

Chapter 1

Quantum Mechanics

1.1 Why Two Types of Mechanics?

We live in a complicated world. Our sense organs provide a steady flow of information regarding the numerous phenomena that surround us. Powerful technological inventions also extend the reach of the human senses, giving us access to information more exact and complete than would otherwise be available.

The world of our sense perception is the macroscopic one. Here physical phenomena are described by *classical physics*, which includes classical mechanics, continuum mechanics (hydrodynamics and the theory of elasticity), thermodynamics, and electrodynamics. Because classical physics deals with phenomena in which microscopic structure plays no significant role, it cannot yield a comprehensive theory of the structure of real substances.

The laws of classical physics govern the motions of objects whose linear dimensions are sufficiently large:

$$R_{\text{cl}} > 10^{-6} \text{ m, say.}$$

Nothing more powerful than an optical microscope will be needed to observe such objects. Classical mechanics, in particular, describes the motions of planets, comets, stars, and galaxies.

But there exists another world, inaccessible to direct observation through our sense organs. This is the amazing world of micro-objects, in which physical phenomena are subject to the laws of quantum mechanics. The dimensions of molecules, atoms, atomic nuclei, and elementary particles are very small and could be characterized as

$$R_{\text{qu}} < 10^{-8} \text{ m.}$$

Thus we have two distinct physical theories. One describes macroscopic phenomena, the other microscopic phenomena. Why do two types of mechanics exist? The answer is far from simple. Let us pursue the question in more detail.

Until the end of the 19th century, practically all physical phenomena were described using classical mechanics. This subject was originally expounded by Sir Isaac Newton in his “*Philosophiae naturalis principia mathematica*”, a monumental work published in 1687. The appearance of each new experimental fact had required only modification to some old equation, or perhaps the introduction of a new one, but had not cast doubt on classical physics itself. Around the turn of the 20th century, however, existing approaches based on classical physics failed to describe new experimental data on atomic and subatomic phenomena. Two of these were *black body radiation* and the *photoelectric effect*.

In physics, the term “absolutely black body” is used for an object that absorbs light (i.e., electromagnetic radiation) but does not reflect it. A model of a radiating absolutely black body is a closed box with impenetrable walls, in which a tiny hole is made so that we can observe the radiation. In 1860, Gustav Kirchhoff established that the intensity of black body radiation depends on temperature and frequency only, and not on the substance from which the walls are made.

In 1896 Wilhelm Wien found that the radiation energy per unit volume and unit frequency (the density of black body radiation) decreases according to an exponential law

$$\rho(\omega, T) \sim \omega^3 \exp(-a\omega/T),$$

where ω is the frequency, T is the temperature, and a is a constant. *Wien’s law* holds for large frequencies ($a\omega/T \gg 1$). In 1900 Lord Rayleigh discovered that at low frequencies ($a\omega/T \ll 1$) the radiation density has the form

$$\rho(\omega, T) \sim \omega^2 T.$$

It was obvious that this latter relation, known as the *Rayleigh–Jeans law*, cannot hold for large frequencies; indeed it permits an absurd result later termed the “ultraviolet catastrophe” — an unlimited increase in the radiation density $\rho(\omega, T)$. The behavior of the radiation density in the intermediate range of frequencies was still an open question.

At the end of the 19th century the German physicist Max Planck inves-

tigated the dependence of the electromagnetic radiation intensity of a black body on frequency. Naturally, Planck was an adherent of classical physics because there was no other science at that time. However, in 1900 he established that the available experimental data could be explained only under the assumption that black body radiation is emitted in discrete portions. The energy of such a portion is given by the formula $E = \hbar\omega$, where

$$\hbar = \frac{h}{2\pi} = 1.05457266 \cdot 10^{-34} \text{ J} \cdot \text{s}. \quad (1.1)$$

The constant \hbar (here the bar reminds us of the division by 2π) is called *Planck's constant*, and the energy portions are called *quanta* (from the Latin word “quantum”, meaning “how much”). Planck's constant has the dimension of action (or of angular momentum); i.e., \hbar is a quantum of action.

This result was really overwhelming, as a discrete character for energy is impossible in classical physics. Planck's law meant a qualitatively new approach which required new physical and conceptual levels of understanding. This was the first “brick” to be laid in the foundation of an emerging physical theory: quantum mechanics. For his work on the discovery of energy quanta, Planck was awarded the Nobel Prize in Physics for 1918.

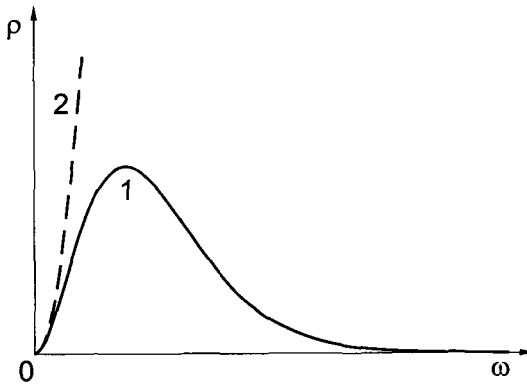


Fig. 1.1 Blackbody radiation density ρ versus frequency ω .

On the assumption that black body radiation is emitted in quanta, Planck obtained a formula for the radiation density, $\rho(\omega, T)$, which agrees with the known limiting cases for small and large frequencies and also stands in good agreement with the experimental data over the whole frequency

range (Fig. 1.1):

$$\rho(\omega, T) = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/kT} - 1)}, \quad (1.2)$$

where k is Boltzmann's constant.

If the frequency is small ($\hbar\omega \ll kT$), then from (1.2) we can obtain the Rayleigh–Jeans law

$$\rho(\omega, T) = \omega^2 kT / (\pi^2 c^3).$$

For large frequencies ($\hbar\omega \gg kT$) Planck's formula (1.2) turns into Wien's law

$$\rho(\omega, T) = (\omega^3 \hbar / \pi^2 c^3) \exp(-\hbar\omega/kT)$$

See Fig. 1.1.

The next major step was taken by Albert Einstein in 1905. He introduced the concept of a quantum of light and established that the photoelectric effect (the emission of electrons from a substance by light, a phenomenon discovered by Hertz in 1887) obeys the following law:

$$E_m = \hbar\omega - A, \quad (1.3)$$

where E_m is the maximum energy of an emitted electron, $\hbar\omega$ is the energy of the absorbed quantum of light, and A is the photoelectric work function (i.e., the energy required for emission of electrons from the substance). Einstein received the Nobel Prize in Physics for this discovery in 1921. Note that the term *photon* for a quantum of electromagnetic radiation was introduced by J. Lewis in 1926.¹ Today the photon is regarded as an elementary particle with zero rest mass and spin 1, possessing energy $\hbar\omega$ and momentum $\hbar\omega/c$.

In 1911 Ernst Rutherford discovered the atomic nucleus, whose linear size turned out to be four orders of magnitude smaller than that of the atom. It became clear that an atom is a system in which electrons somehow move around a very small nucleus, and the *planetary model* of the atom arose. There was, however, an important contradiction with classical physics. An electron moving along a curved trajectory through an electric field must

¹Lewis regarded the quantum of light as an indivisible atom, an assumption which did not stand the test of time. Nonetheless, his term "photon" has become a standard term in physics and has even entered the popular lexicon. What would Star Trek be without *photon torpedoes*?

emit electromagnetic waves. The resulting loss of energy should be accompanied by a decay in the electron's orbit, this process continuing until the electron finally falls into the nucleus and the atom collapses. According to classical physics then, any atom must cease to exist after some finite time interval. But it was already known that atoms are actually stable and have no such constraint on their lifetimes.

This contradiction was resolved by Niels Bohr in 1913 in his famous work "About the structure of atoms and molecules". Bohr theorized that an electron in an atom can remain for an infinitely long time only in certain states characterized by discrete values of the total energy. So the idea of discreteness, already known for electron energy, was used to explain atomic structure. Bohr obtained a correct formula for the electron energies in the hydrogen atom (the *energy spectrum* for this atom). For this purpose he used the equality of the centrifugal and electric forces acting on a charged particle (an electron) moving along a circular orbit in a Coulomb field:

$$\frac{mv_n^2}{r_n} = \frac{e^2}{r_n^2}. \quad (1.4)$$

Here e is the electronic charge (traditionally, in formula (1.4) the CGSE system of units is used), m is the electron mass, and r_n and v_n are the orbital radius and velocity values. It is assumed that the mass of the nucleus is infinitely large in comparison to that of the electron. The number n (*main quantum number*) takes positive integer values $n = 1, 2, 3, \dots$ and serves to index the possible states of the atom (the electron orbits).

Furthermore, Bohr supposed that the angular momentum of the electron can take only discrete values:

$$mv_n r_n = n\hbar. \quad (1.5)$$

While the relation (1.4) is usually referred to as "classical," the relation (1.5) is referred to as "quantum mechanical" as it involves the quantization of angular momentum (the requirement that angular momentum values be discrete). However, in today's established theory of quantum mechanics, the angular momentum cannot be written as a product of the momentum mv_n and distance r_n . So the left-hand side of (1.5) is a purely classical expression for the moment of momentum.

From (1.4) and (1.5) one can readily obtain the quantities r_n and v_n :

$$r_n = \frac{\hbar^2}{me^2} n^2, \quad v_n = \frac{e^2}{\hbar n}. \quad (1.6)$$

The total energy E_n of an electron in the hydrogen atom is a sum of the kinetic energy $mv_n^2/2$ and the potential energy $-e^2/r_n$. Therefore, we find that

$$E_n = -\frac{me^4}{2\hbar^2 n^2}. \quad (1.7)$$

The negative sign signifies that the electron exists in a bound state. The first five energy levels in the spectrum of the hydrogen atom are shown in Fig. 1.2, where one can see the levels getting closer as the binding energy of an electron decreases. Later, the energy expression (1.7) was obtained on the basis of quantum mechanics via solution of the Schrödinger equation.

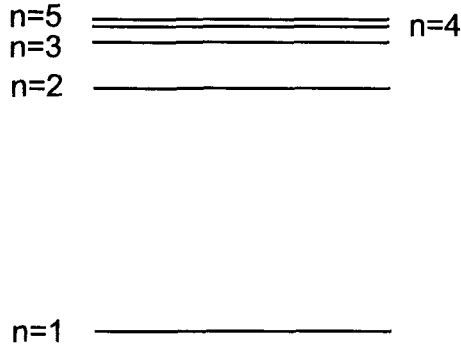


Fig. 1.2 The first five levels of the energy spectrum of the hydrogen atom.

While deriving (1.7) Bohr postulated the concept of *permissible orbits* of the electron, having radii r_n , as well as the possibility of emission and absorption of a certain portion (quantum) of electromagnetic radiation by the electron during its transition between orbits. This is the essential content of Bohr's two famous postulates. It is obvious that the energy $\hbar\omega$ of such a quantum must equal the difference of the electron energies for these orbits. If the electron passes from an orbit with energy E_n to one with energy E_s (we assume that $n > s$), it emits a quantum (a photon) with energy

$$\hbar\omega = E_n - E_s. \quad (1.8)$$

Formula (1.8) is the Bohr frequency relation. Substituting the expres-

sion for the energy (1.7) into it, we obtain

$$\omega = \frac{me^4}{2\hbar^3} \left(\frac{1}{s^2} - \frac{1}{n^2} \right). \quad (1.9)$$

By fixing the number s we obtain a series of frequencies (the *Lyman series* for $s = 1$, the *Balmer series* for $s = 2$, the *Paschen series* for $s = 3$) within which the number n defines a certain line. For each series we must keep $n > s$. Bohr noted that from (1.9) it is impossible to obtain other series of frequencies that were ascribed to hydrogen at the time. He was right to point out that these series belong to helium.

Bohr's frequency relation (1.8) or (1.9) was important because the radiation frequency ω did not coincide with that of the electron orbit. For this reason the result differed in principle from, for example, those of Nicolson (who assumed these two frequencies must be equal). For his investigation of the structure of atoms and the radiation emanating from them, Bohr was awarded the Nobel Prize in Physics for 1922.

Let us emphasize that in the works of Planck, Einstein, and Bohr, which formed the basis for the development of quantum mechanics, the corresponding final formulae were correct, but the methods by which they were found are subject to criticism. In fact, these scientific giants used their powerful intuition to guess the necessary results and causes of the corresponding physical phenomena. However, these results cannot be obtained in the framework of classical physics as it existed at that time. Quantum mechanics, as elaborated later, made it possible to derive these formulae by means of quite different and strict methods. In other words, the methodology for obtaining quantum results used by Planck, Einstein, and Bohr was not correct, and is now of mainly historical interest. Let us also note that the energy formula (1.7) obtained by Bohr is valid only for the hydrogen atom, but for more complex atoms Bohr's approach does not hold. Moreover, the approach is valid only for circular orbits, and cannot be applied to elliptical ones in this form.

The explanation of radiation from an absolutely black body, the photoelectric effect, and the structure of the hydrogen atom required the introduction of new concepts that turned out to be incompatible with classical physics. This quandary was the impetus for the creation of a new theory: quantum mechanics. So it is important to ascertain whether there exist general properties of our world which require the existence of two types of mechanics.

In physics, there is a relation between the fundamental symmetry prop-

erties of space and time and the laws of conservation of certain physical quantities. For example, the equivalence of all moments of time (temporal homogeneity) leads to the law of energy conservation for a closed (isolated) system. Similarly, the equivalence of all points of space (spatial homogeneity) leads to the law of conservation of momentum, and the equivalence of all directions in space (the isotropy of space) leads to the law of conservation of angular momentum. These symmetry properties are geometrical symmetries since they are not connected with concrete types of interaction.

Space possesses one more geometrical symmetry, namely that of similarity (scale invariance), which is connected with changes in spatial scale. However, the laws of nature turn out not to be invariant with respect to a similarity transformation (scale change). In reality then, the principle of similarity is not valid. This can be formulated as follows: the absolute size of a body *does* matter; two systems that are geometrically similar but different in scale are fundamentally different in mechanical behavior.

The failure of the similarity transformation was known to Galileo. He understood that if the sizes of animals and men were to be essentially increased, a significant increase in the firmness of their bones would become necessary; otherwise the bodies of giants would collapse under their own weight. Another example is a model of a building (e.g., a house) made of matchsticks. If such a model were scaled up to realistic dimensions, it would collapse similarly. So the principle of similarity is not valid at macroscopic scales.

The existence of the smallest natural building blocks — molecules and atoms — which have finite sizes, the existence of the elementary electric charge, and the limiting speed of signal propagation (the speed of light), all imply a failure of the similarity principle and non-invariance of natural laws with respect to scale transformations. Hence follows the existence of two types of mechanics: (1) the classical version, valid at distances large compared with the linear sizes of molecules and atoms, and (2) the quantum version, whose laws describe physical processes at distances comparable with the linear sizes of molecules and atoms or smaller.

1.2 Main Ideas and Principles of Quantum Mechanics

In 1923 the French physicist Louis de Broglie suggested that a material particle having nonzero mass also possesses wave properties uniquely related to its mass and energy. He ascribed a wavelength to a free particle, which

is related to its momentum p by the formula

$$\lambda = \frac{\lambda}{2\pi} = \frac{\hbar}{p}. \quad (1.10)$$

The quantity λ (or λ) has been called the *de Broglie wavelength* for the particle. The concept of the wave nature of particles with nonzero rest mass was developed by de Broglie in a series of works in 1924. He received the Nobel Prize in Physics for the year 1929.

No one in France could grasp the profundity of de Broglie's idea. His doctoral committee (consisting of the physicists Perrin and Langevin, the mathematician Cartan, and the crystallographer Mauguin) may not have even conferred his degree. But Langevin sent the thesis to Einstein for evaluation, and the answer was "He has lifted an edge of the great curtain." De Broglie's formula (1.10) deserves a place alongside Planck's formula $E = \hbar\omega$ and Einstein's formula $E = mc^2$.

With de Broglie's discovery it became clear that material particles possess physical properties similar to those of quanta of electromagnetic radiation (photons): they have both wave properties and particle properties. The discovery of the wave nature of microparticles was a brilliant guess which led to a true upheaval in our understanding of the physics of microparticles.

In 1927 the Americans Davisson and Germer and, independently, the Englishman G. Thompson, discovered electron diffraction by crystals experimentally. This was a remarkable corroboration of the predicted wave properties of material particles. Davisson and Thompson received the Nobel Prize in Physics for 1937.

We should stress that wave properties are inherent even in a single material microparticle. If one passes an electron beam of very low intensity through a crystal, so that individual electrons fly through the crystal independently of one another, then with sufficiently long exposure the same diffraction pattern will be observed as for a beam of high intensity. Since an individual electron causes the blackening of only one grain of the photoemulsion on the screen, electron diffraction means that one can only indicate a probability that an electron will reach some point of the screen. But it is impossible to calculate its trajectory.

Thus, in 1924 two basic ideas of quantum mechanics were already known. The first was that of *quantization*: the possibility that a physical quantity will take a discrete series of values under certain conditions. The second was the *wave-particle duality* of quantum objects, i.e., that any

quantum object (photon, electron, atom, molecule, etc.) is both a corpuscle (particle) and a wave at the same time. Under different conditions the particulate or wave properties of a certain object can be manifested to a greater or lesser extent.

Both quantum ideas were in absolute contradiction with classical physics, in which physical quantities take only continuous values. Moreover, in classical physics a particle and a wave are incompatible, whereas in quantum mechanics a particle and a wave cannot be separated. We stress that the wave-particle duality of a microobject should be understood as its potential ability to manifest its particulate or wave properties, depending on the conditions under which it is observed. Particles and waves are different forms of the same physical reality.

In 1925 the German physicists Heisenberg, Born, and Jordan elaborated matrix mechanics, which was the first variant of quantum mechanics. In 1926 the Austrian physicist Erwin Schrödinger developed wave mechanics, based on an equation later named after him. To describe a microobject state, Schrödinger introduced the wave function (the ψ -function). In 1926–1927 the Englishman Paul Dirac made a great contribution to the elaboration of mathematical techniques for quantum mechanics, and in 1927 proposed the method of secondary quantization. For the creation of quantum mechanics, Werner Heisenberg was awarded the Nobel Prize in Physics for 1932. Schrödinger and Dirac were awarded the Nobel Prize in Physics for 1933 for the discovery of new productive forms of atomic theory.

In 1926 Born gave a statistical interpretation of the wave function $\psi(\mathbf{r})$ as a probability amplitude such that the quantity $|\psi(\mathbf{r})|^2 d^3r$ yields the probability that the particle resides in volume d^3r in the vicinity of the point \mathbf{r} . We stress that the wave function can be a complex quantity. It can also depend on time. For his fundamental research in quantum mechanics, especially for his statistical interpretation of $\psi(\mathbf{r})$, Born was awarded the Nobel Prize in Physics for 1954.

Although the mathematical body of quantum mechanics was mainly elaborated in 1925–1926, it remained unclear why Heisenberg's matrix mechanics and Schrödinger's wave mechanics yield the same results. Development of mathematical techniques has made it clear that these are simply two equivalent variants of the same science, subsequently called quantum mechanics. The probabilistic character of the laws of quantum mechanics is due to the intrinsic randomness of the behavior of microobjects. Quantum mechanics can only predict the probability that a physical quantity will take a given value. For this reason, the probability concept plays a

fundamental role in quantum mechanics. However, probability in quantum mechanics differs substantially from this notion elsewhere in physics.

The wave function is a probability amplitude and, generally speaking, can be a complex quantity. As distinct from other parts of physics, in quantum mechanics the wave functions (the probability amplitudes) are added to each other, but not the probabilities themselves. In this way, interference terms arise in the total probability.

Let us illustrate the interference of probability amplitudes as an example of the superposition principle, which plays an important role in quantum mechanics. In the simplest variant it can be formulated as follows. If a given physical system can be in a state described by a wave function ψ_1 and the same system can be in another state described by a wave function ψ_2 , then it can also be in the state described by the function

$$\psi = a_1\psi_1 + a_2\psi_2, \quad (1.11)$$

where a_1 and a_2 are constants (generally speaking, complex) satisfying the condition $|a_1|^2 + |a_2|^2 = 1$.

Let a measurement, carried out with the system under consideration in the state described by wave function ψ_1 , yield result 1, and a similar measurement, carried out with the system being in the state described by wave function ψ_2 , yield result 2. Then the same measurement, carried out with the same system being in the state described by the wave function (1.11), will yield result 1 with probability $|a_1|^2$ and result 2 with probability $|a_2|^2$.

The superposition principle leads to an important consequence: the equations of quantum mechanics must be linear. Indeed, the Schrödinger equation, which is the basic equation of nonrelativistic quantum mechanics, is linear.

If the number of states in which the system can exist is greater than two, then the superposition principle appears as follows:

$$\psi = \sum_n a_n \psi_n, \quad (1.12)$$

where the sum is extended over all possible states of the system. The number of system states can be infinite. Then the sum in (1.12) will contain an infinite number of addends.

The probability that the system is in the state described by the wave

function (1.11) is defined by the expression

$$w = |\psi|^2 = |a_1|^2 |\psi_1|^2 + |a_2|^2 |\psi_2|^2 + (a_1 a_2^* \psi_1 \psi_2^* + a_1^* a_2 \psi_1^* \psi_2). \quad (1.13)$$

The probability w consists of three terms. The first two are the probabilities of finding the system in the states 1 and 2, respectively, while the third represents an interference of the probability amplitudes. The occurrence of interference terms is characteristic of quantum mechanics. It is connected with the intrinsic wave properties of microobjects. The interference existing in quantum mechanics is the main conceptual difference between the classical and quantum descriptions of objects and processes. It can be illustrated by considering a particle motion along different paths from point 1 to point 2. In classical physics the probabilities of the particle motion along each path are summed. In quantum mechanics the amplitudes associated with these paths are summed, and then the squared modulus of the resulting amplitude is the probability of the particle having passed from point 1 to point 2.

In 1927 Heisenberg established the *uncertainty relation*. Let us consider its simplest form. If we introduce the root-mean-square deviations (*dispersions*) for the coordinate and momentum

$$\langle (\Delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2, \quad \langle (\Delta p_x)^2 \rangle = \langle p_x^2 \rangle - \langle p_x \rangle^2, \quad (1.14)$$

then it turns out that their product for any microobject cannot be smaller than the value $\hbar/2$:

$$\langle (\Delta x)^2 \rangle \langle (\Delta p_x)^2 \rangle \geq \frac{\hbar^2}{4}. \quad (1.15)$$

The angle brackets in (1.14) and (1.15) denote the averaging of quantities in the microobject state under consideration.

Formula (1.15) is the uncertainty relation for the position and momentum. It means that the more precisely we determine the position of a microobject, the greater the indeterminacy of its momentum component along the same axis will be, and vice versa.

The uncertainty relation (1.15) shows that in quantum physics the simultaneous use of the notions of position and momentum is nonsense, although the very notions of position and momentum do have physical meaning. Uncertainty relations similar to (1.15) also exist for other pairs of quantities which are called *conjugate pairs* (for example, the energy of a microobject and the time of its interaction with a measuring device, the

projection of the angular momentum onto the axis of quantization and the azimuthal angle, etc.).

Physical laws have, as a rule, the form of certain exclusions (restrictions) imposed upon physical quantities under consideration. However, only three exclusions hold throughout physics.² The first is the uncertainty relation, which forbids the product of the dispersions of two conjugate physical quantities to be less than $\hbar^2/4$. The second is the impossibility of existence of perpetual motion of the first and second kinds. The third does not allow a particle with nonzero rest mass to increase its velocity from a value $v < c$ to a value $v > c$ (the impossibility of crossing the light cone).

The universal character of the uncertainty relation means that it is also valid in classical physics. However, owing to the smallness of the quantity \hbar , it is never manifested in classical physics because in this case the uncertainties (dispersions) $\langle(\Delta x)^2\rangle$ and $\langle(\Delta p_x)^2\rangle$ are so small that they cannot be noticed. Let us emphasize that the existence of the uncertainty relation is a general law of quantum mechanics which is not connected with a method or accuracy of measurement of the corresponding quantities.

The uncertainty relation (1.15) leads to an important consequence. Since it is impossible to determine simultaneously a position of a microobject and the corresponding component of momentum with arbitrary accuracy, then this implies the absence of any trajectory of the microobject, which is defined in classical physics as a function $\mathbf{p}(\mathbf{r})$. In other words, if we know the particle position at a given moment of time, we cannot know it at any subsequent moment. Rather, we can only determine the probability of finding the particle at a point in space at any subsequent moment.

The absence of trajectories of microobjects turns out to be of great importance for systems of identical particles. If at a given moment of time we number identical particles, then at any following moment we cannot indicate where each of the numbered particles is.

This property of systems of identical particles can be formulated in the following way: the physical properties of a quantum system of identical particles are invariant with respect to the permutation of any pair of particles in this system. In particular, the probability defined by the squared modulus of the wave function of a system of identical particles, $|\psi|^2$, is invariant with respect to such a permutation. However, it is clear that in this case the wave function ψ itself may be not invariant with respect to the permutation of a pair of identical particles.

²It is worth noting that the familiar conservation laws are not among these so-called *global exclusions*.

There are two possibilities: the wave function is *symmetrical* and does not change under the permutation of a pair of identical particles, or it is *antisymmetrical* and does change sign. It is proved in quantum mechanics that, if the wave function of a system of identical particles is symmetrical (antisymmetrical) at a given moment of time, then it will be symmetrical (antisymmetrical) for all time.

Thus, a general problem arises to determine which systems of identical particles are described by symmetrical wave functions and which are described by antisymmetrical ones. It turns out that the symmetry properties of a wave function are completely defined by the spin (intrinsic angular momentum) of particles which, like the orbital moment, is measured in units of Planck's constant \hbar . If the spin of a particle is an integer (possibly zero), then the wave function is symmetrical, and if the spin is half-integral, then the wave function is antisymmetrical.

Spin is an inherent characteristic of a particle, like its mass and charge. It is a purely quantum characteristic, having no analogue in classical physics. In quantum mechanics the orbital moment of a particle, which is related to motion in space, takes only discrete values that are multiples of \hbar . For this reason, in quantum mechanics it is convenient to measure angular momentum in units of Planck's constant \hbar . The spins of different particles can have (in units of \hbar) both integral and half-integral values. The spins of electrons, protons, neutrons, and neutrinos are equal to $1/2$, the spins of π -mesons are equal to zero, and the spin of a photon equals 1.

If identical particles possess an integral spin, then they obey the *Bose-Einstein statistics* and are called *bosons*. If identical particles have a half-integral spin, then they obey the *Fermi-Dirac statistics* and are called *fermions*. A system of identical bosons is described by a symmetrical wave function, and a system of identical fermions by an antisymmetrical wave function.

In 1924–1925 Wolfgang Pauli investigated the properties of systems of identical particles, and established what was later to be called the *Pauli exclusion principle*:

Two identical fermions cannot be in the same state.

For this discovery Pauli was awarded the Nobel Prize in Physics for 1945. Although Pauli provided a proof of the exclusion principle, the latter had been used intuitively by Bohr in 1921–1922 when he classified atoms on the basis of their electron structure.

The validity of the exclusion principle can be easily understood as fol-

lows. Let us assume that two identical fermions are in the same state. Then their permutation does not change the wave function of the system, since in this case the system turns into itself by virtue of the indistinguishability of the identical particles. On the other hand, the wave function of the system of two identical fermions must be antisymmetrical, i.e., it must change its sign under their permutation. For this reason, the wave function in the case under consideration equals zero. In other words, the probability of two identical fermions being in the same state is equal to zero.

If one considers a system of identical particles at low temperature, then it turns out that a system of identical bosons behaves in a different way as compared with a system of identical fermions. At absolute zero all the identical bosons would be in their lowest energy state. In this case, *Bose condensation* arises.

In the case of a system of identical fermions, no more than one particle can be in each state, even at absolute zero, owing to Pauli's exclusion principle. For this reason, a certain distribution of fermions over energies arises. Since atoms of different elements have different chemical properties, and since electrons in metals, even near absolute zero, have large energies, it is obvious that various physical characteristics of systems of identical bosons and fermions near absolute zero must be different.

Let us consider another interesting quantum effect. The solution of the Schrödinger equation for the potential of a harmonic oscillator shows that a quantum oscillator with frequency ω has the lowest energy $E_0 = \hbar\omega/2$, which is called its *zero-point vibrational energy*. In other words, the quantum oscillator differs from the classical one in the fact that it does not rest in its lowest energy state, but performs these zero-point vibrations with energy E_0 . In quantum mechanics, it can be shown that the energy E_0 is the minimum energy of the oscillator which is permitted by the uncertainty relation.

This amazing property of quantum oscillators has been verified experimentally. In particular, it means that, when approaching absolute zero, atoms situated at points of the crystal lattice of a solid perform zero-point vibrations. One can investigate the scattering of electromagnetic radiation (photons) on atoms of the lattice. From the character of this scattering at temperatures close to absolute zero one can determine whether atoms at the points of the lattice rest or vibrate. These experiments have uniquely shown that atoms at the lattice points perform the zero-point vibrations with the energy E_0 , because the zero-point vibrations substantially change the character of the photon scattering.

Zero-point vibrations are also performed by any physical field even in the absence of the particles that are quanta of this field. For example, zero-point vibrations of an electromagnetic field lead to a small change of the energy levels of atoms as compared to those calculated without taking them into account. This phenomenon was for first time observed experimentally by the American physicist Willis Eugene Lamb in 1947. It has been called the *Lamb shift*, and has been explained by calculations carried out on the basis of quantum electrodynamics. For this discovery Lamb was awarded the Nobel Prize in Physics for 1955.

The concept of a field plays a very important role in modern quantum physics. A field can exist even in the case when the corresponding particles are absent. This state of a field is called vacuum. However, there are zero-point vibrations that occur in a vacuum, which sometimes leads to a spontaneous creation of a pair consisting of particle and antiparticle. This pair is subsequently annihilated. For example, in an electromagnetic field electron-positron pairs can be spontaneously created and annihilated. It is for this reason that in quantum theory the distinction between a particle and field disappears.

The successes of classical physics had led to a profound confidence in the possibility of unambiguous predictions of various physical events. The most prominent representative of this conception of necessity was the famous French scientist Pierre-Simon Laplace. In his words,

“An intellect, which could know all the forces in the nature and the relative situation of the things composing it at a given instant and which could be powerful enough to subject them to an analysis, would be able to comprehend the motions of the greatest bodies in the Universe and of a smallest atom by a single formula, and nothing would remain unknown, both the past and the future would be open to him.”

So Laplace believed that a knowledge of the positions and velocities of all particles of a physical system made it possible to predict its behavior. This statement expresses the “Laplace determinism” that underlies classical physics.

In quantum mechanics the situation is different: it is impossible to know the positions and velocities of particles simultaneously, by virtue of the uncertainty principle. One can only define the wave function at an initial moment of time, which is the most complete description of the state of a

physical system. In quantum mechanics, which uses the wave function, only a probabilistic description of events is possible. Laplace's classical causality does not hold for microobjects, whose behavior is inherently random.

The probabilistic character of the quantum description of the behavior of microobjects is also rooted in the interaction of a microobject with the rest of the world. It is just the attempt to isolate a microobject or some physical system from the rest of the world that leads to the randomness in their behavior. In the words of Heisenberg,

“It is necessary to draw one's attention to the fact that a system which should be considered according to the methods of quantum mechanics actually is a part of much larger system and, ultimately, of the whole world. It is in an interaction with this large system, and we have also to say in addition that microscopic properties of the large system are, at least to a great extent, unknown”.

That is why, by virtue of the probabilistic character of the quantum laws, the physical processes of microobjects are characterized by a probabilistic form of causality, of which Laplace's determinism is only a limiting case.

However, one should not think that quantum mechanics is not a deterministic theory. It is deterministic in the sense that it defines the law of the wave function variation with time. Elements of unpredictability and randomness arise only with attempts to interpret a microobject on the basis of classical conceptions about its positions and velocities.

1.3 Measuring the Physical Characteristics of Microobjects

An important criterion for the validity of a physical theory is agreement between calculated predictions and experimental data. Another is the ability to predict results of future measurements from those already available. Hence the measurement of physical characteristics of microobjects is an issue of great importance in quantum mechanics.

We are macroscopic objects living in the macroscopic world. This necessitates that any device (*detector*) used for quantum measurements also be a macroscopic object, so that we can perceive the information it obtains. Such a device must also, as a macroscopic system, exist in a rather unstable state that can be easily changed under the influence of a microobject. For example, in a bubble chamber a superheated transparent liquid instantly

boils when a charged particle flies through it (Sect. 5.5). The resulting bubbles of vapor allow us to see the particle trajectory. Here the thickness of the trajectory is large in comparison to the atomic scale, so the uncertainty relation is not violated. By such a rough approximation a microobject can be considered classically.

The detector necessarily and significantly changes the state of the microobject with which it interacts. In classical physics it is assumed that the influence of a measurement device on an object can be made arbitrarily small. During a quantum measurement, one cannot in principle neglect the interaction between the detector and the microobject. Consequently, the uncontrollable character of the interaction between detector and microobject leads to the necessity of the probabilistic description of quantum processes, since the measurement destroys the initial quantum state of the microobject in an unpredictable way.

The process of a quantum measurement is irreversible: as a result of the measurement, the wave function of the microobject changes abruptly; i.e., a reduction or “collapse” of the wave function occurs. Reconstruction of the initial state of the microobject is absolutely impossible after the measurement process has occurred. Thus the *irreversibility* of the process of measurement plays a fundamental role in quantum physics.

By virtue of this irreversibility, an irreproducibility of a single measurement arises. Since in each act of measuring the interaction between the detector and microobject occurs in a different way, the measured results will be different. Only a sufficiently large number of measurements will give a certain stable pattern of the distribution of results. This pattern can also be obtained in another series including a sufficiently large number of measurements.

The state of a microobject is not defined before a measurement. A series of measurements performed with the same detector over identical, as one would think, microobjects yields a set of different results. If a beam of electrons passes through a slit in a screen, then different electrons will reach different points of a photoplate and a certain diffraction pattern will appear. In this case one can only determine the probability that the electrons arrive at different points of the photoplate. In other words, a certain statistical distribution of electrons on the photoplate arises which is not chaotic. The task of quantum mechanics is to determine the probability distribution for various physical quantities characterizing microobjects.

Let us now direct the electron beam onto a screen with two slits A and B (Fig. 1.3). If A is open and B is closed, then on the screen-detector we

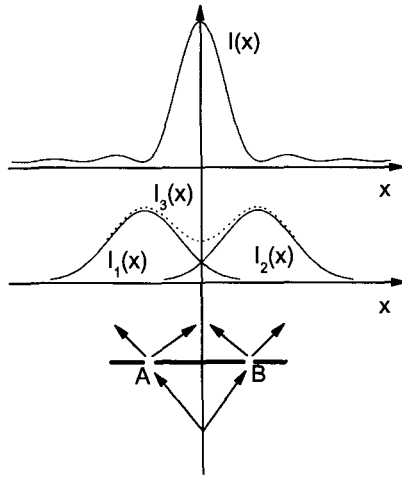


Fig. 1.3 Distribution of electrons passing through a screen with two slits.

observe the electron distribution $I_1(x)$ that corresponds to a wave function ψ_1 . If only B is open, the electron behavior will be described by a wave function ψ_2 and we will observe the electron distribution $I_2(x)$. If A and B are open, then the wave function of the electrons is $\psi = \psi_1 + \psi_2$. In this case each electron, owing to its wave nature, will pass through both slits simultaneously. The probability density of this process is defined by the expression

$$w = |\psi|^2 = |\psi_1|^2 + |\psi_2|^2 + (\psi_1^* \psi_2 + \psi_1 \psi_2^*). \quad (1.16)$$

The third term in (1.16) describes the interference of waves passing through the two slits simultaneously. This case corresponds to the electron distribution $I(x)$.

If the electrons are allowed to pass through the slits opened in turn, then the probability distribution of this process will be defined by a sum of the probabilities of electron passage through each slit separately:

$$\tilde{w} = |\psi_1|^2 + |\psi_2|^2. \quad (1.17)$$

In this experiment the interference disappears; i.e., controlling the electron passage through a certain slit in the screen destroys the interference. In other words, *quantum measurement* destroys interference and we observe

the electron distribution $I_3(x)$.

Quantum measurements are characterized by the union of a measuring device and the microobject over which the measurement is performed. It is this inseparable union of a macroscopic (classical) device and the microobject under analysis that leads to their uncontrollable interaction changing the microobject state.

Quantum mechanics is a probabilistic theory, which makes it principally different from classical physics. However, it turns into classical mechanics in the limiting case when Planck's constant \hbar becomes negligible. Formally, the transition to classical mechanics is performed when one lets this quantity decrease to zero: $\hbar \rightarrow 0$. Quantum mechanics is grounded in, and therefore irrevocably linked to, classical mechanics.

The limit $\hbar \rightarrow 0$ can be understood as follows. If $n \gg 1$, then, according to (1.5), the angular momentum of an electron in an atom becomes very large relative to Planck's constant: $mv_n r_n \gg \hbar$. In other words, in the case $n \gg 1$ the constant \hbar can be neglected and the discreteness of the angular momentum disappears. Thus, quantum mechanics turns into classical mechanics when quantum numbers are large — this is the *correspondence principle* initially formulated by Bohr.

In particular, this means that for large quantum numbers the frequency of radiation emitted by an atom at the transition from one state to another asymptotically coincides with the frequency predicted by the classical theory. If the atom passes from an excited state with energy E_{n+1} to a state with energy E_n , then the radiation frequency equals $\omega_n = (E_{n+1} - E_n)/\hbar$, where E_n and E_{n+1} are defined by (1.7). Then

$$\omega_n = \frac{me^4}{2\hbar^3 n^2} \left[1 - \left(\frac{n}{n+1} \right)^2 \right].$$

For $n \gg 1$ we get $1 - (n/n+1)^2 \approx 2/n$. Thus, we find that

$$\omega_n = \frac{me^4}{\hbar^3 n^3} = \frac{v_n}{r_n},$$

where the electron velocity v_n and the "orbital radius" r_n are defined by the formulae (1.6).

In classical mechanics the frequency $\tilde{\omega}$ of the electron revolution with velocity v along the orbit of radius r is equal to $\tilde{\omega} = v/r$. We can see that for $n \gg 1$ the quantum result coincides with this. Moreover, the distance between neighboring energy levels of the hydrogen atom tends to zero for

large quantum numbers ($n \rightarrow \infty$). In this limiting case, the discreteness of the energy spectrum becomes less significant and the atomic system behaves like a classical one.

Because of its probabilistic approach to the explanation of microprocesses, quantum mechanics was rejected by some scientists during its development. One of the opponents of the quantum theory was Einstein, who helped sow the seeds for its creation. He proposed various arguments to prove that quantum mechanics is not valid. However, Bohr, also a giant of Physics, parried with cogent counterarguments in support of quantum mechanics.

In 1935, a paper by Einstein, Podolsky, and Rosen appeared, in which they put forward their famous paradox. According to their initials, it became known as the EPR-paradox. We will not expound the essence of this or certain other paradoxes that were put forward during the development of quantum mechanics in order to prove its incompleteness. We shall only note that the outstanding intellect of Niels Bohr resolved all these paradoxes.

Discussions regarding the completeness of the quantum-mechanical description of microobjects have led to the conjecture that the uncertainty in the behavior of a quantum object is explained by the existence of some "hidden" parameters, about which observers know nothing. Just the presence of these hidden parameters could lead to the probabilistic behavior of microobjects and to the uncertainty of the results of measurements. It followed from this approach that knowledge of the hidden parameters could allow one to predict exactly the microobject behavior, i.e., that the determinism of classical physics would triumph.

The first proof of the nonexistence of hidden parameters was given by J. von Neumann. However, a formulation of the proposition was required which could experimentally corroborate the absence of hidden parameters. Finally, in 1965, Bell proposed a statement (the Bell theorem) that made it possible to ascertain experimentally the distinction between the predictions of quantum mechanics and the theory of hidden parameters.

Experiments based on the Bell theorem were carried out by Clause and Freedman at the University of California in 1972, as well as by Aspect, Dalibard, and Roger at the Paris Institute of Optics in 1982. These and other experiments have proved the validity of quantum mechanics and the failure of the hidden parameter theory. Undoubtedly, experimental studies in this direction will continue. However, the theory of hidden parameters, at least in its present form, does not agree with experimental data.

1.4 Structure of Atoms

Bohr's 1913–1922 works, which investigated the structure of atoms, gave correct results which were, in fact, guessed, since the methods used to obtain them were incorrect. Later, these results were obtained on the basis of quantum mechanics.

Bohr's formulas (1.6) and (1.7) are of value because they allow us to predict a characteristic linear size for the hydrogen atom, as well as a characteristic velocity and energy for the electron in the atom. For $n = 1$ (the ground state of hydrogen) we have

$$\begin{aligned} r_1 &= \frac{\hbar}{me^2} \approx 0.53 \cdot 10^{-10} \text{ m}, \\ v_1 &= \frac{e^2}{\hbar} \approx 2.2 \cdot 10^6 \text{ m/s}, \\ E_1 &= -\frac{me^4}{2\hbar^2} \approx -13.6 \text{ eV}. \end{aligned} \quad (1.18)$$

The quantity r_1 has been called the radius of the *first Bohr orbit*. It shows that the characteristic linear size of the atom, by order of magnitude, is $R_{at} \sim 10^{-10}$ m. Using the constants m , e , \hbar , one can compose the quantity $\tau_{at} = \hbar^3/(me^4) \approx 2.5 \cdot 10^{-17}$ s, which is a characteristic atomic time.

To understand to what extent the smallest particles of substance are small, let us consider a continuous chain composed of atoms lined up side by side. If every inhabitant of our planet had put one atom into this chain, its length would be equal to several centimeters.

By virtue of the uncertainty relation (1.15), a microobject described by quantum mechanics has no trajectory. This means that there exist no electron orbits in atoms, only probabilities of different distances between the electron and nucleus. For example, for the ground state of the hydrogen atom ($n = 1$) this probability equals

$$w_1(r) = Cr^2 \exp\left(-\frac{2r}{r_1}\right), \quad (1.19)$$

where C is a constant.

The probability (1.19) has a maximum at $r = r_1$, i.e., the radius of the first Bohr orbit is the most probable distance between the electron and nucleus in the ground state of the hydrogen atom. We stress that in reality the electron does not revolve around the atomic nucleus like a planet moving around the Sun. It simply exists in the atom and can be found at any point

of the atom with a strictly definite probability. According to quantum mechanics, a hydrogen atom in the ground state has spherical symmetry, whereas in classical mechanics a system consisting of two charged particles cannot be in a spherically symmetrical state.

We note that the initial approach of Bohr, which is based on classical mechanics, gave the correct formula (1.7) for the energy of an electron in a hydrogen atom, but it led to an incorrect value of the electron orbital moment l . Indeed, for $n = 1$ (the ground state of the hydrogen atom), formula (1.5) yields the value $l = 1$ (in units of \hbar); according to quantum mechanics, it should be $l = 0$.

The structure of atoms is mainly defined by the electric interaction of atomic electrons with the nucleus and between the electrons themselves. If there are two or more electrons in the atom, then this is already a complex system of mutually interacting particles. However, it turns out that it is a good approximation for a complex atom to consider that each electron moves in an effective central field which is created by the nucleus and other electrons. Such a field is called the *self-consistent field*.

A state of an electron in a central field (in a spherical atom) is characterized by the main quantum number n , the orbital moment l , and the projections of the orbital moment m and of the spin σ on the quantization axis. The electron spin projection can take two values: $\sigma = \pm \frac{1}{2}$. The electron orbital moment projection (the magnetic quantum number) can take $2l + 1$ values, since it takes integer values with the limits $-l \leq m \leq l$. The electron orbital moment can take the values $l = 0, 1, 2, \dots$, and the main quantum number $n = l + 1; l + 2; l + 3; \dots$

According to the Pauli principle, only one electron can be in the state with quantum numbers n, l, m, σ . All of the atomic electrons with the same quantum numbers n and l are said to be in the same *electron shell* of the atom. Such electrons possess the same energy and are said to be *equivalent*. An electron shell can contain $2(2l + 1)$ equivalent electrons having different values of m and σ . All of the $2(2l + 1)$ atomic electrons having the same n and l values form a perfectly occupied (*closed*) shell.

Table 1.1 Electron states.

1	0	1	2	3	4	5	6
	s	p	d	f	g	h	i
$2(2l+1)$	2	6	10	14	18	22	26

To denote the states of electrons with definite values of orbital momentum, the literal notation presented in Table 1.1 is used. The lower line of Table 1.1 contains the total number of electrons in a closed shell with the given values of quantum numbers n and l . The state of the atomic electron with definite values of n and l is denoted by a figure indicating the value of n and a letter indicating the value of l . Sometimes, one refers to electrons with main quantum numbers $n = 1; 2; 3; 4; 5; 6; \dots$, as being in the $K-$, $L-$, $M-$, $N-$, $O-$, $P-$, $Q-$, \dots , shells, respectively.

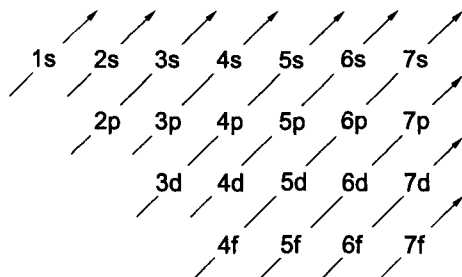


Fig. 1.4 The order in which atomic shells are filled is shown by arrows.

The order of filling of the atomic shells is shown by the arrows in Fig. 1.4. However, one should remember that there are some exceptions to this rule. The distribution of electrons in shells is called the *electron configuration* of the atom. Let us consider this for some particular atoms.

A hydrogen atom contains one electron in the state $1s$. In a helium atom there are two equivalent electrons in the same state, which differ by the values of their spin projections on the quantization axis. For this reason, an atom of helium has electron configuration $1s^2$, where the superscript denotes the number of equivalent electrons in this state.

A lithium atom has the electron configuration $1s^2 2s$, while a beryllium atom has the configuration $1s^2 2s^2$. In the boron atom the first electron with $l = 1$ (the p -electron) appears. The electron configuration of this atom has the form $1s^2 2s^2 2p$. Further, the filling of the $2p$ -shell continues: a carbon atom has the electron configuration $1s^2 2s^2 2p^2$, a nitrogen atom has the configuration $1s^2 2s^2 2p^3$, and so on. An atom of neon, which is an inert gas, has all shells filled: $1s^2 2s^2 2p^6$.

In atoms of sodium and magnesium, the $3s$ -shell is filled. This is also true for the $3p$ -shells in atoms of the elements from aluminum to argon. In atoms of potassium and calcium the $4s$ -shell is filled, and

in atoms of the elements from scandium to zinc the $3d$ -shell is filled. Here, the first violations of the successive filling of the electron states in the shells appear: a chromium atom has the electron configuration $1s^2 2s^2 2p^6 3s^2 3p^6 4s 3d^5$ instead of $\dots 4s^2 3d^4$, and a copper atom has the configuration $1s^2 2s^2 2p^6 3s^2 3p^6 4s 3d^{10}$ instead of $\dots 4s^2 3d^9$.

Atoms of the elements in which the $3d$ -shell is filled (from scandium to zinc, inclusive) form the *ferrum series*. Atoms of the elements in which the $4d$ -shell is filled (from yttrium to cadmium, inclusive) form the *palladium series*. Atoms of the elements in which the $5d$ -shell is filled (from lanthanum to mercury, inclusive, except for lanthanoids) form the *platinum series*.

The first f -electron appears in the atom of cerium. The elements with the atomic number Z in the range $58 \leq Z \leq 71$ (from cerium to lutetium, inclusive), in which the $4f$ -shell is filled, are called the *lanthanoids* or *rare-earth elements*. The elements with $90 \leq Z \leq 103$ (from thorium to lawrencium, inclusive), in which the $5f$ -shell is filled, are called the *actinoids*. In the *inert gases* helium ($Z = 2$), neon ($Z = 10$), argon ($Z = 18$), krypton ($Z = 36$), xenon ($Z = 54$), and radon ($Z = 86$), all shells are completely filled.

Calculations show that the average distances from the nucleus to the electrons in the d - and, especially, f -shells are substantially smaller than those to the electrons in the s - and p -shells. The chemical properties of elements are mainly defined by the electrons situated in the outer domains of atoms where the s - and p -electrons can most probably be found. For this reason the elements of the ferrum series, in which the $3d$ -shells are filled, have fairly similar chemical properties. Still more similar properties are possessed by the lanthanoids, in which the $4f$ -shell is filled. The same holds for the palladium and platinum series, and for the actinoids. Thus, the periodicity of chemical properties of elements discovered by Mendeleev means, from the quantum-mechanical point of view, the replicating of the structure of the electron shells being filled up.

The elements in whose atoms the s - and p -shells are filled are called the elements of *basic groups*, and the elements in whose atoms the d - and f -shells are filled are called the elements of *transition groups*. There are also some exceptions here. For instance, copper and zinc belong to the elements of transition groups, but not to the basic ones.

Let us briefly touch upon the notion of valency of elements from the point of view of quantum mechanics. This notion can be used basically in simple compounds. Valency defines the ability of atoms to join with each other. In 1927, Heitler and London related this ability to the value of total

spin of an atom. This was the beginning of quantum chemistry. According to their investigations, the valency is the doubled spin of an atom joining a compound. The same atom can manifest different valencies depending on the state in which it enters into a compound. Atoms enter into compounds in such a way that their spins should mutually compensate. Let us consider the valencies of elements of the basic groups.

- (1) Atoms of elements of the first group (Li, Na, K, Pb, Cs, and Fr, being alkaline metals) have spin $1/2$ in the ground state and manifest the valency 1. Their first excited state lies far from the ground state. For this reason they usually enter into compounds in the ground states.
- (2) Atoms of elements of the second group (Be, Mg, Ca, Sr, Ba, and Ra, being the alkaline-earth metals) have zero spin in the ground state and do not enter into compounds in this state. However, they have an excited state whose last shell configuration is sp (instead of s^2) with spin 1 and this is energetically close to the ground state. So atoms of these elements can enter into compounds in these excited states and manifest the valency 2, in this case.
- (3) Atoms of elements of the third group (B, Al, Ga, In, and Tl) have electron configuration of the last shell s^2p and the spin $1/2$ in the ground state. Close to the ground state they have an excited state with the configuration of the last shell sp^2 and the spin $3/2$. For this reason, the elements of this group manifest valencies 1 and 3. Exceptions are the first two elements B and Al, which are always trivalent.
- (4) Atoms of elements of the fourth group (C, Si, Ge, Sn, and Pb) have the electron configuration of the last shell s^2p^2 with spin 1 in the ground state, and close to it there is an excited state of configuration sp^3 with spin 2. Hence they manifest valencies 2 and 4. The first two elements of this group, C and Si, mainly manifest valency 4 (an exception is, for instance, carbon monoxide CO).
- (5) Atoms of elements of the fifth group (N, P, As, Sb, and Bi) in the ground state have the configuration of the last shell s^2p^3 with spin $3/2$. The closest excited state has configuration sp^3s' , where s' denotes the electron state which has main quantum number being greater by one than in the state s . The spin of this excited state is equal to $5/2$. For this reason, the elements of the fifth group manifest valencies 3 and 5.
- (6) Atoms of elements of the sixth group (O, S, Se, Te, and Po) in the ground state have the configuration of the last shell s^2p^4 with spin 1. Besides, they have excited states with the configurations s^2p^3s' and

$sp^3s'p'$ with spins 2 and 3, respectively. For this reason, the elements of the sixth group manifest valencies 2, 4, and 6. An exception is oxygen, which is always bivalent because its atoms enter into compounds only in the ground state.

- (7) Atoms of the seventh group (F, Cl, Br, I, and At, being the halogens) in the ground state have the configuration of the last shell s^2p^5 with spin 1/2. They can also enter into compounds in the excited states with configurations s^2p^4s' , $s^2p^3s'p'$, $sp^3s'p'^2$, having spins 3/2, 5/2, 7/2, respectively. For this reason, they can manifest valencies 1, 3, 5, 7. An exception is fluorine, which is always univalent.
- (8) Atoms of elements of the eighth group (He, Ne, Ar, Kr, Xe, and Rn, being the inert gases) in the ground state have perfectly filled shells with zero spin. For this reason, they are practically always inert. Some can enter into compounds; this stems from the transition of electrons from the outer filled shell to the states of the unfilled d - and f -shell, which are close in energy.

Quantum mechanics also makes it possible to explain the valencies of elements of the transition groups. In atoms of these elements the filling of the d - and f -shells occurs. Electrons in the d - and f -shells of atoms are situated deeper than the outer s - and p -electrons. For this reason, the interaction of these atoms with other atoms and molecules is usually weaker than for the atoms of basic group elements. In particular, among the compounds of transition group elements, molecules with nonzero spins are often encountered. Atoms of these elements can manifest both even and odd valencies, which are defined both by the interaction of outer electrons and by the possibility to enter into compounds in excited states when the deep-lying electrons pass over into the s - and p -shells.

When atoms join a molecule, the electron densities in the filled shells change only slightly. However, in this case the electron densities in the unfilled shells change substantially. From this point of view, there are two extreme cases. The first is the *heteropolar* or *ionic bond*, in which all valence electrons pass over from some atoms to others and the molecules consist of charged ions with perfectly filled shells. For example, in the molecule NaCl, the sodium atom gives its s -electron to the chlorine atom, and the positively charged ion Na^+ together with the negatively charged ion Cl^- form a molecule of sodium salt. Another extreme case is the *homopolar* or *covalent bond*, in which atoms remain neutral and the valence electrons become "collective". Examples occur in the molecules H_2 , Cl_2 , and so on.

Naturally, there also exist intermediate types of bond which are partially ionic and partially covalent in character.

Let us now consider some aspects of the atom structure. The wave function of an electron in a central field, i.e., in a spherical atom, can be written in the form

$$\psi_{nlm}(r, \theta, \varphi) = R_{nl}(r)Y_{lm}(\theta, \varphi), \quad (1.20)$$

where $R_{nl}(r)$ is the radial wave function, $Y_{lm}(\theta, \varphi)$ is the *spherical harmonic* function whose definition can be found in mathematical reference books, and r, θ, φ are the spherical coordinates of the electron. We present the mathematical expressions for the first several normalized spherical harmonics:

$$Y_{00}(\theta, \varphi) = \frac{1}{\sqrt{4\pi}}, \quad Y_{10}(\theta, \varphi) = i\sqrt{\frac{3}{4\pi}} \cos \theta,$$

$$Y_{1,\pm 1}(\theta, \varphi) = \mp i\sqrt{\frac{3}{4\pi}} e^{\pm i\varphi} \sin \theta,$$

$$Y_{20}(\theta, \varphi) = \sqrt{\frac{5}{16\pi}} (1 - 3 \cos^2 \theta),$$

$$Y_{2,\pm 1}(\theta, \varphi) = \pm \sqrt{\frac{15}{32\pi}} e^{\pm i\varphi} \sin 2\theta,$$

$$Y_{2,\pm 2}(\theta, \varphi) = -\sqrt{\frac{15}{32\pi}} e^{\pm 2i\varphi} \sin^2 \theta.$$

The spherical function $Y_{lm}(\theta, \varphi)$ does not depend on the form of interaction between the electron and the self-consistent field of the atom. All the information about this interaction is contained in the radial function $R_{nl}(r)$. The probability density of the electron being found in a state with the quantum numbers n, l, m in the point having the coordinates r, θ, φ is equal to

$$w_{nlm}(r, \theta, \varphi) = |\psi_{nlm}(r, \theta, \varphi)|^2 = |R_{nl}(r)|^2 |Y_{lm}(\theta, \varphi)|^2. \quad (1.21)$$

The squared modulus of the spherical function, $|Y_{lm}(\theta, \varphi)|^2$, is a standard mathematical expression that contains no information about interaction of the electron with the atom field. Unfortunately, in some textbooks this quantity is called the “electron cloud” and pupils learn the pictures

representing $|Y_{lm}(\theta, \varphi)|^2$. The notion of “electron cloud” is an unsuccessful attempt to build up a visual model of a microobject (the atomic electron).

It is conventional to apply the term “cloud” to a dense mass of small floating particles (e.g., ordinary clouds of atmospheric water vapor condensed high in the sky). For this reason, the notion of “electron cloud” for one electron in an atom has neither physical nor semantic meaning. We stress that the quantity $|Y_{lm}(\theta, \varphi)|^2$ is only a part of the probability (1.21), which contains no information about the interaction of the electron with another part of the atom. It is the same for all spherical atoms.

The state of an atomic electron with definite values of the quantum numbers n, l, m is not observable, because the electron energy cannot depend on the orbital moment projection m ; indeed, otherwise a rotation of the coordinate system could change the orbital moment projection value and, together with it, the electron energy. The observable electron states in an atom are those with definite values of n and l , whose energies are E_{nl} . The wave function of such a state is defined by the expression

$$\psi_{nl}(r, \theta, \varphi) = R_{nl}(r) \sum_{m=-l}^l c_m Y_{lm}(\theta, \varphi), \quad (1.22)$$

where the c_m are some constant (generally speaking, complex) coefficients. The quantity $|c_m|^2$ defines the probability of the electron being found in an unobservable state with the definite value of m .

Since the electron energy E_{nl} in the state n, l does not depend on the quantum number m , it is usually said that this state is *degenerate* in m . The multiplicity of this degeneracy is equal to $2l + 1$. Degeneracy of the state with the energy E_{nl} with respect to the electron spin projection σ is equal to 2 since σ can only take two values $\pm 1/2$. For this reason, the degeneracy multiplicity of the state with the energy E_{nl} is equal to $2(2l + 1)$. This is just the maximum number of equivalent electrons in the shell with given values of n and l , which are indistinguishable.

The probability density for the electron in an observable state with quantum numbers n, l and energy E_{nl} to be at a point with coordinates r, θ, φ is equal to

$$w_{nl}(r, \theta, \varphi) = |\psi_{nl}(r, \theta, \varphi)|^2 = |R_{nl}(r)|^2 \left| \sum_{m=-l}^l c_m Y_{lm}(\theta, \varphi) \right|^2. \quad (1.23)$$

The angular dependence of the probability $w_{nl}(r, \theta, \varphi)$ is the squared mod-

ulus of a superposition of spherical functions, but not of a single spherical function. The radial portion of probability $|R_{nl}(r)|^2$ contains all the information about interaction of the electron with the field of the atom and defines, in particular, the average distance between the electron and nucleus. If the quantity $r^2|R_{nl}(r)|^2$ has one maximum, then this state corresponds to a circular orbit, as a classical analog. If it has two or more maxima, then the classical analogs for these cases are elliptic orbits.

The electron energy in a hydrogen atom (1.7), as distinct from the electron energies in other atoms, depends only on the main quantum number n and not on the orbital moment l of the electron. Thus, an electron state in a hydrogen atom, having the energy E_{nl} , is degenerate in both m and l . For this reason, the wave function of an observable electron state in a hydrogen atom with energy E_{nl} has the form

$$\psi_n(r, \theta, \varphi) = \sum_{l=0}^{n-1} \sum_{m=-l}^l c_{lm} R_{nl}(r) Y_{lm}(\theta, \varphi), \quad (1.24)$$

where c_{lm} are some constant (generally speaking, complex) coefficients. The quantity $|c_{lm}|^2$ defines the probability that the electron is in an observable state with definite values of the quantum numbers l and m .

The probability that the electron in a hydrogen atom, having energy E_n , is at the point r, θ, φ is equal to

$$w_n(r, \theta, \varphi) = |\psi_n(r, \theta, \varphi)|^2 = \left| \sum_{l=0}^{n-1} \sum_{m=-l}^l c_{lm} R_{nl}(r) Y_{lm}(\theta, \varphi) \right|^2 \quad (1.25)$$

It is evident that, for the electron in a hydrogen atom which is in an observable state with the definite energy value E_n , it is impossible to separate a factor depending on the angles only. In other words, for a state of a hydrogen atom with the definite energy value E_n , the probability that the electron is located at the point r, θ, φ is not separable into radial and angular factors.

Thus an “electron cloud” is not a probability distribution describing the presence of an atomic electron at certain points in space for an observed state with definite energy, and does not correspond to modern concepts about atomic structure. This is simply an unsuccessful attempt to create a visual model of a microobject, which is impossible in principle. In serious textbooks on quantum mechanics, the notion of an “electron cloud” is, naturally, not used.

1.5 Structure of Matter

An explanation of the structure and various properties of solids, liquids, and gases turned out to be possible on the basis of quantum mechanics. Full consideration of these questions requires substantial analysis using complicated mathematical techniques that lie beyond the scope of this book. For this reason, we shall restrict ourselves only to the consideration of some important properties of the condensed media, which are connected with their microscopic structure.

Many solids are *crystals*. A crystal is a form of substance in which a regular arrangement of atoms having a three-dimensional periodicity is observed. A crystal structure is defined by the geometry of the crystal lattice in whose points atoms, ions, and molecules are situated. In biological crystals, the lattice points are occupied by proteins, nucleic acids, or even viruses. Other solids are *amorphous*. These are characterized by an isotropy in their properties, i.e., by an absence of any strict periodicity in the arrangement of their atoms, ions, molecules, or groups thereof.

Crystalline substances can be divided into three main groups with respect to their ability to conduct electric current: *metals* (conductors), *dielectrics*, and *semiconductors*. Metals have electrical conductivity in the range 10^8 – 10^6 $(\Omega\text{m})^{-1}$. The corresponding range for dielectrics is 10^{-8} – 10^{-10} $(\Omega\text{m})^{-1}$. Semiconductors occupy an intermediate position between metals and dielectrics in terms of conductivity. We should note that the properties of substances with respect to the conduction of electric current can be substantially modified by external conditions such as temperature and pressure.

Metals are widespread in Nature. It is enough to notice that under normal conditions 83 of the first 106 elements are metals. Metals exhibit a practically free conduction of electric current. Their conductivity decreases as temperature increases.

A metal is a crystal whose lattice points contain ions, because some atomic electrons (the valence electrons) break away from their atoms. Collectively, these *free electrons* can be considered as a gas within the metal. At the metal's boundary, there exists a potential barrier that prevents electrons from flying away from the metal.

If one applies a high-intensity electric field $\mathcal{E} \sim 10^8$ V/m to a metallic cathode, a *cold emission* current results. This is a purely quantum macroscopic phenomenon, which is connected with the sub-barrier passage of electrons known as the *tunnel effect* (Fig. 1.5). As a result, electrons

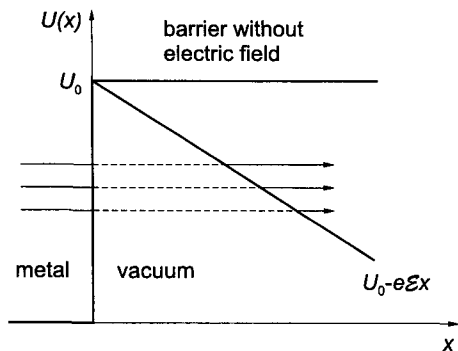


Fig. 1.5 The tunnel effect.

fly away from the metal and a current is observed. The dependence of this current $I(\mathcal{E})$ on the electric field intensity \mathcal{E} is defined by an expression that can be obtained only on the basis of quantum theory:

$$I(\mathcal{E}) = I_0 \exp(-\mathcal{E}_0/\mathcal{E}), \quad (1.26)$$

where I_0 and \mathcal{E}_0 are constants.

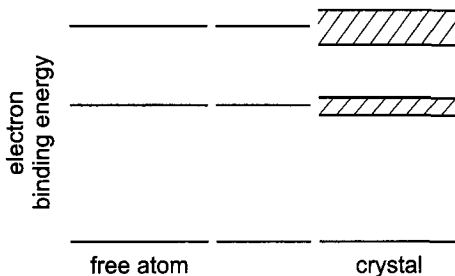


Fig. 1.6 Energy levels of electrons within a crystal.

Since electrons are fermions, they obey the Pauli principle. For this reason, electrons that have broken away from atoms and are practically free over the volume of the crystal occupy different energy levels (Fig. 1.6). Distinct from a free atom, in a crystal the permitted values of electron energies form bands as shown in Fig. 1.6. These bands are separated by *forbidden regions*. Each band contains a large number of energy levels that can be occupied by electrons. Widths of the bands increase as the electron binding energy decreases, i.e., the deepest atomic energy levels correspond

to very narrow bands in the crystal. Let us note that in Fig. 1.6 the simplest case is shown when the bands do not overlap. In reality, band overlapping is possible.

Now consider the mechanism of electric current flow through a crystal. In an unexcited crystal all free electrons tend to occupy the lowest permitted energy levels. By the Pauli principle, electrons occupy all levels up to some highest occupied level, whose energy is called the *Fermi energy* E_f . Fig. 1.7 shows two cases of the highest occupied level situation in a crystal. From this viewpoint only two bands are of interest: the upper one, which is the *conduction band*, and the lower one — the *valence band*. Energy levels in all the bands lying below are completely occupied, and those in all the higher-energy bands are empty.

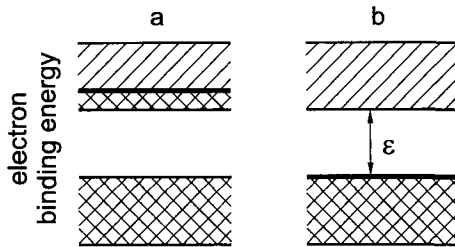


Fig. 1.7 The topmost energy level bands in a crystal.

If the lowest empty level is situated far from the upper edge of the band (Fig. 1.7a), then under the action of an electric field applied to the crystal, the electrons situated near the empty levels increase their energy and pass over to the nearby empty levels. Their movement creates an electric current in the crystal.

Such crystals are metals, and the electrons creating the electric current in them are called *conduction electrons*. Any metal is characterized by a certain electrical resistance. It is caused by scattering of the conduction electrons on thermal vibrations of the ions, forming the crystal lattice, and on structural inhomogeneities of the lattice (the impurity atoms and lattice defects). The resistance of a metal decreases with decreasing temperature and increases with increasing temperature.

If the band is completely occupied (Fig. 1.7b), then the first empty level lying in the conduction band is separated from the last occupied level by a forbidden energy band of width ϵ . The quantity ϵ is usually equal to several eV. For example, for diamond it equals $\epsilon = 5.5$ eV. In this case,

the voltage applied to the crystal cannot transfer electrons from the valence band to the conduction band, i.e., such a crystal is a dielectric and does not conduct electric current. Only a very high voltage (the material's *breakdown voltage*) can transfer electrons to the conduction band.

If a substance has a narrow forbidden region ($\varepsilon \lesssim 1$ eV), then thermal motion can cause a small number of electrons in the crystal to pass from valence band to conduction band. The positions left empty in the valence band are called *holes*. These behave like positively charged particles with a mass equal to the electron mass and with the charge $+e$. The number of electrons having sufficient energy for the transition from valence band to conduction band is proportional to $\exp(-\varepsilon/kT)$ for a wide range of temperatures T . It is obvious that the conductivity of the material will have a similar temperature dependence. Such substances are called *semiconductors*. Their electrical conductivity increases with temperature (in contrast to the behavior of metals as mentioned above), because of the increase in vibrational amplitude of the ions forming the crystal lattice. This mechanism for conductivity is characteristic of pure semiconductors. We should note that at sufficiently high temperatures some dielectrics become semiconductors.

In a semiconductor, electrons are constantly making the transition from valence band to conduction band. The reverse transition occurs as well; here a conduction electron and a corresponding hole disappear as mobile charge carriers. This process is called *recombination*. At any given temperature an equilibrium arises between the numbers of electron transitions to and from the conduction band. In other words, the number of electrons in the conduction band and the number of holes in the valence band is conserved at any given temperature. Note that in semiconductors, electrical conductivity is due to both the motion of negatively charged electrons and the motion of holes in the opposite direction.

Some substances, with regard to their electrical properties, occupy an intermediate position between metals and semiconductors. They have a comparatively small number of electrons in the conduction band even at very low temperatures, i.e., they can conduct only small currents. Such substances are called the *semimetals* (examples include Bi, Sb, and As).

For practical aims one usually employs not pure semiconductors but deliberately impure ones. They contain a small amount of admixtures which substantially change their physical properties. The impurities are of two types: *donors* and *acceptors*. A donor impurity will donate surplus electrons to a semiconductor and create electron conductivity — the result is

an *n-type semiconductor*. An acceptor impurity will capture some electrons from the valence band and create hole conductivity, resulting in a *p-type semiconductor*.

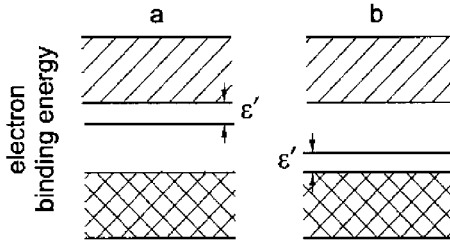


Fig. 1.8 Creation of conductivity in doped semiconductors.

Fig. 1.8 shows how conductivity is created in impure or doped semiconductors. In the donor semiconductor (Fig. 1.8a) the electron to be released is on the energy level of a doped atom situated close to the conduction band. For this reason, even at a low temperature, such an electron can pass to one of the lowest empty levels of the conductivity band. Typical examples of donor semiconductors are Ge and Si containing sparse numbers of P, As, or Sb atoms that have replaced atoms of the main element at some lattice points. The ionization energy of the doping atoms is very small: $\epsilon' \approx 0.01$ eV for germanium and 0.04 eV for silicon. For this reason, even at 77 K nearly all doping atoms are ionized, and in this case the number of conduction electrons is determined by the donor concentration.

In an acceptor semiconductor (Fig. 1.8b) the doping atom has an empty energy level close to the valence band. The capture of an electron from the valence band on this level of the acceptor leads to the rise of a hole in the valence band and to the creation of hole conductivity. Typical examples of acceptor semiconductors are Ge and Si containing sparse numbers of B, Al, or Ga atoms.

It is possible to grow a semiconductor (e.g., germanium) crystal consisting of two individually doped regions: one *n*-type and the other *p*-type. Such a *pn* crystal, if it has a narrow boundary region between the *n*- and *p*-regions, can be used as a rectifier (*diode*). If positive and negative voltages are applied to the *p*- and *n*-regions, respectively, electrons will pass from the *n*-region to the *p*-region and holes will move in the opposite direction. So a continuous current will flow through the crystal. For an applied voltage of opposite polarity, the current will be practically nil.

More complicated devices can be constructed from semiconductor crystals of the *npn*-type. Here a narrow *p*-type layer, a few hundred-thousandths of a meter wide, is placed between two *n*-type regions. If, for example, a positive voltage is applied to the left end of such a semiconductor and a negative voltage to its right end, then current flows. The left portion (*emitter*) is similar to a triode filament — it emits electrons — while the middle (*base*) and right (*collector*) portions are analogous to the triode grid and anode, respectively. This *npn*-semiconductor (*transistor*) can function as an amplifier, increasing the voltage and power of a signal delivered to its emitter. A *pnp*-semiconductor in which hole conduction takes place will operate analogously.

Let us consider the magnetic properties of solids. Those substances that can be magnetized in an external magnetic field are said to be *magnetic*. Inside a magnetic substance during magnetization, an intrinsic magnetic field arises. There exist weak-magnetic substances: *diamagnets* and *paramagnets*. Diamagnetic substances include He, Ar, Au, Zn, Cu, Ag, Hg, water, and glass. Paramagnetic substances include O, Al, Pt, the alkali metals, the alkaline-earth metals, and the rare-earth elements.

In the absence of an external magnetic field, the atoms and molecules of diamagnets do not possess any intrinsic magnetic moments. If a diamagnet is placed in a magnetic field, then in each atom or molecule an additional current is introduced which creates a magnetic moment. According to Lenz's law, the current induced by an external magnetic field is always directed in such a way that it decreases the field inside the substance. For this reason, the induced magnetic moments are aligned opposite the direction of the external magnetic field, and the diamagnet is magnetized. When the external magnetic field is removed, the diamagnet is demagnetized.

The atoms and molecules of a paramagnet have some intrinsic magnetic moments in the absence of an external magnetic field. In this case, however, random thermal motion leads to a chaotic orientation of their magnetic moments; i.e., the paramagnet does not possess an intrinsic magnetic field when an external magnetic field is absent. In an external magnetic field, the magnetic moments of atoms and molecules align along the field, and the paramagnet is magnetized.

As already noted, when an external magnetic field is absent, the magnetic moments of atoms and molecules in a paramagnet are oriented chaotically. However, there exist substances which have an ordered structure in absence of an external magnetic field (Fig. 1.9). If in such a structure the mean magnetic moments of atoms are oriented in the same direction (Fig.

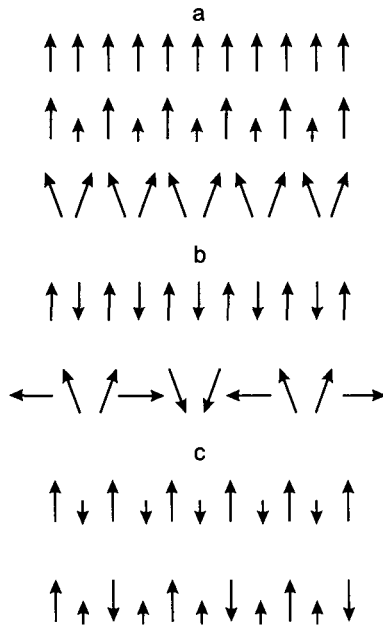


Fig. 1.9 Orientation of magnetic moments in structured materials.

1.9a) so that a macroscopic volume magnetization arises, then the material is said to be *ferromagnetic*. But if the ordering is such that the overall magnetic moment equals zero (the magnetic moments of neighboring atoms are directed oppositely) and the macroscopic magnetization is absent, it is said to be *antiferromagnetic* (Fig. 1.9b). Finally, if the magnetic moments can have the opposite direction of ordering (Fig. 1.9c) but the magnetization is different from zero, the material is *ferrimagnetic*. The critical temperature above which the magnetic ordering disappears is called the *Curie temperature* or *Curie point* T_c in the ferro- and ferrimagnetic cases, and the *Néel temperature* or *Néel point* T_n in the antiferromagnetic case (after the French physicist Néel who, independently of Landau, explained the phenomenon of antiferromagnetism).

Ferrimagnetism can be considered as the most general case of magnetic ordering. From this point of view, ferromagnetism and antiferromagnetism are particular cases of ferrimagnetism. Examples of ferromagnetics are Fe, Co, and Ni; antiferromagnetics include MgO, FeO, and NiO, while ferrimagnetics include Fe_3O_4 , CoFe_2O_4 , and NiFe_2O_4 . Note that the critical temperature values for various magnetic substances lie in a very wide range.

For instance, the ferromagnetics Dy, Ni, and Fe have Curie temperatures of 37 K, 627 K, and 1043 K, respectively. Above the critical temperature, ferromagnetics and antiferromagnetics become paramagnets.

At temperatures $T < T_c$, a ferromagnetic material consists of *domains*. A domain is a small volume of substance having linear sizes on the order of 10^{-4} – 10^{-5} m, inside which all magnetic moments of atoms are co-oriented, i.e., the domain has maximum possible magnetization. In the absence of an external magnetic field, the magnetic moments of domains inside a ferromagnetic sample are oriented chaotically so that the overall magnetic moment of sample is zero. Under the action of an external magnetic field, however, the magnetic moments of the domains align and the sample as a whole becomes magnetized. When the magnetic field is removed, some of the magnetic moments remain in alignment. For this reason, a *residual magnetism* can arise and permanent magnets can be created.

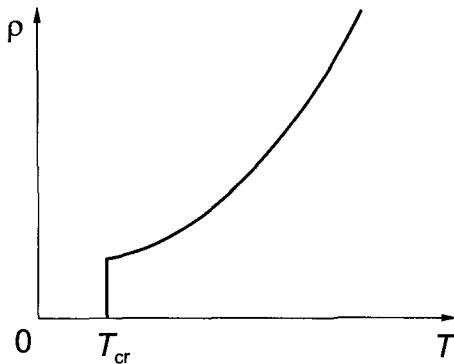


Fig. 1.10 Superconductivity in mercury.

Now let us consider phenomena that occur at very low temperatures. These are the *superconductivity* of some metals (e.g., Pb, Ta, Sn, Al, and Nb) and alloys (e.g., Nb_3Sn), and the *superfluidity* of liquid helium. Superconductivity was discovered by the Dutch physicist Kamerlingh-Onnes in 1911, when he investigated the electrical resistance of mercury at low temperatures. It turned out that at the critical temperature $T_{cr} = 4.15$ K, the resistance of mercury fell abruptly to zero (Fig. 1.10). Further, superconductivity of other substances was discovered. Each of them has its own critical temperature T_{cr} for transition into the superconducting state. For his investigation of the properties of matter at low temperatures, which led to the production of liquid helium, Kamerlingh-Onnes was awarded the

Nobel Prize in Physics for 1913.

If an electric current is induced in a superconducting metal ring, it will circulate in this ring without any damping, because its flow through the superconductor is not accompanied by any heat evolution. This property of superconductors is used for the construction of superconducting magnets and other devices. It has also been established that a weak magnetic field does not penetrate a superconductor. The expulsion of magnetic field from a superconductor (the *Meisner effect*) means that in an external magnetic field a superconductor is a perfect diamagnet.

The nature of superconductivity was explained by the American physicists Bardeen, Cooper, and Schrieffer in 1957. They were awarded the Nobel Prize in Physics for 1972 as a result. According to their theory, two conduction electrons in the crystal lattice of a metal create a bound state, the *Cooper pair*, which has zero total moment (spin). Let us consider the mechanism of creation of the Cooper pair in more detail.

Two electrons experience Coulomb repulsion. However, an electron can excite or absorb a quantum of the lattice vibrations, which is called a *phonon*. If an electron excites a phonon and another one absorbs it, then such an exchange of a phonon leads to an attraction arising between the electrons; moreover, the attractive force in a superconductor turns out to be greater than the force of Coulomb repulsion. It is just this attractive force, arising from the presence of the crystal lattice, that leads to the pairing of electrons. Such a bound electron pair behaves like a particle. For this reason it is called a *quasiparticle*, which is a boson. At temperatures close to absolute zero, these quasiparticles experience Bose condensation. The Bose condensate, consisting of the electron pairs, moves freely (experiencing no resistance) through the crystal, which leads to superconductivity.

The states of electrons in a superconductor are continuously changing and, for this reason, the compositions of electron pairs are continuously changing as well. Since the linear size of an electron pair is on the order of 10^{-6} m, this is an example of the long-range coupling of particles.

At $T = 0$ K, all conduction electrons are paired. If $T \neq 0$, the probability of breaking an electron pair differs from zero. For this reason, at $T \neq 0$ the unpaired electrons form a normal electron "liquid" in the crystal, and the paired ones form a superconducting "liquid". Above $T = T_{cr}$ the quasiparticles (Cooper pairs) become completely broken, and superconductivity disappears.

Superfluidity is observed in liquid helium, which leaks through very narrow channels and capillaries without friction. This phenomenon was

discovered by the Soviet physicist Kapitsa in 1938. He established that at temperatures below $T_\lambda = 2.17$ K (the *lambda point*), liquid ^4He becomes superfluid. Kapitsa received the Nobel Prize in Physics for 1978 for this discovery and for his fundamental research on low temperature physics.

For temperatures $T > T_\lambda$, liquid ^4He is called He I; for $T < T_\lambda$, it is called He II. At the temperature $T = T_\lambda$ in liquid ^4He , a phase transition of the second kind occurs, i.e., in this case its internal structure abruptly changes. Note that phase transitions of the second kind are undergone by ferromagnetics and antiferromagnetics at the Curie point, as well as by superconductors at the critical point.

An explanation for superfluidity of liquid He II was given by Landau in 1941. According to his theory, there are two types of elementary excitations (quasiparticles) in He II: phonons and rotons. At finite temperatures, a part of He II behaves like a normal viscous liquid and another part behaves like a superfluid which possesses no viscosity. There is no friction when these liquids move through each other. The Soviet physicist Landau was awarded the Nobel Prize in Physics for 1962 for his pioneering theories regarding condensed matter, especially liquid helium.

Consideration of the normal and superfluid components in He II is a convenient way to describe phenomena occurring in a quantum Bose liquid. This does not mean that the liquid is actually separated into two fractions. In reality, in He II two types of motion can occur simultaneously: one of them identical to a viscous liquid, and the other corresponding to a superfluid liquid. Both types of motion occur without transferring momentum from one to the other.

In 1972–1974, superfluidity of ^3He was discovered, which is observed at temperatures of several mK. In this case two atoms of ^3He , which are fermions, create a pair which is a quasiparticle (boson) and the Bose condensation of these quasiparticles occurs, which leads to superfluidity of liquid ^3He .

Finally, we will note that quantum mechanics has made it possible to ascertain the physical laws of various processes and phenomena observed in condensed media, which in principle cannot be explained on the basis of classical physics. Studies of the microscopic structure of solids allowed people to create devices and mechanisms widely used in modern technology.