Chapter 1

Risk Management and Financial Derivatives

We begin with a brief and straightforward introduction to the basic concepts, properties and pricing principles of financial derivatives, and a clear statement of the main subject of the book——the valuation problem of option pricing.

1.1 Risk and Risk Management

Risk——uncertainty of the outcome.

Risk can bring unexpected gains. It can also cause unforeseen losses, even catastrophes.

Risks are common and inherent in the financial markets and commodity markets: asset risk (stocks...), interest rate risk, foreign exchange risk, credit risk, commodity risk and so on.

There are two totally different attitudes toward risks:

1. Risk aversion: quantify an identified risk and control it, i.e., to devise a plan to manage the exposed risk and convert it into a desired form. Basically, two kinds of plans are available: *a. Replace the uncertainty with a certainty* to avoid the risk of adverse outcomes even if this requires giving up the potential gaining opportunity. *b. Be willing to pay a certain price for the potential gaining opportunity, while avoiding the risk of adverse outcomes.*

2. Risk seeking: willing to take the risk with one's money, in hope of reaping risk profits from investments in risky assets out of their frequent price changes. Acting in hope of reaping risk profits from the market price changes is called **speculation**.

Financial derivatives are a kind of risk management instrument. A derivative's value depends on the price changes in some more fundamental

underlying assets.

Many forms of financial derivatives instruments exist in the financial markets. Among them, the three most fundamental financial derivatives instruments are: **forward contracts**, **futures**, and **options**. If the underlying assets are stocks, bonds, foreign exchange rates and commodities etc., then the corresponding risk management instruments are: stock futures (options), bond futures (options), currency futures (options) and commodity futures (options) etc.

In risk management of the underlying assets using financial derivatives, the basic strategy is **hedging**, i.e., the trader holds two positions of equal amounts but opposite directions, one in the underlying markets, and the other in the derivatives markets, simultaneously. This risk management strategy is based on the following reasoning: it is believed that under normal circumstances, prices of underlying assets and their derivatives change roughly in the same direction with basically the same magnitude; hence losses in the underlying assets (derivatives) markets can be offset by gains in the derivatives (underlying assets) markets; therefore losses can be prevented or reduced by combining the risks due to the price changes.

The subject of this book is pricing of financial derivatives and risk management by hedging.

1.2 Forward Contracts and Futures

Forward contract —— an agreement to buy or sell at a specified future time a certain amount of an underlying asset at a specified price.

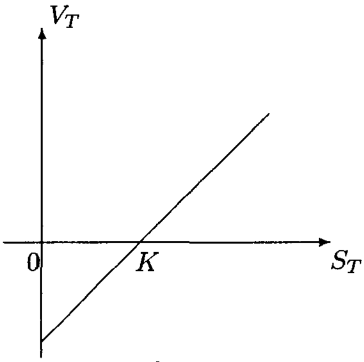
A forward contract is an agreement to replace a risk with a certainty.

The buyer in the contract is said to hold a **long position**, and the seller is said to hold a **short position**. The specified price in the contract is called the **delivery price** and the specified time is called **maturity**.

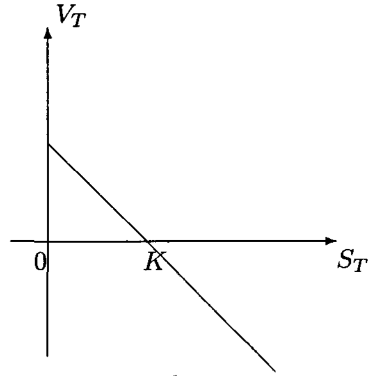
Let K——delivery price, and T——maturity, then a forward contract's **payoff** V_T at maturity is:

$$\begin{aligned} V_T &= S_T - K, \quad \text{(long position)} \\ V_T &= K - S_T, \quad \text{(short position)} \end{aligned}$$

where S_T denotes the price of the underlying asset at maturity $t = T$.



long position



short position

Forward Contracts are generally traded OTC (over-the-counter).

Future——same as a forward contract, an agreement to buy or sell at a specified future time a certain amount of an underlying asset at a specified price. Futures have evolved from standardization of forward contracts. Futures differ from forward contracts in the following respects:

a. Futures are generally traded on an exchange.

b. A future contract contains standardized articles.

c. The delivery price on a future contract is generally determined on an exchange, and depends on the market demands.

1.3 Options

Options—— an agreement that the holder can buy from, or sell to, the seller of the option at a specified future time a certain amount of an underlying asset at a specified price. But the holder is under no obligation to exercise the contract.

The holder of an option has the right, but not the obligation, to carry out the agreement according to the terms specified in the agreement. In an options contract, the specified price is called the **exercise price** or **strike price**, the specified date is called the **expiration date**, and the action to perform the buying or selling of the asset according to the option contract is called **exercise.**

According to buying or selling an asset, options have the following types:

call option is a contract to buy at a specified future time a certain amount of an underlying asset at a specified price.

put option is a contract to sell at a specified future time a certain amount of an underlying asset at a specified price.

According to terms on exercise in the contract, options have the following types:

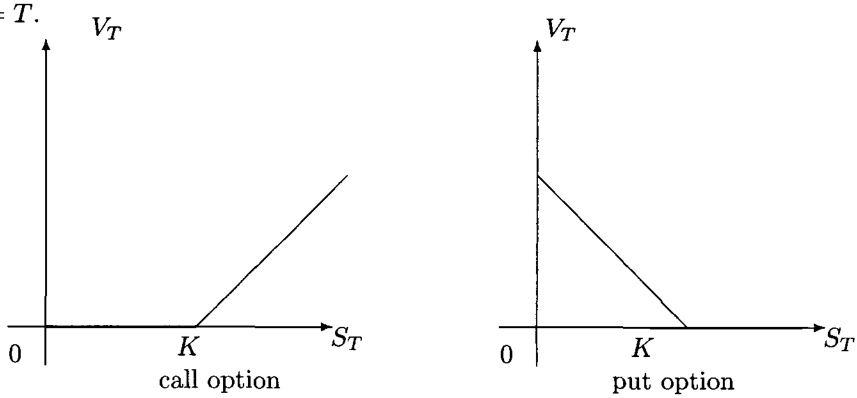
European options can be exercised only on the expiration date.

American options can be exercised on or prior to the expiration date.

Define K—— strike price and T—— expiration date, then an option's payoff (value) V_T at expiration date is:

$$V_T = (S_T - K)^+, \qquad \text{(call option)}$$
$$V_T = (K - S_T)^+, \qquad \text{(put option)}$$

where S_T denotes the price of the underlying asset at the expiration date $t = T$.



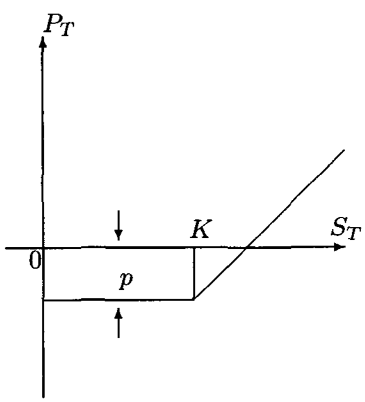
Option is a contingent claim. Take a call option as example. If S_T, the underlying asset's price at expiration date, is higher than the strike price K, then the holder of the option can exercise the rights to buy the asset at the strike price K(to gain profits). Otherwise, the option is a worthless paper. Thus, to price an option is essentially to set a price to this kind of contingent claims. The significance of this fact goes well beyond the scope of derivatives pricing, and applies to many other industries such as investment and insurance etc.

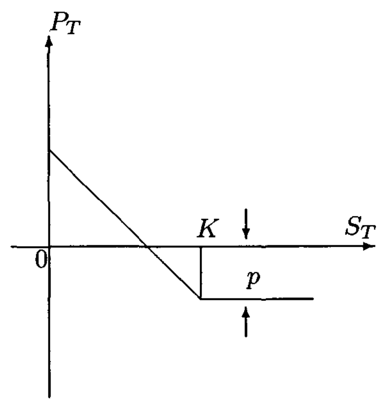
The price paid for a contingent claim is called the **premium**. The needs of clients vary. Correspondingly, there exist a variety of options-the financial products developed by financial institutions. Every type of options requires pricing. Option pricing is the main subject discussed in this book. Taking into account the premium p, the total gain P_T of the option holder at its expiration date is

[Total gain] = [Gain of the option at expiration] − [Premium],

i.e.

$$P_T = (S_T - K)^+ - p, \qquad \text{(call option)}$$
$$P_T = (K - S_T)^+ - p. \qquad \text{(put option)}$$





1.4 Option Pricing

As a derived security, the price of an option varies with the price of its underlying asset. *Since the underlying asset is a risky asset, its price is a random variable. Therefore the price of any option derived from it is also random. However, once the price of the underlying asset is set, the price of its derived security (option) is also determined.* i.e., if the price of an underlying asset at time t is S_t, the price of the option is V_t, then there exists a function $V(S,t)$ such that

$$V_t = V(S_t, t),$$

where$V(S,t)$is a deterministic function of two variables.*Our task is to determine this function by establishing a model of partial differential equations.*

V_T, an option's value at expiration date, is already set, which is the option's payoff:

$$V_T = \begin{cases} (S_T - K)^+, & \text{(call option)} \\ (K - S_T)^+. & \text{(put option)} \end{cases}$$

The problem of option pricing is to find $V = V(S,t), (0 \leq S < \infty, 0 \leq t \leq T)$, such that

$$V(S,T) = \begin{cases} (S-K)^+, & \text{(call option)} \\ (K-S)^+. & \text{(put option)} \end{cases}$$

In particular, if a stock's price at the option's initial date $t = 0$ is S_0, we want to know how much to pay for the premium p, i.e.

$$p = V(S_0, 0) = ?$$

The problem of option pricing is hence a **backward problem.**

1.5 Types of Traders

There are three types of derivatives traders in the security exchange markets:

1. Hedger

Hedging: *to invest on both sides to avoid loss.* Most producers and trading companies enter the derivatives markets to shift or reduce the price risks in the underlying asset markets to secure anticipated profits.

Example A US company will pay 1 million British Pound to a British supplier in 90 days. Now it faces a currency risk due to the uncertain USD/Pound exchange rates. If the Pound goes up, it will cost the company more for the payment, thus will hurt the company's profits. Suppose the exchange rate is currently 1.6 USD/Pound, and the Pound may go up, the company may consider the following hedging plans:

Plan 1 Purchase a forward contract to buy 1 million Pound with 1, 650, 000 Yuan 90 days later, and thus lock the cost of the payment in USD.

Plan 2 Purchase a call option to buy 1 million Pound with 1, 600, 000 USD 90 days later. The company pays a premium of 64, 000 USD (assuming a 4% fee) for the option.

The following table shows the results of the above risk-avoiding plans.

current rate (USD/Pound)	90 days later rate(USD/Pound)	no hedging (USD)	forward contract hedge(USD)	call option hedge(USD)
1.60	up 1.70	1, 700, 000	1, 650, 000	1, 664, 000
	down 1.55	1, 550, 000	1, 650, 000	1, 614, 000

One can see from this example: if the company adopts no hedging plan, its payment will increase if the Pound rate goes up, and thus will hurt its total profits. If the company signs a forward contract to lock the cost of the 90 days later payment, it has avoided a loss if the Pound goes up, but it has also given up the opportunity of gaining if the Pound goes down. If the company purchases a call option, it can prevent loss if the Pound goes up, and it can still gain if the Pound goes down, but it must pay a premium for the option.

2. Speculator

Speculation: an action characterized by willing to risk with one's money by frequently buying and selling derivatives (futures, options) for the prospect of gaining from the frequent price changes.

A speculator assumes the price risk, hoping to gain risky profits by holding certain positions (long or short).

Speculators are indispensable for the existence of hedging business, and they came into markets as a necessary result of the growth of the hedging business. It is speculators who take over the price risks shifted from the hedgers, and thus become the major bearers of the risks in the derivatives markets. Speculation is an indispensable lubricant in the derivatives markets. Indeed, frequent speculative transactions make hedging strategies workable.

Comparing to investing in an underlying asset, investments in its options are characterized by **high profits and high risk**, since an investment in options markets provides a much higher level of leverage than an investment in the spot markets. Such an investor invests only a small amount of money (to pay the premium) but can speculate on assets valued dozens of times higher than that of the invested money.

Example Suppose the price of a certain stock is 66.6 USD on April 30, and the stock may go up in August. The investor may consider the following investing strategies:

A. The investor spends 666, 000 USD in cash to buy 10, 000 shares on April 30.

B. The investor pays a premium of 39, 000 USD to purchase a call option to buy 10, 000 shares at the strike price 68.0 USD per share on August 22.

Now examine the investor's profits and **returns** in two scenarios (ignoring the interests).

Situation I The stock goes up to 73.0 USD on August 22.

Strategy A. The investor sells the stocks on August 22 to get 730, 000

USD in cash.

$$\text{return} = \frac{730\ 000 - 666\ 000}{666\ 000} \times 100\% = 9.6\%;$$

Strategy B. The investor exercises the option to receive a payoff:

$$\text{payoff} = 730\ 000 - 680\ 000 = 50\ 000\text{USD}$$

$$\text{return} = \frac{50\ 000 - 39\ 000}{39\ 000} \times 100\% = 28.2\%.$$

Situation II The stock goes down to 66.0 USD on August 22.

Strategy A. The investor suffers a loss:

$$\text{loss} = 666\ 000 - 660\ 000 = 6000\text{USD},$$

$$\text{return} = \frac{660\ 000 - 666\ 000}{666\ 000} \times 100\% = -0.9\%;$$

Strategy B. The investor receives a payoff:

$$\text{payoff} = (660\ 000 - 680\ 000)^+ = 0.$$

The investor loses the entire invested 39, 000 USD, hence a loss of 100%.

3. Arbitrageur

Arbitrage: based on observations of the same kind of risky assets, taking advantage of the price differences between markets, the arbitrageur trades simultaneously at different markets to gain riskless instant profits. Arbitrage is not the same as speculation: speculation is to seek profits promised by **predictions** of the future prices, and is thus **risky**. Arbitrage is to snatch profits originated in the **reality** of the price differences between markets, and is therefore **riskless**.

An opportunity for arbitrage cannot last long. Since once an opportunity for arbitrage arises, the market prices will soon reach a new balance due to actions of the arbitrageurs and the opportunity will thus disappear. Therefore, all discussions in this book are founded on the basis that arbitrage opportunity does not exist.