

Preface

At one time I worked on the first heart sound produced by the left ventricular. The existence of a high frequency component was one of the predictions of the new theory, and I was very interested to get an experimental verification of this prediction. It was a reason for my talk for the group of cardiologists in one of the hospitals. I started my talk by saying: “Let us consider the left ventricular as a sphere or cylinder...”. At that moment I lost my audience who knew (as I did) that the left ventricular has neither the form of a sphere nor that of a cylinder. On the other hand, I knew how to build the model for the simplest geometry of sphere or cylinder. When an average physicist comes up against a new problem, he immediately starts to apply to this problem all the methods and ideas he knew from his previous experience¹.

The first step is the choice of some simplified model (geometric form of the left ventricular in my case). The simplest, the most general and the most used model is that of a harmonic oscillator. This model is used for the description of different phenomena in mechanics, optics, acoustics, electronics, engineering, etc. [1]. In fact, it has been applied everywhere, from quarks to cosmology. Moreover, a person who is worried by oscillations of prices in the stock market can be relaxed by classical music produced by the oscillations of string instruments.

¹Of course, the great physicists do not come under these headings - recall A. Einstein with general relativity, P. Dirac with relativistic quantum mechanics, or L. Onsager with the two-dimensional Ising problem. In fact, they created not only the new physics, but also the new mathematics

Wandering about the Internet one can find many curious facts. Although the ancient Greeks already had a general idea of oscillations and used them in musical instruments, the first practical application of an oscillating system took place in 1602 by a physician in Venice named Santorio who had heard from his great friend Galileo about the general laws of the oscillations of a pendulum. Mr. Santorio called this system “pulsilogium” and used it to measure the pulse of his patients. Many other applications have been found in the last 400 years...

Two main features characterize this book. Firstly, the book contains a comprehensive description of all “oscillator-like” stochastic differential equations which were studied until 2004, and it can serve as a starting point for researchers and engineers who meet these equations in their work. The second characteristic feature of this book is its simplicity and small volume.

Dozens of existing books and many hundreds of articles create a serious problem that confronts the author of a new book on stochastic differential equation. The problem consists of a difficult decision about restricting the consideration to some specific problem. My decision was to concentrate on a single stochastic one-dimensional classical oscillator, omitting thereby the quantum problem, interactions of oscillators and higher dimensions as well as the fascinating problem of deterministic chaos.

I decided not only to avoid all rigorous mathematical proofs and statements, but also to drop the large body of applications and the traditional comprehensive introduction to the mathematical theory of random processes. All necessary explanations are given in the appropriate sections of the book. If the material presented demands complicated calculations, I covered only the qualitative results referring for details to the original articles. Only the general knowledge of mathematical physics is required of the reader.

This book is devoted to “noisy” equations, i.e., equations which contain random forces, although for the reader’s convenience, chapter 1 contains a short review of the deterministic equations and the types of the second order (underdamped) differential equations involving additive and/or multiplicative noise which are considered,

along with their simplified versions (first order differential equation) in great detail in this book. Chapter 2, which is devoted to the short general descriptions of noise, is required for understanding the ensuing material. In chapter 3 we consider the Brownian motion, thereby paying respect to Einstein [2], Smoluchowski [3] and Langevin [4] who were the first to introduce a random force in deterministic equations for velocity and for coordinate. The following chapters contain detailed analyses of the overdamped (chapters 4 – 7) and underdamped (chapters 8 – 12) noisy oscillators.