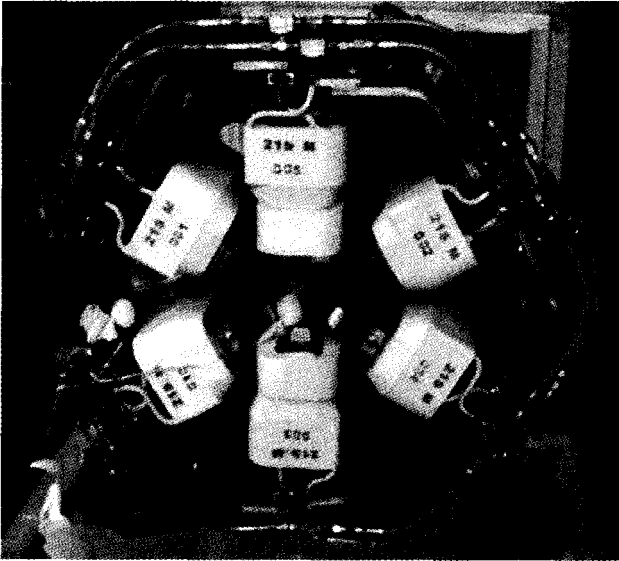


Figure 5 SPEAR3 Sextupole Magnet with Skew Quadrupole Trim Coils

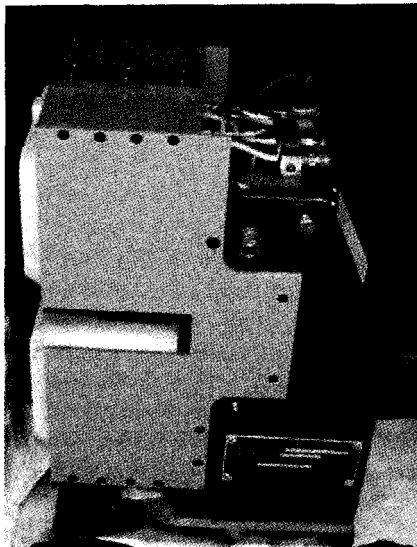


Figure 6 SPEAR3 Combined Horizontal and Vertical Steering Corrector

Fig. 1 illustrates a PEP-II Low Energy Ring dipole. A dipole magnet has two poles and has a constant field in the magnet aperture. Fig. 2 illustrates the SPEAR3 gradient dipole magnet. In the gradient dipole, the field shaped by the pole combines a constant dipole field with a field linearly distributed along the transverse and vertical directions. The SPEAR3 gradient magnet simultaneously bends and horizontally defocuses the charged beam.

Fig. 4 illustrates a PEP-II Low Energy Ring quadrupole magnet. Fig. 3 illustrates a SPEAR3 quadrupole magnet. The two illustrated quadrupoles differ in design and construction details. The PEP-II magnet uses a welded core. Its conductors are large and require separate busses for the coil to coil electrical connections. The SPEAR3 quadrupole employs a glued core. The coil conductors are bent to an electrical manifold at both ends of the magnet, used to make the coil to coil electrical connections. Alternate coils are manifolded together at each end of the magnet to reduce the congestion due to the crossover topology. A quadrupole has four poles and a null field at its center. Its field magnitude varies linearly with the distance from the magnet center.

Fig. 5 illustrates the SPEAR3 sextupole magnet. A sextupole magnet has six poles and a null field at its center. Its field magnitude varies quadratically with the distance from the magnet center. Two extra coils installed on the vertical poles of the sextupole are added to produce a skew quadrupole trim field (a linearly distributed field rotated so that the field directions are parallel to the horizontal and vertical magnet axes).

Fig. 6 illustrates a combined horizontal and vertical corrector magnet. The corrector is a low field dipole magnet whose function is to correct the angle orbit of the charged particle beam by steering the beam horizontally and vertically. For the illustrated corrector example, the vertical field is shaped by the poles. In this example, the horizontal field is shaped by the placement of individual conductors.

#### 1.4 Conventions

The right hand rule describes positive directions in vector relationships. Positive current flows from the positive (+) lead of a power supply to the negative (-) lead of a power supply. The flux direction due to positive current flowing in a coil surrounding a magnet pole is determined by the right hand rule. This convention also determines the polarity. Magnetic flux flows from the positive to the negative pole of a magnet. In the two dimensional illustrations shown in fig. 8, a  $\odot$  is used to describe the positive charge direction out of the page and an  $\times$  is used to describe the positive charge direction into the page.

#### 1.5 The Field from a Line Current (Biot-Savart law)

The amplitude of the magnetic field is computed using the line integral form of the magnetic field equation (see page 98)

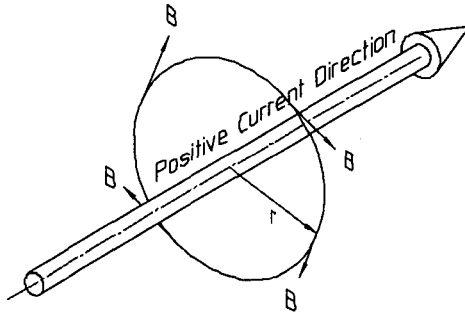


Figure 7 Magnet field due to a line current

$$\oint \vec{H} \cdot d\vec{l} = \frac{B}{\mu_0} 2\pi r = I,$$

$$B = \frac{\mu_0 I}{2\pi r}.$$

The direction of the magnetic field is described in fig. 7

## 1.6 Magnetic Force on a Line Current

The vector expression for forces on a particle beam is given by the vector cross product equation

$$\vec{F} = e\vec{v} \times \vec{B}. \quad (1.3)$$

### 1.6.1 MKS Units

Unless otherwise specified, all expressions in this text are expressed in the MKS system of units

$$\begin{aligned} \vec{F} &= \text{Newtons,} \\ e &= \text{coulombs,} \\ \vec{v} &= \frac{m}{\text{sec}}, \\ \vec{B} &= \text{Tesla.} \end{aligned}$$

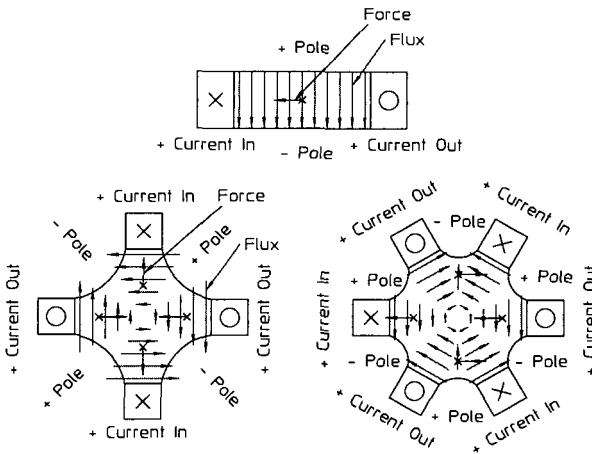


Figure 8 Coil Currents, Polarities and Force Directions for a Positive Beam Current

### 1.6.2 Force Directions

The right hand rule force direction on a positive particle beam is described in vector form in eq. (1.3) and is illustrated for the different magnet types in fig. 8. In this illustration, the coils have been simplified so that the dipole, quadrupole and sextupole appear to have one, two and three coils. Later chapters will show that the number of coils is equal to the number of magnet poles or multiples thereof.

### 1.6.3 Dipole Magnet

The dipole magnet has two poles and a uniform field. Applying the right hand convention for the coil current flowing in the indicated direction, the magnetic flux flows downward. Since the convention requires the magnetic flux to flow from the positive to negative poles, the upper pole is positive and the lower pole is negative. For positive beam current into the page, force direction, using the right hand convention, is to the left.

**Gradient Magnets** Gradient magnets are specialized dipole magnets which, in addition to a bend field at its center, has a linear gradient. This magnet is a combined function magnet which simultaneously defocuses (or focuses) and bends the beam.

### 1.6.4 Quadrupole Magnet

The quadrupole magnet has four poles and a zero field at its center. The field is normal to the horizontal and vertical centerlines and its distribution is linear with

distance from the center. Applying the right hand convention for the coil current flowing in the indicated directions, the magnetic flux flows outward from the poles at  $\frac{\pi}{4}$  and  $\frac{5\pi}{4}$  and inwards to poles at  $\frac{3\pi}{4}$  and  $\frac{7\pi}{4}$ . Since the convention requires the magnetic flux to flow from the positive to negative poles, the poles at  $\frac{\pi}{4}$  and  $\frac{5\pi}{4}$  are positive and the poles at  $\frac{3\pi}{4}$  and  $\frac{7\pi}{4}$  are negative. For the illustrated positive beam currents at 0 and  $\pi$  planes into the page, along the horizontal centerline, the force direction is toward the center of the magnet. For the illustrated positive beam currents at  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$  planes into the page, along the vertical centerline, the force direction is away from the center of the magnet. The illustrated magnet is considered an  $F$  quadrupole for positively charged beam. Using the conventional definition, the  $F$  quadrupole focuses the beam in the horizontal plane and defocuses the beam in the vertical plane. The  $D$  quadrupole defocuses the beam in the horizontal plane and focuses the beam in the vertical plane.

### 1.6.5 Sextupole Magnet

The sextupole magnet has six poles and a zero field at its center. The field is normal to the horizontal centerline and centerlines at angles  $\frac{\pi}{3}$  and  $\frac{2\pi}{3}$ . Its distribution is quadratic ( $\propto r^2$ ) with distances from the center. Applying the right hand convention for the coil current flowing in the indicated directions, the magnetic flux flows outward from the poles at  $\frac{\pi}{6}$ ,  $\frac{5\pi}{6}$  and  $\frac{3\pi}{2}$ . The magnetic flux flows inwards to poles at  $\frac{\pi}{2}$ ,  $\frac{7\pi}{6}$  and  $\frac{11\pi}{6}$ . Since the convention requires the magnetic flux to flow from the positive to negative poles, the poles at  $\frac{\pi}{6}$ ,  $\frac{5\pi}{6}$  and  $\frac{3\pi}{2}$  are positive and the poles at  $\frac{\pi}{2}$ ,  $\frac{7\pi}{6}$  and  $\frac{11\pi}{6}$  are negative. For the illustrated positive beam currents at 0 and  $\pi$ , along the horizontal centerline, the force direction is to the left of the magnet. For the illustrated positive beam currents at  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ , along the vertical centerline, the force direction is to the right of the magnet.

The function of a sextupole magnet is to correct for the chromatic aberration due to dispersion in a dipole caused by the momentum spread in the beam. For a beam with energy spread traversing a dipole magnet, the higher energy particles are bent less than the lower energy particles, causing the dipole magnet to disperse a beam with point distribution into a beam with line distribution along the horizontal plane. The line beam leaving the illustrated dipole magnet (bending the beam to the left) is populated with higher energy particles on the right side of the beam and the lower energy beam on the left side of the beam (looking in the beam direction). The effect of a quadrupole on this dispersed beam is to longitudinally spread the focal point of the quadrupole lens, focussing the higher energy beam downstream and the lower energy beam upstream from the desired focal point. The sextupole magnet is designed to compensate for this effect. The illustrated magnet is designated an  $F$  sextupole for positively charged beam. The  $F$  sextupole selectively bends the beam on the right side of the magnet towards the beam centerline (shortening the focal length) and the beam on the left side of the beam away from the beam centerline (lengthening the focal length), restoring the desired single focal point. Since the sextupole also steers the vertically displaced beam in the opposite direction, it is

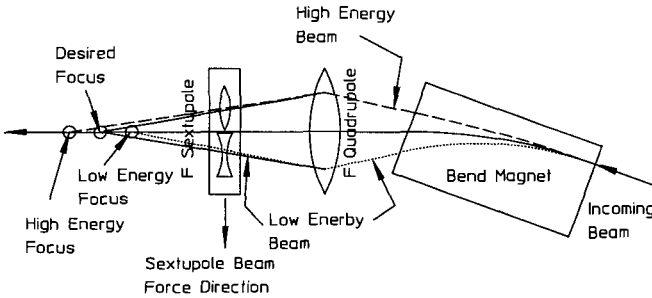


Figure 9 Dispersion and Chromatic Aberration Correction

usually located in positions in the synchrotron lattice where the beam is vertically small. Using the conventional definition, the  $F$  sextupole bends the beam toward the center of the accelerator ring and the  $D$  sextupole bends the beam away from the center of the accelerator ring. The orbits are illustrated in fig. 9.

#### 1.6.6 Corrector Magnets

##### Steering Correctors

Steering corrector magnets are used to provide minor horizontal and vertical steering ( $\lesssim 1.5$  mrad) of the charged particle beam. They are normally located upstream from main ring quadrupoles and are used to steer the beam to the center of the quadrupole. Unwanted beam steering occurs when the charged particle beam is not centered in the quadrupole. In order to conserve lattice space and to simplify operation, horizontal and vertical steering are often combined in a single magnet.

##### Skew Quadrupole Correctors

Skew quadrupole corrector magnets are used to correct for the integrated effects of rotationally misaligned main ring quadrupoles. When the main ring magnets are rotationally misaligned, radial components of the magnetic field proportional to the sine of the misalignment angle are introduced along the horizontal and vertical planes. These small radial fields rotate the beam and mix the horizontal and vertical beam phases, causing instabilities. The skew quadrupole trims are used to compensate for these radial field errors. Corrector magnets are normally powered with bipolar power supplies so that corrections can be made in all directions and angle misalignments for different magnet polarities. Schematic illustrations of the three types of corrector magnets are shown in fig. 10.

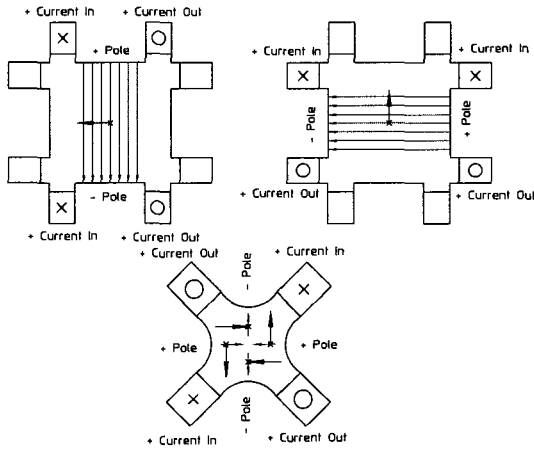


Figure 10 Horizontal/Vertical and Skew Quadrupole Correctors

1.6.7 Specialized Magnets

Specialized magnets are used for injection and/or extraction of charged particle beams into/out of accelerator rings. Normally, injection requires the buildup of current into the ring and occurs after previously stored beam is circulating in the ring. In order to ensure that the injected beam is stored (incorporated with the previously injected beam), it must be injected as transversely close to the central orbit of the circulating beam as possible. This is accomplished by using a combination of several bump magnets to momentarily move the circulating beam transversely close to the injected beam. When the bump magnets are turned off, the beam is restored to its central orbit. A good explanation of this process is described in an article by Dr. Gottfried Mülhaupt of Grenoble (in section 3.6.1 of reference [4]) covering synchrotron injection taking advantage of the transverse phase space in an accelerator with a fractional tune of approximately 1/4.

Bump Magnets

The bump magnet is a dipole usually with a laminated yoke or a yoke made from high resistive permeable material. This material is selected since it does not carry eddy currents. The magnetic field in the bump magnet must be raised to its required field and reduced again to zero in the shortest possible time (usually the time required to make a single orbit around the ring). Power supply voltage constraints often limit the rate at which the magnetic field can be changed. Thus, the previous article by Mülhaupt describes a system which takes advantage of the accelerator fractional tune

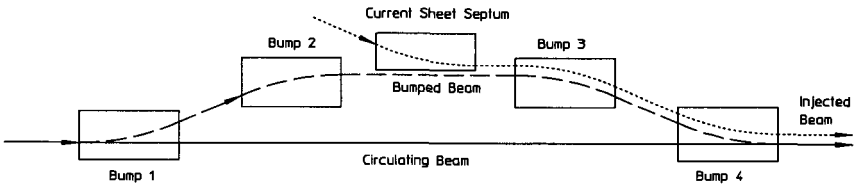


Figure 11 Bumped Beam Orbit

to increase the required time to change the bump magnet field.

The rapid change in the magnetic field excites eddy currents in low electrically resistive permeable material. Because of the rapid change in the magnetic field, the bump magnets are normally installed in boxes which are part of the vacuum chamber. The field for bump magnets with metallic vacuum chambers installed within the gap will be excluded from the interior of the chamber (the beam space) because of eddy currents. A simplified illustration of the bumped beam orbit is shown in fig. 11.

### Current Carrying Septum

The external field (fringe field) of the injection septum must be as small as possible to avoid affecting the circulating beam. The current in the outer coil of the current carrying septum magnet divides the high field region of the magnet from the low field region. This type of magnet is often operated in the persistent mode and left on for long periods after injection. The cross section of a current carrying septum, typically operated in the persistent mode, is shown in fig. (12). The fringe field in the current carrying septum can adversely affect the orbit of the circulating beam. Its magnitude is dependent on the design of the iron yoke and is discussed in a later chapter (see discussion on page 103).

### Eddy Current Septum

Another injection septum design (the eddy current septum) is employed for higher fields requiring higher currents and a very limited space for the septum. The eddy current septum magnet, illustrated in fig. (13), employs a pulsed current in the coil around the back leg of the yoke. Eddy currents generated by the rapidly changing magnetic flux exclude the field from regions outside of the high conductivity copper eddy current box. In particular, the currents generated in the septum minimizes the field penetrating into the region of the bumped and circulating beam. In practice, the eddy current septum has a longer time constant than the pulse width of the

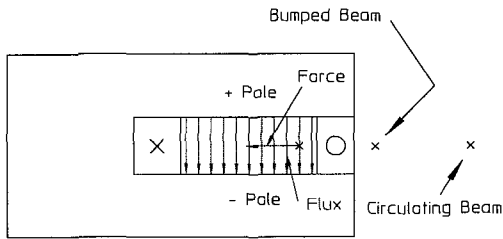


Figure 12 Current Carrying Septum

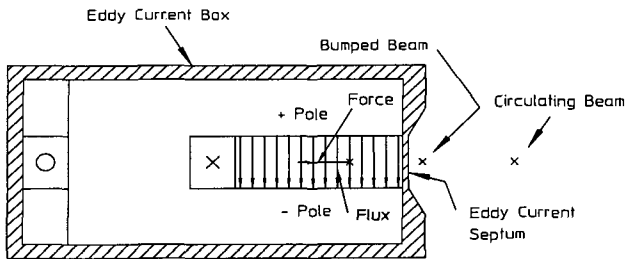


Figure 13 Eddy Current Septum

excitation and the fringe field persists longer than the magnet field.

### Lambertson Septum

The Lambertson septum magnet is used to inject the beam from above or below the plane of the accelerator. Its cross section is shown in fig. (14). As in the case of the current carrying septum, the magnet is left on and its external field must be as small as possible in the region of the circulating beam. Its fringe field is horizontal rather than the vertical fringe field for the current carrying and eddy current septa. Thus, the unwanted vertical steering is a bit more difficult to deal with than the horizontal orbit perturbations from the horizontally steering septa. Means of estimating the size of the fringe field is covered in a later chapter in this book (see discussion on page 104). In general, a larger opening angle in the septum causes an intensification of the gap H-field in the septum. Since the H-parallel field is continuous across the iron air boundary, this intensification causes a larger fringe field.

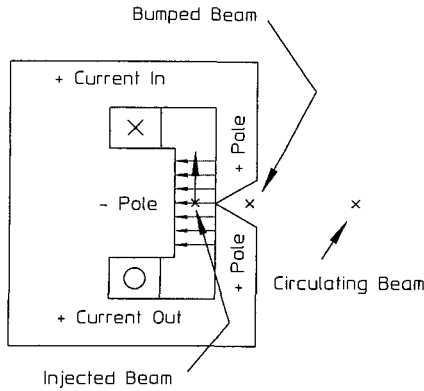


Figure 14 Lambertson Septum

### 1.6.8 More Polarity

All the previous figures showing the direction of magnetic flux and the direction of forces for positive beam flowing “into the paper” employ the conventions outlined in the beginning of this chapter. However, it is often difficult to remember and/or employ all the conventions when connecting a power supply or determining the correct intercoil bussing to connect the two coils for a dipole, the four coils for a quadrupole or the six coils for a sextupole. Moreover, since all the conventions were defined for a positively charged circulating beam, one can become confused when considering electron accelerators with negatively charged beam. One has to remember that the right hand rule becomes a left hand rule to determine the direction of the magnetic forces or to reverse the power supply leads assuming positively charged particles to obtain the correct magnet polarities for electrons. One of the most common errors in the final stages of accelerator construction projects is the reversing of polarities on isolated magnets. Another less common error is the mis-connection of the leads connecting the separate coils of a single magnet.

A simpler means of determining the correct magnet polarities exists.

### Magnetostriction

Using the right hand rule force and magnetic flux direction convention, it can be seen that the magnetic flux and force directions for a series of conductors carrying parallel positive currents in the same direction are as shown in fig. 15. The forces on parallel conductors, carrying the same charges in the same direction, are towards each other. This phenomenon is familiar to those dealing with multi-strand cables. The separate

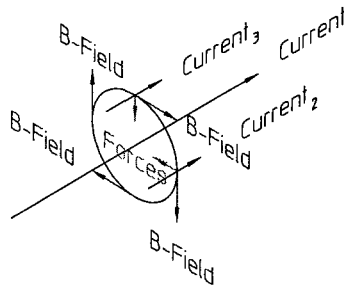


Figure 15 Forces on Parallel Conductors

strands of a cable attract each other and the cable cross-section compresses when the power is turned on. One can take advantage of this behavior and develop a simpler set of polarity conventions.

#### 1.6.9 Alternate Polarity Convention

- Currents with the same charge travelling in the same direction attract.

#### Corollaries

- Currents with opposite charges travelling in the same direction repel.
- Currents with the same charge travelling in opposite directions repel.
- Currents with opposite charges travelling in the opposite direction attract.

By considering the charged particle beam as a line current with appropriate signs, one can use this alternate convention and/or its corollaries to determine the direction of the power supply currents through the different coils of the magnet to ensure the proper magnet polarities for positively or negatively charged beams. One does not have to figure out the polarities of the magnet poles.

## 1.7 Chapter Closure

This chapter introduces and describes the functions and characteristics of different types of conventional iron dominated electro-magnets used for low and medium energy charged particle accelerators and beam transport lines. The main magnet types are the dipole, quadrupole and sextupole whose fields are uniform, linear and quadratic whose functions are to bend, focus and correct the chromaticity of the beam. Other magnets are described which correct beam orbits and compensate for the effects of installation/alignment errors. Some specialized magnets required for beam injection/extraction are described. Descriptions of the charged particle beam

orbits through the different magnet types is beyond the scope of this work. The physics of beam orbits through magnets can be found in texts by Wiedemann [2] and Lee [3].

Conventions and definitions are introduced to identify the force direction on moving positively charged particle beams. These conventions are restated using the principle of magnetostriction to simplify the concepts and to avoid confusion when attempting to establish the correct polarity for beams with different charges.

For modern accelerators, especially for storage rings and colliders whose beam lifetimes are measured in hours, the properties of magnets are crucial to the accelerator performance and beam lifetime. Magnets need to be installed and aligned precisely, the excitation of individual magnets must be precise and predictable and the magnetic field shape must be free of errors to satisfy the requirements needed by the physics of accelerators. The quality of the magnets depends on the extent that the fields have the desired shapes. The shapes of the magnetic fields depend on the iron pole shapes, the mechanical fabrication precision of the construction of the poles and the assembly of the parts making up the magnet yoke. Later chapters describe means of satisfying the physics requirements for high quality accelerator magnets. The task of the magnet designer is to design magnets which satisfy the physics requirements and can be translated into mechanical components and magnet assemblies accurately, reliably and economically.

## 1.8 Problems

### Problem 1.1 (Solution)

Using the eq. (1.3) and the MKS system of units, show that the units for force are expressed in Newtons.

### Problem 1.2 (Solution)

Why are there no forces between two line currents a distance  $d$  apart and perpendicular to each other?

### Problem 1.3 (Solution)

What is the expression for the magnitude of the force per unit length on one conductor due to the magnetic field generated by a parallel conductor where the distance separating the two conductors is given by  $d$ ?