1: Why Quantum Fluctuations of Spacetime?

The primary function of physics is to understand the universe according to a single set of principles that can be stated and formally manipulated in a consistent manner. The history of physics illustrates grand syntheses in theoretical physics from Copernicus to Einstein. On the theoretical front there are cases of individuals who performed grand intellectual feats, such as Newton, Maxwell and Einstein, as well as other instances where the unification of physical reality is more of a collaborative effort, such as with quantum mechanics. It is also important to keep in mind those crucial experimental results which played a vital role as well, from Galileo' s observation of Jovian moons to the Michelson-Morely experiment. The most important experiments are those that counter the conventional thinking of the day.

For quantum gravity to become serious physics, as opposed to abstract mathematics and theory, experimental tests must be performed. These measurements may involve gravity waves or neutrinos detected from the early universe, subtle effects in particle physics or sensitive tests of quantum gravity fluctuations. If the future presents the physics community with no experimental tests of theories, or with the futility in attempting to make such observations, then the topics of quantum gravity and the very early cosmology are not properly topics of physics, but are arenas of mathematics with interpretations that border on theology. Only with experimentation will quantum gravity be developed as an arena of physics in manner similar to the history of the founding of quantum mechanics.

This problem lies with the nature of the Planck length. It is sometimes illustrative to go back and see where this fundamental scale comes from. For a black hole with a mass M its Schwarzshild radius is known to be $r = 2GM/c^2$. We ignore the issue of rotating black holes. An obvious question is what is a scale where a black hole has a deBroglie wave length $\lambda = h/p$, p the momentum of the black hole, equal to its diameter defined by the Schwarschild radius. This is a spacetime momentum that is $p = E/c = Mc$. Further, the mass of the black hole is equated to $M = h/c\lambda$. The analyst who puts this all together then finds that the deBroglie wavelength of this black hole is,

$$\lambda_P = \sqrt{\frac{Gh}{c^3}} = 4.051 \times 10^{-33} cm. \tag{1.1}$$

This is generalized for a two dimensional wave on the event horizon to obtain the Planck length $L_p = \sqrt{G\hbar/c^3} = .1616 \times 10^{-33}$cm. If one converts this length into energy units, since energy is based on $E = hc/\lambda$, the energy associated with a wave quanta on this scale is 10^{19}GeV. As a comparison the Fermi length for a nucleon is 10^{-13}cm, so the energies required to probe these lengths are 20 orders of magnitude larger that what is used in nuclear physics.

From an experimental physics viewpoint this is terrible news. The current accelerators push particles to $10^3 GeV$. This means that based on current technology with accelerators that have diameters of several kilometers, such machines must be scaled up by a factor of 10^{16}. A light year distance is approximately $10^{13} km$, which means that an accelerator would have to have the diameter of some 10^3 light years. Such an accelerator would encompass a significant portion of our Milky Way galaxy. So this is indeed terrible news. One might switch to cosmic ray physics, yet the highest energy recorded so far is some is 10^9GeV. The author has addressed the question of whether an implosive shock wave from a nuclear explosion might produce Planck scale energy at its core. This appears more realistic by some measures. However, estimates are not encouraging. Under highly idealized conditions it would require a nuclear explosive of some 10^5 megatons, more than the current nuclear explosive capacity currently arrayed in the world. Rayleigh-Taylor instabilities of shock wave fronts bring about serious questions about the feasibility of this. This means such a device would likely have explosive power several magnitudes beyond this to achieve a shock wave front that could implode material into a quantum black hole. Based on the accrued costs of nuclear weapons research $\simeq$ $\$10^{13}$ by both the United States and Russia, it is unlikely that a "Planck bomb" will be built, where such would likely be politically unpopular. It is left to the reader familiar with nuclear explosives to calculate an estimated energy required to produce a quantum black hole whose energy output is greater than the explosive energy necessary to produce it.

So the question must be asked on how quantum gravity can experimentally explored. At the same time, one might ponder how well quantum gravity is theoretically understood. Einstein stated that nature was subtle, but not malicious. It might well be that our ideas about quantization are not appropriate for the subject of quantum gravity. The above argument for the Planck scale is based upon rather elementary concepts of quantum waves and their relationship with the area of a black hole event horizon. The viewpoint rests upon the idea that gravitation is quantized field. It may be best to assume the point of view that gravitation is a field that is unified with quantum mechanics into a larger field theory. If this is so quantum mechanics and gravitation are low energy results from a more general symmetry at very high energy.

Canonical spacetime operators may be derived which satisfy a finite temperature theory. Here the temperature is determined by the gravity associated with a quantum fluctuation. Further these fluctuations exhibit gravitational self-squeezing. A self-parametrically down shifted or squeezed vacuum state may then determine the tunneling probability vacuum modes may tunnel into a nascent cosmology. This is then developed further to illustrate how this fits into a conformal field theory, where this may be derived by coupled fluctuations of a string with a two dimensional membrane on a black hole membrane.

This may demonstrate how the Planck scale is a fundamental cut off in measurable physics. Any wavelength shorter than the Planck scale will be concealed behind the event horizon of a Planck mass black hole. So physics with energy greater than the Planck energy is not measurable or observable. This argument rests upon the assumption the gravitation coupling parameter G, Newton's gravitational constant, is truly constant. It also assumes that the Planck unit of action is truly constant. The Planck and gravitational constants are amongst the smallest of physical constants. They also point to a curious difference between quantum gravity and other quantized fields. Gauge theories fields are quantized with their coupling parameters held constant, where renormalization of that constant is a derived result. With quantum gravity there exist two limits: one where $\hbar \rightarrow 0$ and the other where $G \rightarrow 0$. These two limits recover classical gravitation and quantum mechanics respectively. Quantum gravity may well be a domain of physics where both of these constants may be renormalized to larger values for interaction energies less than the Planck energy scale. If both of them are renormalized to larger values as the interaction energy increases this would mean that the Planck length is in reality larger than what is currently thought.

Andre Sakharov suggested a foundation of gravitation as a metric elasticity [1]. This metric elasticity was based upon a microscopic structure analogous to molecular structure behind material elasticity. Wheeler proposed pregeometry as more fundamental to physics than geometry. In this viewpoint the gravitational coupling parameter is seen to emerge from a grand sum over many pregeometric entities which have oscillation modes. A renormalization of physics view may then be invoked so very high frequency oscillations are removed from this contribution, at least for physics at low energy. This suggests the foundations of gravitation lay in the quantum physics of the vacuum.

It is then reasonable to propose a Lagrangian that describes the zero-point energy of the geometrodynamic vacuum. This indicates a Lagrangian as a power series in spacetime curvature

$$\alpha\hbar\int k^3dk ~+~ \beta\hbar(\int kdk)R^{(4)} ~+~ \hbar\int\frac{dk}{k}(\gamma R^{(4)^2} ~+~ \delta R^{\mu\nu}R_{\mu\nu}) ~+~ O(R^{(4)^3}). \qquad (1.2)$$

Here α, β, γ, δ are coupling coefficients of order unity and k are the modes of the pregeometric entities. The corrections to Einstein's general relativity advanced by Sakharov are a reflection of departures at high energy due to the breakdown of spacetime. The first term is expected to be removed by renormalization arguments. From the second term it is seen that $c^3/16\pi G ~=~ -\hbar\beta\int kdk$. This leaves open the question of whether the speed of light or the gravitational constant is determined by this factor. Since the speed of light is an aspect of the metric signature of spacetime $(-1,~1,~1,~1)$ this might be assumed to be the ultimate invariant in nature.

This introduction is written as the question, "why quantum fluctuations of spacetime?" This question is ultimately at the heart of problems that needs to be addressed in the future. From a theoretical perspective the physical vacuum of the standard model is $\sim$ 246GeV, while the cosmological constant suggests that it should be no more than about $10^{-44}GeV^4$. This is an incredible observational conflict of some 120 orders of magnitude. The standard model of electroweak interactions has a good track record of experimental tests and astronomy has produced decent constraints on the cosmological constant. So these two observational results lead to a theoretical contradiction. The second reason for the question is that quantum fluctuations may well be experimentally testable with sufficiently sensitive measurements. It is possible to detect metric fluctuation to verify their existence, where later the nature of quantum gravity fluctuations may be probed to obtain data that may help resolve this issue by supporting or refuting theories of quantum gravity.

The approach of this monograph is to consider the nature of quantum gravity, more specifically quantum fluctuations in spacetime, and in a context that holds some prospect for experimental tests or astronomical observations. This is also written with the expectation that our views on the foundations of the universe are likely to change radically over the first two decades of the 21^{st} century. This may come from the LIGO observation of the gravity wave universe, as well as possible experimental detection of quantum gravity fluctuations. This will further mean that many of our ideas about quantum gravity, strings and unification will be radically changed. As such this monograph is directed significantly towards the theory and phenomenology of spacetime fluctuations and possible experimental measurements. This is done in order to provide a practical direction for research in the future, research that holds the prospect of undoing much of what is canonized as theoretically tentative.

John Wheeler stated there are three levels of gravitational collapse: black holes, cosmological big bang and quantum gravity fluctuations. In each of these cases there exist event horizons. Event horizons are themselves not detectable, nor is it a surface that matter impacts, but the physics near them is. In the case of black holes the gravitational field removed from the event horizon is such that its influence on incoming matter is detected. black holes has been largely detected, where they further appear to be rather astrophysical aspect of the universe. black hole event horizons also have the curious property of freezing matter just above them. Another event horizon is very easy to "see,"but still difficult to recognize. This is the night sky, which outside the light of stars, exhibits a blackness of a horizon. This is a result of the cosmological horizon due to the big bang. The red shifting of light emitted by galaxies and quasars and radiation released during the transition to the current matter dominated period, as seen with the microwave background, accumulate near this horizon. The quantum gravity fields that decoupled from other fields around $t > 10^{-40}$sec into the big bang should exist as gravity waves. These have not yet been detected.

Event horizons are two dimensional achronal surfaces. The achronal nature of event horizons means that observable fields are time dilated and length contracted as they accumulate near them. Recognition of this has lead to the holographic principle. The curvature of spacetime in the environment of an event horizon and this accumulation of fields near them leads to the striking prospect that all fields may be described according to surfaces of two dimensions that accumulate near event horizons. This means that the data for fields and gravity need only to be specified in two dimensions rather than within three. From a computational viewpoint, in particular if a problem is being run on a computer, this is a great saving in the number of calculations required. Further, it suggests a new paradigm in spacetime physics where the most fundamental object of dynamics is the surface within a Planck unit of distance from an event horizon. The field theoretic content of this membrane is purely gravitational, as all other fields with wave lengths larger than the Planck length have been red shifted to near infinity. As such this surface in two dimensions is regarded as a quantum membrane. It is a D-brane under a spacetime target map and interacts with strings according to $U(n)$ Chan-Paton factors.

This monograph is in part concerned with D-branes of two dimensions and other D-branes of other dimensions correlated with D2-branes by duality principles. Higher dimensional D-branes will be discussed, but not as extensively developed. The initial thrust here will be quantum gravity in a weak coupling limit, where the stronger coupling limit is considered later. If aspects of quantum gravity are ever to be experimentally explored it will certainly be this domain that is tested first. In the case of the universe future observations of gravity waves produced in the big bang will reveal structure very close to the initial event. The gravity waves will be due to the tunneling of the universe out of the vacuum or from the decoupling of gauge fields from quantum gravity. If Light Interferometric Gravitational Observatories (LIGO) are able to probe this aspect of the universe then measurements of quantum gravity, at least in the weak limit, are possible. In this case future astronomers may make observations of the affect of a D-brane in two dimensions.

Before engaging in a discussion of spacetime quantum fluctuations it is best to consider quantum fluctuations in nonrelativistic quantum mechanics. This will lay the foundation for discussions of spacetime fluctuations. The approach that is taken is to use the Bohm approach to quantum mechanics to derive stochastic fluctuations. This is done not to promote any "interpretation" of quantum mechanics or to advocate for hidden variables, but since this formalism is a convenient approach to initiate a discussion of quantum fluctuations. In this presentation it will also be indicated that this viewpoint on quantum mechanics is fundamentally no different from the standard approach. The derivation of a path integral is given as a demonstration that this "particle-pilot wave" approach to quantum mechanics is not fundamentally different from the more standard Hilbert space and operator approach.

A fluctuation of the spatial three dimensional manifold $\Sigma^{(3)}$ will accelerate a test mass. If the particle horizon is a length L^2/L_p from the test mass this acceleration will be on the order of c^2L_p/L^2, which can become quite large. It is then apparent that this mass under this acceleration will have a particle horizon within $c^2/a \simeq L^2/L_p$ of the particle. This accelerated particle will couple to this fluctuation, with a corresponding event horizon, which has a finite temperature. From a physical viewpoint it makes sense that a particle under any acceleration, constant or not, will interact with a thermal vacuum. The vacuum such a particle interacts with will have a finite temperature. Whether the zero point energy for quantum gravity is finite temperature up to the Planck length is a question to be addressed later. Yet apparently the quantum gravity vacuum for scales that approach the Planck length will be finite temperature. This temperature will then be associated with the event horizon.

The Unruh effect demonstrates now quantum theory does not have a one to one correlation with a state in Hilbert space and a particle. The existence of a particle is not completely correlated with a Hamiltonian of the form $H = \hbar\omega a^\dagger a$. A vacuum according to an accelerated observer contains particles in a thermal distribution. Such a vacuum will have a Hamiltonian with terms $a^\dagger a^\dagger$ and aa, where this additional term to the Hamiltonian is essentially a squeezed state operator.

These properties are known with quantum fields in curved spacetime. An open question is whether these properties of the quantum gravity vacuum apply to gravitation itself. Intuitively it makes sense that it should, at least in the weak limit. The one major difference is that the vacuum in this case interacts with itself. This presents some problems, but some preliminary results are developed here. This self-interaction results in a generalized uncertainty principle for quantum gravity. A most interesting find from this generalized uncertainty principle is that the quantum gravity vacuum acts as its own parametric amplifier. The vacuum is then parametrically down shifted and that the Planck length is shifted to a larger value. The Planck energy is down shifted to energies approaching $\sim 10^{16}$GeV. This puts quantum gravity at the same energy as GUT unification energy. This also raises the experimental prospect that particles accelerated by parametrically amplified coherent states of photons may interact by entanglement with the quantum gravity vacuum squeezed to high values. The experimental prospect then exists to detect departures from standard quantum uncertainty principle by effectively renormalizing $L_p = 1.6 \times 10^{-33}$cm to larger values. Further, this self-squeezing of the quantum gravity vacuum is also a mechanism that may permit virtual cosmologies to tunnel out of the vacuum state.

For spacetime quantum fluctuations associated with event horizons the relevant physical fields involved with this fluctuation are those tied to the event horizon. These fields then determine a virtual two dimensional membrane. These membranes are then identified with virtual black holes, Cauchy horizons and Lanzcos junctions with wormholes. In the case of a virtual cosmology they are identified with cosmological horizons. These are the three levels of collapse indicated by Wheeler. As these two dimensional D-branes are due to the holographic principle and they capture the three levels of collapse, it is then advanced that they codify the dynamics of quantum gravity in the weak limit.

The quantum gravity fluctuation is developed with teleparallel connections and Finsler geometry. Finsler geometry, with its Lagrangian-like structure, is a natural way to develop this theory of fluctuations. Teleparallel connections define paths on a manifold that are not geodesic in nature, such as the lines of latitude on a globe. Physically these describe accelerated motion, which provides a framework for the description of quantum gravity fluctuations. From here the physical basis for quantum gravity fluctuations is developed, where various implications for the physical world are discussed. In particular possible implications for GUT theories and quark physics are discussed.

If quantum fluctuations of spacetime exist they should be measurable. The dimension of an atom is such that quantum gravity fluctuations should be measurable with sensitive enough technology. For a region with length scale L the curvature induced by quantum fluctuations is [2]

$$\Delta R \sim \frac{L_p}{L^3}, \tag{1.3}$$

where $L_p = \sqrt{G\hbar/c^3} \sim 1.6 \times 10^{-33} cm$. For the region contained in an atom, $L \simeq 10^{-8} cm$, these curvature fluctuations will be on the order of $\Delta R \simeq 10^{-9} cm^{-2}$. This appears small, but when compared to the curvature near the Earth $G\rho/c^2 \simeq 4 \times 10^{-28} cm^{-2}$ it is in fact quite large. An atomic physics experiment should detect spacetime fluctuations. The comparison of the gravity force due to metric fluctuations with electrostatic forces in an atom is quite small with a ratio of $F_{fluc}/F_e \simeq 10^{-21}$. So the effect is subtle, but potentially within the bounds of a possible measurement.

Of all the fields that exist in the universe the best understood, which has spawned much technology as well, is the electromagnetic field. Further, the interaction of photons and atoms is one of the most highly developed arenas of quantum physics. It is then reasonable to consider this as a potential probe of quantum gravity fluctuations. The physics required to detect quantum spacetime fluctuations is an atom-photon interaction laser. A theoretical basis for this is developed in the theory of coherent states. The matters of stochastic processes from quantum and thermal noise are addressed. It is found that the technology is quite possible and that this experiment could be performed.

With very high powered lasers an electron may be accelerated up to $a = 10^{22} cm/sec^2$ or beyond. This acceleration may then lead to a parametric down shifting of the quantum state of the electron. This may be further "primed" into action by the employment of squeezed laser states, which enter into an entanglement with the electron. It is possible a greater uncertainty in state measurement may be detected. If this is the case with higher accelerations this process may be extended to measure these departures further. In effect the Planck length and equivalently the Planck constant are renormalized so that $\hbar \rightarrow \hbar^*$ with $\hbar^* > \hbar$. The pursuit of such experiments and long term improvements in high powered lasers hold the prospect that the Planck length could be renormalized to 10^{-13}cm. This would in the long run result in the ultimate source of energy where 100% of matter can be converted to energy. In effect a virtual black hole could be renormalized to the scale of a nucleus and nuclei could be converted into positrons and photons. If this energy technology does emerge in the future it will provide both abundant energy, as well as a way of completely disposing of nuclear wastes from the nuclear fission age of the 21^{st} century.

The nature of quantum fluctuations and virtual black holes is examined in the light of the horizon, or a surface one Planck length from the horizon, as a D2-brane under a target map. This construction is tied to the horizon algebra for the black hole and its supersymmetric extension. From here the topology of the virtual black holes is examined. The moduli space for the horizon algebra is found where the singularities in the moduli space correspond to the horizon. In a pseudoEuclidean metric the nature of the cones in projective space is addressed where a Virasoro sector is derived with weighted projective spaces. Here the weighting of projective lines $z_i \rightarrow z_i^n$ results in a periodic structure from which a Virasoro algebra emerges. These singularities suggest structure beyond the string scale, where an internal structure to the string emerges from the blow up of a point. A gauge theory with a quaternionic Polyakov action is seen to emerge from this construction. This leads to the prospect of a theory beyond string theory at high energy and at the Hagedorn temperature.

This squeezed state renormalization suggests high energy physics at the TeV scale might be impacted by the nature of quantum fluctuations and virtual black holes. The compactification of space on the horizon leads to the existence of gauge connections threaded within wrapped dimensions pinned to the horizon. This leads to the possibility the Higgs field may be related to quantum black holes. The Higgs field may have a quantum entanglement with a quantum black hole, where there is a small probability that the Higgs field may act as a black hole that violates baryon numbers. Under the renormalization of the Planck scale at high energy the possibility exists that black holes could be produced in the laboratory. The major departure with such similar schemes is that ordinarily the black hole exists on a much higher energy scale than the Higgs, but under circumstances where the experiment can be conducted with the parametric amplification of states the Planck scale renormalization can bring the black hole down the TeV scale. This brings forth large extra dimensions at experimentally testable scales.

This then is the start of a more formal investigation into the nature of the quantum gravity vacuum. A new uncertainty principle, partly indicated prior to this point, is found. This vacuum is described by a multilinear bracket structure outlined by Peano. This vacuum is described by an orthomodular lattice that puts relativity and quantum theory on the same footing. Gravitation and quantum mechanics are then aspects of the same theory. With this uncertainty principle it is argued that the divergences in quantum gravity and the strong coupling limit in string theory may be absorbed into the uncertainty. Such divergences are the energy cut-off in emergent one-loop quantum gravity, where in the vacuum these divergences may be absorbed.

The octonions form the most accessible multilinear bracket structure as a nonassociative algebra. This extension of field theory as a model for quantum gravity is argued for on physical grounds. The mathematical structure of this theory also indicates that nature may be fundamentally unitary. This will preserve quantum information in black holes. The octonions have connections to E_8 lattice codes where quantum information is ultimately processed by the quantum gravity "computer" in a way the preserves quantum information. This further has connections to string theory and the adS/CFT correspondence. Octonionic field theory when topologically gauged also naturally leads to a supersymmetric form of gauge fields. The fundamental discrete structure for this theory are the Mathieu groups, the first 5 elementary sproadic groups. It is possible that the other sporadic groups, the Janko group and the Conway group for instance might provide the permutation structure for higher multilinear structures of Peano that lay beyond the octonions. This may extend up to the monster group.

Beyond this a more philosophical question is addressed whether the ultimate fundamental laws of physics are knowable. Quantum theories involve the propagation of a field within a space. Even string theories invoke a background structure. Yet this approach approach to physics may be terribly limited. In the case of the quantum cosmology in its first few Planck units of time one clearly has quantum gravity modes propagating on themselves. There is the possibility that quantum gravity may exhibit axiomatic incompleteness due to Gödel' s theorem. In effect the problem of fields propagating on themselves is similar to a quantum computer that uses its own programming structure as its data input. If this is the case structures that approach the Planck scale may indeed explode on us and these structures may emerge for no particular reason at all. As such we may never come up with a final theory of quantum gravity. In fact maybe the "final theory" of quantum gravity is that there is no complete axiomatic based final theory.

In the past 25 year considerable progress has been made theoretically towards an understanding of quantum gravity. From Hawking black hole radiation, strings, loop space variables and now with membranes a formalistic framework has been developed whereby quantum gravity can be discussed. Of course as yet nothing empirical has been found, so much of this machinery is today just formalistic possibilities. Yet physics may be near where theoretical work could begin to gel into a more consistent whole, where initial experimental and observational tests are possible. It this does occur within the next 25 years, then quantum gravity may emerge as a realistic physical theory that may begin to rightfully call itself science. It is the hope that the work presented here will at least in some small way contribute to this long term effort.

1.1 Quantum Fluctuations and Spacetime

Now consider the elementary notion of quantum fluctuations. Elementary courses in modern physics illustrate how the uncertainty principle of Heisenberg $\Delta E \Delta t \simeq \hbar$ indicates a small fluctuation in the energy of a particle associated with a probability that particle may pass over a potential energy barrier. This quantum effect is well documented in physics such as radioactive decay. This suggests a quantum particle in motion exhibits stochastic deviations from its classical motion. This is seen in the Ehrenfest theorem

$$\frac{d}{dt}\langle\, |\mathbf{r}|\,\rangle = \frac{1}{m}\langle\, |\mathbf{p}|\,\rangle, \qquad (1.4)$$

where both sides of the equation have a sum over all states. The velocity of any particle is an expectation from a sum over velocities, where most deviate from a classical result. The inference is that if one makes a momentum measurement of the particle there is a probability that the particle is under a momentum fluctuation. A measurement has a probability of finding the momentum of the particle different from the classical result.

Quantum mechanics indicates quantum wave functions are nonlocal, where different states or probable outcomes are entangled in a nonlocal fashion. The Einstein-Podolsky-Rosen (EPR) and Bell's theorem illustrate this is an inescapable aspect of quantum mechanics. Early in the 20^{th} century this was troubling to many, including Einstein. David Bohm presented the Schrödinger equation in a real part that is a modified Hamilton-Jacobi equation and an imaginary part that is a continuity equation. The modified Hamilton-Jacobi equation contains a "quantum potential" interpreted as due to a pilot wave. However, this formalism did not eliminate the strangeness of nonlocality, for under an experiment the quantum potential must nonlocally readjust itself to fit the outcome of any experiment.

There are some of situations where Bohm's approach to quantum mechanics may be useful. One of those is with quantum fluctuations. With a wave function in polar form $\psi = \rho e^{iS/\hbar}$ the application of the momentum operator $\hat{p} = -i\hbar\nabla$ gives the result

$$\hat{p}\psi = (\nabla S - i\hbar\nabla\rho)\psi, \tag{1.5}$$

which is a classical momentum plus an imaginary term interpreted as a fluctuation in the momentum. This imaginary term will be illustrated how this term is interpreted as a fluctuation through the continuity equation. The expectation of the momentum is

$$\langle\, |\hat{p}|\,\rangle = \langle\, |\nabla S|\,\rangle + \langle\, |\delta p|\,\rangle, \tag{1.6}$$

where the expectation of the momentum is $\langle\, |\hat{p}|\,\rangle = \langle\, |\nabla S|\,\rangle$ and so $\langle\, |\delta p|\,\rangle = 0$. The expectation value of the square of fluctuations is nonzero and is delta function correlated

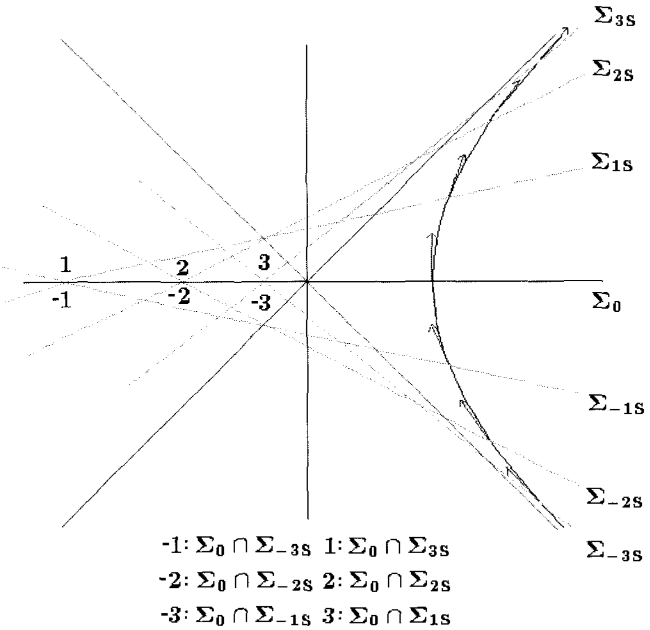
$$\langle\, |\delta p(t)\delta(t')|\,\rangle = (\Delta p(t))^2\delta(t - t'). \tag{1.7}$$

The term $i\hbar\nabla\rho$ is demonstrated to obey the same Markovian result. This development is examined further, where fluctuations are independent of the expectations $\langle\, |\nabla S|\,\rangle$ and canonical transformations on the expectations and fluctuations are also independent. This leads to the fact that a canonical transformation of the fluctuation momentum can change the quantum potential in the modified Hamilton-Jacobi equation. As such the modified Hamilton-Jacobi equation only describes one path in the entire "sum over histories" in a path integral. Hence a path integral emerges from Bohm's quantum theory.

The purpose of this exercise is to introduce the notion of the quantum fluctuations and to illustrate that this Bohmian starting point does not infer any "quirky" notions of hidden variables or some underlying "subquantum" causality to quantum fluctuations.

The exercise simply uses this approach to quantum mechanics as a tool to develop quantum spacetime fluctuations, not to promote any sort of hidden variable agenda.

The Unruh effect is the thermalization of quantum vacua as measured on an accelerated frame. This is due to the inability to specify fields on the other side of the split horizon $\mathcal{H}_1 \oplus \mathcal{H}_2$. The instantaneous planes of simultaneity, Σ_n, the accelerated observer passes through intersect one another on the other side of $\mathcal{H}_1 \oplus \mathcal{H}_2$. As such fields the observer measures are thermally distributed to prevent future information from being observed in the past. Given time increments s, not all fields on Σ_{-3s} are causally tied to fields on Σ_{3s} as detected by the accelerated observer. Figure 1.1 illustrates the intersection of instantaneous planes of simultaneity on the plane of simultaneity of an inertial observer Σ_0, where the accelerated frame comes to an instantaneous stop. This means there are nonlocal entanglements between states on different planes of simultaneity which are hidden by the particle horizon. These hidden entanglements result in a decoherence of the particle states.



Geometry of the accelerated reference frame.
Figure 1.1

As indicated above quantum gravity fluctuations are likely to be associated with a horizon analogous to the Unruh effect. It is argued that this is in general the case. The first situation examined is a rotating open string, where the quantum vacuum is examined.

The thermal vacuum for a rotating string is equivalent to a string that interacts with a black holes with a Hawking temperature. This situation is argued to obtain for general accelerations. The time period for this fluctuation T and the frequency of modes ν must satisfy $T \sim .1/\nu$. The situation where $T \sim 1/\nu$ is likely to occur near the strong coupling domain of physics, where this approach to quantum fluctuations of spacetime is likely to break down. This domain is discussed in chapters 7 through 9.

Gravitation has an uncertainty in the energy-momentum content of a 3-d volume of space. A spatial surface evolves according to the diffeomorphisms of spacetime physics. This makes it difficult to assign a region that contains a certain mass-energy content. Then one has the uncertainty principle of quantum mechanics, which gives a commutator structure for the uncertainty between conjugate observables. The "fusion" of these uncertainties illustrates how the vacuum dynamics has an uncertainty in $a^\dagger a$ or particle number and in topological numbers. This leads to a generalized uncertainty in the commutation of variables [3]. Under sufficiently large gravity fields the uncertainty in two conjugate variables will both increase. The raising and lowering operators on either side of an acceleration induced horizon satisfy

$$\alpha_i^b = \hat{a}_j^b + a_i^b e^{-\pi\mathcal{H}/g},\ {\alpha^b}_i^* = \hat{a}_j^{b\dagger} + a_i^{b\dagger} e^{-\pi\mathcal{H}/g}, \tag{1.8}$$

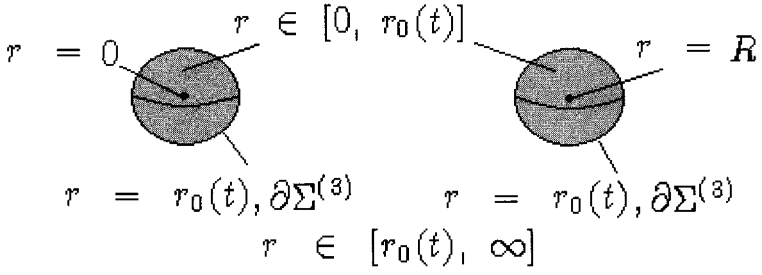
where $\hat{a}_j^b$ and $\hat{a}_j^{b\dagger}$ are operators for fields on the observer's side of the horizon with $[\hat{a}_j^b,\ a_j^{b\dagger}] = [\hat{a}_j^{b\dagger},\ a_j^b] = 0$. This generalized uncertainty principle is given by the commutation between raising and lower operators according to

$$[\alpha_j^b,\ \alpha_k^{c\,*}] = \big(1 + e^{-2\pi\mathcal{H}/g}\big)\delta_{jk}\eta^{bc} - \frac{\pi}{g} e^{\pi\mathcal{H}/g}\big(\alpha_j^{0b^*}\alpha_k^{0c} + \alpha_j^{0b}\alpha_k^{0c^*}\big)e^{-\pi\mathcal{H}/g}, \tag{1.9}$$

where $H = \hbar\omega \sum_j a_j^\dagger a_j$.

The gravitationally induced squeezing of the vacuum state may permit a virtual cosmology to tunnel out of the vacuum state. The uncertainty in the particle number, which results from the vacuum not being strictly a dynamical eigenstate of a diagonal Hamiltonian, permits the creation of a cosmology out of "nothing". Freeman Dyson suggested, in a critique of Feynman's quantum electrodynamic theory, the following artificial "toy theory". Suppose the electric charge transform as $e \rightarrow ie, i = \sqrt{-1}$. Virtual quanta by $e^2 \rightarrow -e^2$ will not recombine, but rather the virtual charges repel. Consequently the vacuum would be terribly unstable and spew electron-positron pairs or radiation. This is a quantum electrodynamic analogue of a process where the vacuum is unstable so that fields and particles may be generated.

Figure 1.2 illustrates two wormhole openings. At $r = r_0$ is the Lanzcos junction defined by the Reissnor-Norstrom metric. The horizon is identified from the exterior region $r > r_0$ and the interior region $r < r_0$. This interior region is topologically identical to sewing two three dimensional balls together, through the identification of points on their respective two dimensional spherical boundaries, which defines a 3-ball. Across a Lanzcos junction on the 2-spherical boundaries is a jump in G^{00}. This junction is similar to a horizon, or a shock wave front, that changes $t \rightarrow -t$ and $r \rightarrow -r$. This is analogous to the situation Dyson illustrated. In the old "ict" way of doing relativity this also is equivalent to an effective $m \rightarrow im$, for $m =$ mass, where if this virtual cosmology is flat enough this run away situation similar to Dyson's will occur. The fluctuation, which is the wormhole itself, leads to a flattening of this virtual cosmology by gravitational self-squeezing of the vacuum state. This situation is unstable and the uncertainty in particle quantum number, eg. the Witten topological index or Atiyah-Singer index, leads to the generation of a spacetime cosmology literally out of nothing. Further, there is no "prime mover" or extra causal agent: the process is completely due to stochastic fluctuations and is completely spontaneous.



Two worm hole openings with a virtual
spacetime cosmology in the interior region.
Figure 1.2

Thomas Aquinas wrote in his Summa contra Gentiles III, "Materia artificialium est a natura, naturalium vero per creationem a Deo". Yet it appears as if nothingness or the quantum gravitational vacuum permits the creation of something from nothing "Creati Ex Nihilio". This will doubtless have a profound impact on physics and cosmology, as well as on the cultural and social context of the world at large. This has the quality of Creati Ex Nihilio, where the nothingness is the quantum gravity vacuum. This vacuum is defined by virtual excitons of the gravity field, black holes, wormholes, cosmologies and gravity wave fluctuations. The quantum uncertainty principle with the inability to locate mass-energy means that fluctuations of energy can tunnel out of the vacuum and spontaneously give rise to cosmologies.

This implies the existence of other cosmologies, where quantum spacetime may give rise to other nascent cosmologies. However, this junction at $r = r_0$ is a causal boundary and we are prevented from ever looking into the new cosmology. The vacuum may be unstable in a way analogous to Dyson's illustration with QED, but the creation of other cosmologies is forever hidden from our view. These other cosmologies are sealed off from us by horizons or achronal boundaries. As such information about them is inaccessible. This is different from standard quantum mechanics where one can prepare identical systems and obtain an empirical table of amplitudes. All that is possible is to study the nature of quantum gravity fluctuations and infer certain signatures that a nascent cosmology was generated, but observers are forbidden from ever determining anything about these cosmologies or to tap their mass-energy.

This is then concluded with a discussion on a potential H-theorem for quantum gravity and spacetime fluctuations with a finite temperature. A potential approach to this is explored according to knot theory and connections to loop space variables.

1.2: Detecting Quantum Gravity Fluctuations

An electron wave function adjusted by a parameter associated with a loop in space will experience a phase shift. The curvature two-form ${\cal R}^\mu{}_\nu = R^\mu{}_{\nu\rho\sigma}dx^\rho\wedge dx^\sigma$ from quantum gravity fluctuations will produce a phase proportional to

$$\int_{\cal A}{\cal R}^\mu{}_\nu \mathbf{e}_\mu\otimes dx^\nu, \tag{1.10}$$

where ${\cal A}$ is the area in space determined by the cyclic evolution of a parameter.

Consider atoms periodically injected through a cavity at a frequency equal to the Rabi frequency ν_R of the atom interacting with photons. Each atom in a cavity during a time $\frac{1}{2}\nu_R$ it will contribute a photon by stimulated emission. The entangled atom and photon states will exhibit a phase shift due to the zero point energy of the quantum gravity vacuum.

Consider a post-Newtonian form of gravity for brevity. The post-Newtonian gravito-electric and gravito-magnetic fields are similar to Maxwell fields and are written as a linear operator expansion without the difficulties of gravitational nonlinearity. The metric in post-Newtonian gravity is expanded around a flat spacetime:

$$g_{\mu\nu} = \eta_{\mu\nu} + g^{(2)}_{\mu\nu} + g^{(4)}_{\mu\nu} + \dots \{\mu = \nu = 0,\ or\ \mu,\nu = i,j\},$$

$$g_{i0} = g^{(3)}_{i0} + g^{(5)}_{i0} + \dots. \tag{1.11}$$

$\eta_{\mu\nu}$ is the flat spacetime metric. Since the spacetime is source free $g^{(2)}_{\mu\nu} = -(2GM/r)\delta_{\mu\nu} = 0$. The next highest terms within the harmonic condition are $g^{(3)}_{0i} = {\cal A}_i$, where ${\cal A}$ is a vector potential and $g^{(4)}_{00} = -2\phi$ a scalar field. The connections to lowest order are [4],

$$\Gamma^{(4)i}{}_{00} = \frac{\partial\phi}{\partial x^i} + \frac{\partial{\cal A}_i}{\partial t},\ \Gamma^{(3)i}{}_{0j} = \frac{1}{2}\Big(\frac{\partial{\cal A}_i}{\partial x^j} - \frac{\partial{\cal A}_j}{\partial x^i}\Big). \tag{1.12}$$

Now define $\mathcal{E}_i = -\Gamma^{(4)i}{}_{00}$ and $\mathcal{B}_i = \epsilon_{ijk}\Gamma^{(3)j}{}_{0k}$ so these fields are:

$$\mathcal{E}_i = -\frac{\partial \mathcal{A}_i}{\partial t} - \nabla_i\phi,\ \mathcal{B}_i = \frac{1}{2}(\nabla \times \mathcal{A})_i. \tag{1.13}$$

In the harmonic condition $\nabla \cdot \mathcal{A} = 0$ it is easy to find the Maxwell type of equations

$$\nabla \times \mathcal{E} - \frac{2}{c}\frac{\partial \mathcal{B}}{\partial t} = 0 \tag{1.14a}$$

$$\nabla \times \mathcal{B} + \frac{1}{2c}\frac{\partial \mathcal{E}}{\partial t} = 0. \tag{1.14b}$$

Now expand the gravito-electromagnetic field into normal modes with the operators b and $b^\dagger$ analogous to QED. The linearized gravity vector potential expanded as spatial eigenmodes quantized in a box is,

$$\mathcal{A}_i = i\epsilon_i \sum_k \sqrt{\frac{\hbar}{2\omega\epsilon_g V}}\left(b^\dagger e^{-ikx} - be^{ikx}\right). \tag{1.15}$$

Here ϵ_g is a vacuum permittivity for gravity, analogous to the permittivity in electromagnetism. The electric and magnetic analogues of the gravity field are quantum operators for linearized gravitons.

Consider a "toy model" of this gravito-electromagnetic field coupling to an atom. Let two atomic eigenstates be given by σ_z and the transition operator between these states be the operators $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$. The coupling between gravitons and the atom is a quadrupole interaction due to the conservation of momentum and the spin 2 property of the graviton. An approximate quadrupole interaction is $H_2 \simeq g' \int yz\mathcal{E}d^3r$, where $g' = g/V$. The quadrupole term written with $\sigma_\pm$, σ_z gives a Hamiltonian in a rotating wave analogous approximation as

$$H = \frac{\hbar\omega}{2}\sigma_z - \hbar g(b\sigma_+ - b^\dagger\sigma_-)\sigma_z, \tag{1.16}$$

where $g \sim (L_P/L^2)c \simeq 10^{-7}Hz$. The gravity fluctuations are assumed constant through the volume. With only virtual gravitons $b^\dagger b|\ \rangle = 0$ and $b^\dagger b$ is dropped from the Hamiltonian. The occurrence of a linear graviton coupled to this system is associated with a $\Delta j = 2$ in the atom, so there is an implicit spin change in the electron. If an electromagnetic field is quantized in the cavity with operators a and $a^\dagger$. The photons are tuned to interact with the atomic state, so the total Hamiltonian is:

$$H = \frac{\hbar\omega}{2}(\sigma_z + 2a^\dagger a) + \hbar\kappa(a\sigma_+ + a^\dagger\sigma_-) - \hbar g(b\sigma_+ - b^\dagger\sigma_-)\sigma_z = H_0 + H_1 + H_2. \tag{1.17}$$

Physically the atom executes a Rabi oscillation from its interaction with the electromagnetic field.

Within the interaction picture the evolution of H_1 gives

$$\frac{\partial\sigma_\pm}{\partial t} ~=~ \mp i\omega\sigma_\pm,~ \sigma_\pm ~=~ \sigma_\pm(0)e^{\mp i\omega t}. \tag{1.18}$$

The parameters which define the quadrupole moment of the linear gravitational interaction then evolve. This then gives the term:

$$\frac{\nabla_{\sigma_+}H_2 \times \nabla_{\sigma_-}H_2}{(\Delta E)^2} ~=~ -(\hbar g)^2\frac{[b,~b^\dagger]\sigma_z^2}{2(\Delta E)^2} ~=~ -(\hbar g)^2\frac{\mathbf{1}_{2\times2}}{2(\Delta E)^2}. \tag{1.19}$$

The expectation of his term evaluated on the area defined by the evolution of the polarization vectors defines a Berry phase. This phase is given by,

$$\phi ~=~ -(\hbar g)^2\int\int_{area}\frac{d^2x}{2(\Delta E)^2}. \tag{1.20}$$

Evidently the electron needs to have a high radial quantum number to get an enhanced quadrupole moment from an elliptical electron orbit and to minimize the ΔE between the atomic states. This will induce a nondynamical phase on the atomic wave function that is entirely due to modes of the zero point energy of the quantum gravity field that couple to the atom.

For a dipole coupling constant $\sim~.1\omega$ and transitions between high Rydberg states $n ~>~ 300$ the energy gap may be as small as $\Delta E ~\simeq~ 10^{-8}ev$, a quick estimate illustrates that this Berry phase is $\phi ~\simeq~ 10^{-29}$ per Rabi oscillation. The total phase shift per second is $\simeq~ 10^{-18}$, where if the experiment is conducted for a year in a one atom laser the phase would be$\simeq~ 10^{-11} - 10^{-10}$.

The quadrupole moment interaction with quantum gravity fluctuations is required for their observation. This suggests that the orbit of the electron should be a Bohr-Sommerfeld type of orbit with a large eccentricity. This is best achieved if the atom is a high Rydberg atom with an electron wave packet in an elliptical orbit. There is considerable work yet to be done with real atomic wave functions that will give precise theoretical expectations for the phase. The tolerances on the apparatus will have to be comparable to tolerances required of the Light Interferometric Gravitywave Observatory (LIGO).

Additional quantum optical experiments of this nature are proposed to test the holographic principle. The larger scale effects of holographic induced fluctuations should be more amenable to such experimental techniques. The phase terms computed are significantly larger. Through the remainder of this text the holographic principle is explored as a rich foundation for deeper foundations of physics.

Experimental tests of quantum fields in spacetime are an emerging reality. Recently it was announced that gravitational Bohr quantization was measured with cold neutrons [5].

This result is analogous to the Bohr atom derived early in the 20^{th} century. Observing spacetime fluctuations is then likely the next step for measurements on the quantum effects of gravitation.

The second approach towards the detection of quantum gravity is the measurement of the departure in the uncertainty between complementary variables in the presence of strong gravity fields. By the Einstein equivalency principle the acceleration of a particle by a force in flat spacetime is equivalent to any force that deviates the geodesic motion of a particle in curved spacetime. With high powered lasers an electron may be accelerated to values as large as $g = 10^{21}cm/s^2$ and beyond in the not too distant future. This electron will interact with a thermal vacuum with a temperature of $T = (g/10^{21}cm/s^2) \times 1K$. The Unruh effects may be observable within the next few decades, where experiments involving Bose-Einstein condensates are proposed. Assume that electrons are accelerated to huge values by a high powered laser beam, or realistically a pulse, with a good degree of coherence to the beam. This laser beam may further require some degree of parametric amplification as a "primer" required to induce squeezed states associated with the accelerated electron. If a beam of electrons is accelerated then the synchrotron radiation emitted by these electrons will induce a stimulated emission of synchrotron radiation. This uses a laser to generate a laser, analogous to the wiggler laser. The employment of a homodyne detector will measure the degree of squeezing of this light, where further if there are departures in the uncertainty in observables.

Obviously this experiment will present some difficulties. This involves quantum optical techniques in the X-ray region of the electromagnetic spectrum. The technology for these types of measurements will doubtlessly have to be considerably improved in the not too distant future.

Another test of quantum gravity may be with the detection of gravity waves. The earliest phase of the universe was one where gravity was quantized and unified with the other gauge fields. The inflationary process set up the decoupling of the forces of nature may well have also involved the decoupling of gravity from the other forces of nature. With this decoupling gravity assumed a classical nature, but where quantum gravity fluctuations may have been imprinted on classical spacetime. With the rapid expansion of spacetime quantum fluctuations in spacetime may have been also expanded rapidly and frozen into the classical manifold. This would then manifest itself as a cosmic background of gravity waves. Planck scale fluctuations may have been frozen out as gravity waves, which may now exist on a scale of $\sim 1 - 100$m. This spectrum of gravity waves should be due to quantum spacetime fluctuations when the observable universe was 10^{-14}cm to 10^{-16}cm in radius. This background spectrum of gravity wave radiation is analogous to the microwave background of electromagnetic radiation due to the end of the radiation dominated period of the universe.

With Light Interferometric Gravity Observatories (LIGO) it may be possible to detect this background of gravitational radiation. This would extend our observation of the universe as close to the cosmological event horizon as is likely possible. It would also be an astronomical indication of the existence of quantum gravity fluctuations in the early universe. The spectrum of this radiation will contain information about the nature of the extremely early universe. This is one the reasons why much of this monograph discusses quantum gravity from a low energy and potential empirical viewpoint. If this remnant from the quantum gravity period of the universe is detected it will doubtless lead to an incredibly different view of the universe, where much of the highly developed theories devised today will be either revised or abandoned.

1.3: Connections to Strings and Membranes

The holographic principle indicates that quantum fields approach an event horizon, pile up near the horizon and are redshifted out of view. This implies that event horizons are associated with a Planck length thick membrane of fields, where only their quantum gravitational modes are observed. These fields have wavelengths at the Planck scale, where all other field amplitudes on a larger scale are redshifted away to "infinity". In the case of a black hole this membrane consists of all available quantum information that composes the black hole. This membrane is then considered as a two dimensional membrane, or D2-brane, under a target map. For a virtual black holes this membrane may have a negative energy fluctuation. In this case the virtual black hole is a virtual wormhole.

The construction of fluctuations up to this point involves Lanzcos junctions that are discontinuities in G^{00} with analogues to the discontinuity in the electric displacement vector at the boundary of a dielectric. Further, if this discontinuity is zero then the negative energy deficit required to sustain this is removed and the 2-d sphere of the Lanczos junction is the event horizon of a black hole. As a quantum fluctuation this will involve the fluctuation in a Planck mass δm_p plus a fluctuation in a negative energy component $-\delta\epsilon$, where on average $|\delta m_p| > |\delta\epsilon|$ according to the quantum interest conjecture. So in general quantum gravity fluctuations larger than $\sim 10 \times L_p$ will involve a preponderance of virtual black holes, where the occurrence of wormholes is a measure of the squeezing of the vacuum state.

The connection to membranes is discussed in the light of the 't Hooft horizon algebra [6]. This algebra is defined by the commutation of momentum and position variables for fields entering and leaving a black hole. For $x_t = x_t(\sigma)$ the transverse coordinates to the horizon, where σ is an arbitrary coordinate, then

$$[\partial_\sigma x^\mu(\sigma),\ \partial_\sigma x^\nu(\sigma')] = i\epsilon^{\mu\nu}{}_\rho \partial_\sigma x^\rho(\sigma')\delta^2(\sigma - \sigma'). \tag{1.21}$$

These commutators define an $SO(2,1)$ algebra, where this conveniently reduces the dimension of the Lorentz algebra. A basis of connection forms and curvature forms is then derived from this algebra. This system is then extended into a supersymmetric form. With the application of ghost fields and Brecci-Rouet-Stora-Tyupin (BRST) quantization this theory may be expressed according to topological quantum numbers. Here the membrane of fields a Planck unit away from the event horizon of a black hole is characterized by topological indices in an elementary manner.

The horizon algebra contains a moduli space of solutions. The principle bundle that defines this moduli space is split at the event horizon $\mathbf{P} \rightarrow \mathbf{P}_1 \oplus \mathbf{P}_2$ with an associated singularity in the moduli space. This dimension of this moduli space is given by the Donaldson theorem, which has curious implications for manifolds in four dimensions. An examination of the moduli space near the singularity leads to a surprising result. A physical ansatz is invoked to define the analog for the elliptic complex for a pseudoEuclidean metric. The moduli space near the singularity assumes the form of a cone in the weighted projective space CP^2_w. A weighted projective space is where the coordinates in CP^2 are mapped into $z \rightarrow z^n$. This weighted projective space naturally has a Virasoro structure with an associated Kac-Moody algebra. This structure has the form of a quantized gauge theory. This gauge theory is later to be examined with respect to Chan-Paton factors for strings. This leads to the suggestion that strings at high energy are in fact made of constituents or have an underlying structure identified with the blow up of a point in the moduli space. This gauge theory, if it is psuedoEuclidean, will in turn have a moduli space with a similar singularity structure. This leads to a fractal-like situation with respect to the foundations of physics.

These structures lead to various approaches with respect to topological quantum numbers and indices. These are subsequently discussed as possible outcomes of this view of the universe.

1.4 Beyond Standard Constructions

Beyond this point it is possible that the universe is structured according to multilinear brackets more general than the standard commutators of quantum theory or the parallel transport construction of general relativity. These lead to a general structure labelled as the B^* algebra based on Peano's multilinear structure of geometry. A possible realization of this according to the octonions is also developed. In this view the foundations of quantum gravity is posited to be due to the 24-cell and its error correction code basis. In this model Planck units of volume in a 24-cell are distinguishable and their relationship is given by a Hamming or Golay code. This construction appears to underlie the 26 bosonic string, where these is an E_8 lattice construction as well. The underlying structure for this is the Mathieu group $C24$, which is the fifth of the 26 sporadic groups. The current superstring theories may then have this sort of underlying basis according to such constructions.

On a scale smaller than the 24-cell it is likely that this approach fails. This might mean the 24 cell is defined for overlapping Planck units of volume, where the distinguishability of states fails. If the 24-cell is not compressed this domain then may involve structures beyond the octonions. However, this implies physics has an algebraic-geometric structure that no longer is a normed division algebra. It might be possible to order more generalized multilinear brackets for the B^* algebra around the Janko or Conway sporadic groups. This path would lead to the monster group. The octonions or some extended B^* structure may be the end of all possible geometry. This breakdown might reflect how quantum gravity and cosmology ultimately involves quantum logical propositions that are no longer decidable. The physical implication is that physical law may ultimately be random. Physical law may then emerge for reasons that are ultimately random. The emergent physical laws may then be due to the undecidability of the truth of quantum logical propositions. This is an even more speculative look at the foundations of quantum gravity, but one that is shown to have a basis in standard quantum mechanics and Chaitan's halting probability

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