PREFACE

This volume concerns two main topics of interest in the theory of automorphic representations, both are by now classical. The first concerns the question of classification of the automorphic representations of a group, connected and reductive over a number field F. We consider here the classical example of the projective symplectic group PGSp(2) of similitudes. It is related to Siegel modular forms in the analytic language. We reduce this question to that for the projective general linear group PGL(4) by means of the theory of liftings with respect to the dual group homomorphism $\mathrm{Sp}(2,\mathbb{C}) \hookrightarrow \mathrm{SL}(4,\mathbb{C})$. To describe this classification we introduce the notion of packets and quasi-packets of representations – admissible and automorphic – of PGSp(2). The lifting implies a rigidity theorem for packets and multiplicity one theorem for the discrete spectrum of PGSp(2). The classification uses the theory of endoscopy, and twisted endoscopy. This leads to a notion of stable and unstable packets of automorphic forms. The stable ones are those which do not come from a proper endoscopic group.

This first topic was developed in part to access the second topic of these notes, which is the decomposition of the étale cohomology with compact supports of the Shimura variety associated with PGSp(2), over an algebraic closure $\overline{F}$, with coefficients in a local system. This is a Hecke-Galois bi-module, and its decomposition into irreducibles associates to each geometric (cohomological components at infinity) automorphic representation (we show they all appear in the cohomology) a Galois representation. They are related at almost all places as the Hecke eigenvalues are the Frobenius eigenvalues, up to a shift. In the stable case we obtain Galois representations of dimension $4^{[F:\mathbb{Q}]}$. In the unstable case the dimension is half that, since endoscopy shows up. The statement, and the definition of stability, is based on the classification and lifting results of the first, main, part. The description of the Zeta function of the Shimura variety, also with coefficients in the local system, follows formally from the decomposition of the cohomology.

The third part – which is written for non-experts in representation theory – consists of a brief introduction to the Principle of Functoriality in the theory of automorphic forms. It puts the first two parts in perspective. Parts 1 and 2 are examples of the general – mainly conjectural – theory described in this last part. Part 3 can be read independently of parts 1 and 2. It can be consulted as needed. It contains many of the definitions used in parts 1 and 2, but is not a prerequisite to them. For this reason this Background part is put at the back and not at the fore. Regrettably, it does not discuss the trace formula. But this would require another book. Part 3 is based on a graduate course at Ohio State in Autumn 2003.

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