Chapter 1

Why Higher Dimensions are Interesting

While there is currently no experimental evidence at all that supports the notion that our universe might possess any spacetime dimensions beyond the four established ones, it is a rather remarkable fact that we actually know of no compelling reason as to why the number of spacetime dimensions should actually be four. Certainly, a universe with four dimensions has some very special properties. For instance, it is only in four dimensions that the trilinear fermion, anti-fermion, gauge boson coupling constant is dimensionless, one of the key features which leads to the renormalizability of the strong, electromagnetic and weak interaction gauge theories. Nonetheless, it is equally possible to formulate such theories in spacetimes with dimension higher than four — they just would not be as easy to deal with — and even studying these theories in spacetimes with dimension lower than four has often been found to be extremely instructive. Consequently, it is legitimate to consider the possible existence of additional dimensions beyond four, with a view to either learning something theoretical about them which proves informative for 4-dimensional theories, or to being able to perform some experiment in four dimensions which might reveal their presence. What makes this enterprise so interesting is that the very entering into the issue of spacetime dimensions immediately entails the involvement of gravity, to thereby provide a possible road to a unification of gravity with the other fundamental forces.

In fact attempts at a higher-dimensional unification were initiated a long time ago, not long after Einstein's introduction of general relativity in 1916, with the most useful approach having been pioneered by Kaluza [Kaluza (1921)] and Klein [Klein (1926a); Klein (1926b)]. In their approach spacetime was taken to be a flat 5-dimensional $M(4,1)$ space with one timelike coordinate and four spacelike ones, so that the associated gravitational field then has 15 components. With respect to the standard 4-dimensional $M(3,1)$ spacetime these 15 degrees of freedom decompose into a 10-component rank-two tensor, a 4-component vector and a 1-component scalar. By identifying the rank-two tensor with the standard gravitational field $g_{\mu\nu}$ and the vector with the standard electromagnetic vector potential A_{μ}, Kaluza and Klein were able to achieve a purely geometrical unification of gravity with electromagnetism, one which long after their work then came back into prominence (see e.g. [Appelquist, Chodos and Freund (1985)]) following the twin realizations that spontaneously broken gauge theories had need for a scalar field as well, a Higgs field whose vacuum expectation value would serve to break the gauge symmetry, and that non–Abelian Yangs–Mills generalizations of Maxwell's theory could also be incorporated if the number of extra dimensions were to be increased even more [DeWitt (1964)]. To avoid the obvious fact that there was no apparent sign of any macroscopic fifth (or higher) dimension, (and to dispense with Kaluza's original assumption that none of the fields in the theory actually depend on the fifth coordinate at all), it was further presupposed in the Kaluza-Klein theory that any extra dimension be compactified into the topology of the 1-dimensional sphere S^1 (to give an overall geometry which was now $M(3,1)\times S^1$), with a compactification radius for S^1 which was taken to be microscopic. Because of the periodicity of this S^1 (whose implications Klein imported from Schrödinger theory which had just been developed at the time), the gravitational fluctuation modes, viz. the eigenmodes of the model, would then be discrete, having masses which would be the larger the smaller the compactification radius was taken to be. These modes, commonly referred to as KK modes, while characteristic of higher dimensional theories, could thus by hand phenomenologically be made heavier than the maximum energies available to high energy accelerators, with the same mechanism which would conceal any extra dimensions from detection thus also serving to conceal the KK modes too.

Proceeding from a totally different direction, a role for extra dimensions also emerged from elementary particle physics. Specifically, studies of high energy hadron scattering in the 1960s led to the development of the dual resonance model [Veneziano (1968)], a model whose spectrum of states was then found to be reproduced by the spectrum of a vibrating string, a theory which for consistency would need to be formulated in 26 spacetime dimensions if bosonic and in 10 if fermionic. Such string theories (see e.g. [Green, Schwarz and Witten (1987); Polchinski (1998)] for a detailed history and bibliography) were initially thought to describe strong interactions and had a fundamental scale, the string tension, which would be a hadronic GeV scale. A shortcoming of these theories was the presence of a massless spin-two mode for which there was no known hadronic particle, and so it was suggested that this mode might instead be associated with the massless spin-two graviton of gravitational theory. With the replacement of the hadronic string scale by a quantum-gravitational $M_{PL} = 10^{19}$ GeV Planck mass scale, string theory was then recast as a theory not of strong interactions at all but of quantum gravity instead, providing the first consistent melding of Einstein gravity with quantum mechanics. While this theory would still be a higher dimensional theory of quantum gravity rather than a 4-dimensional one, because the theory contained a naturally small length scale, viz. the 10^{-33} cm Planck length, in a compactification the extra dimensions would then naturally, i.e. without presupposition, be microscopic. Similarly, the masses of the excitation modes would automatically be of order M_{PL} and thus be totally beyond reach, a twin-edged aspect of the theory which simultaneously prevents it from being ruled out by experiment while making it almost impossible to be ruled in. Nonetheless, since quarks could also be introduced into the theory, such string theories could potentially provide for a unification of all of the fundamental forces into a "theory of everything", so that they nicely possess both the unification ideas of Kaluza and Klein and a consistent formulation of quantum gravity. With extensions of Kaluza-Klein theory having independently been developed to incorporate both Yang-Mills theory and supergravity theory (the supersymmetric extension of ordinary gravity), with the development of modern superstring theory it then became possible to incorporate all of these higher dimensional ideas into one comprehensive framework.

In parallel with these developments, work in Higgs-driven spontaneously broken non-Abelian gauge theories in four dimensions revealed that such theories possessed configurations in which the Higgs field expectation values could have a spatial dependence, leading to models in which elementary particles were to be extended, topological defects rather than point structures (see e.g. [Rebbi and Soliani (1984); Coleman (1985)]). As such, microscopic elementary particles could have structures analogous to superconducting vortices or ferromagnetic domains; with this last option leading by analog to the possibility [Rubakov and Shaposhnikov (1983)] that our entire 4-dimensional universe could itself be a macroscopic domain wall embedded in some higher dimensional world, a model in which the three fundamental strong, electromagnetic and weak interactions could then be confined to the wall. In such a picture, even while being associated with a long range force, it was possible for electromagnetic flux to be confined to the wall, with there being no emission of photons into the extra dimensions. While the extended structure idea could thus conceal higher dimensions from strong, electromagnetic or weak probes, it still would not do so for gravity which could not be confined in this manner, to thus still require the extra dimensions to be microscopic. Subsequently, the domain wall picture was also found to emerge in superstring theory where it was shown [Dai, Leigh and Polchinski (1989); Polchinski (1995)] that superstring theories had to possess analogous topological defects called D-branes (D denotes Dirichlet boundary conditions). Such branes (viz. membranes generalized to arbitrary dimension — with an N-brane possessing N spatial dimensions) would then provide the locations at which strings would terminate, with superstring theory thus becoming a theory of both strings and branes, though again with still microscopic extra dimensions.

While higher dimension theories thus characteristically enjoyed (or suffered from) having microscopic extra dimensions, research started to move in a very different direction when Arkani-Hamed, Dimopoulos and Dvali [Arkani-Hamed, Dimopoulos and Dvali (1998)] discussed a possible advantage to having altogether larger-sized extra dimensions, and posed the question as to how phenomenologically large such dimensions might actually be permitted to be. In an attempt to resolve the longstanding hierarchy problem of understanding why there was such a huge disparity between the $M_{EW} = 10^3$ GeV electroweak and the $M_{PL} = 10^{19}$ GeV gravitational mass scales, Arkani-Hamed, Dimopoulos and Dvali presupposed that we lived in a $(4+n)$ dimensional world in which there were to be n additional spacelike dimensions each one was of which was still to be compactified, but with some common radius R which was to no longer necessarily be microscopic. In such a world $(4+n)$ dimensional gravity was to be controlled by the electroweak scale rather than by a Planck one (to thus unify electroweak and gravitational interactions at the higher dimensional level), with a potential confinement of gravitational flux lines around our 4-dimensional world then converting the gravitational potential $V(r) = m_1 m_2/M_{EW}^{n+2} r^{n+1}$ between two static masses m_1 and m_2 in $M(3+n,1)$ into the $V(r) = m_1 m_2/M_{EW}^{n+2} R^n r$ one of $M(3,1) \times S_n$, leading to an effective 4-dimensional gravitational coupling given as

$$M_{EFF}^2 = M_{EW}^{n+2} R^n \tag{1.1}$$

instead of a fundamental M_{PL}^2 one. In order to not disagree with current gravitational phenomenology (in which M_{EFF} is to be of order 10^{19} GeV), a value of $n = 1$ was found to be excluded, with $n = 2$ leading to an R of order millimeters. While this value was no less than 10^{32} times larger than Planck length sized extra dimensions, what made this particular value so very interesting was that a millimeter was of order the smallest length scale on which the inverse square gravitational law had actually ever been tested; and while there were very tight constraints coming from precision electroweak measurements forbidding the $SU(3) \times SU(2) \times SU(1)$ interactions from spreading out very far into any extra dimension at all (Coulomb's law, for instance, is tested down to 10^{-16} cm), nonetheless there were no such tight constraints on gravity itself, and its flux lines could in principle spread out into any extra dimension and manifest themselves as modifications to standard gravity at the millimeter level. The original Arkani-Hamed, Dimopoulos and Dvali idea subsequently acquired greater generality when it was found [Antoniadis, Arkani-Hamed, Dimopoulos and Dvali (1998)] that it could explicitly be realized in string theory where our 4-dimensional universe would then be a 3-brane embedded in a higher dimensional bulk. Though no sign of any millimeter or sub-millimeter departure from standard Newtonian gravity has yet actually been detected [Adelberger, Heckel and Nelson (2003)], nonetheless the work of Arkani-Hamed, Dimopoulos and Dvali opened up the possibility that extra dimensions need not be microscopic, and showed that it was at least in principle possible to actually experimentally explore extra dimensions at ordinary (as opposed to Planckian) energies and distance scales.

Following on the work of Arkani-Hamed, Dimopoulos and Dvali, Randall and Sundrum [Randall and Sundrum (1999a)] found an alternate way to address the hierarchy problem based on three key ingredients which not only enabled them to construct a phenomenologically viable model with only one large extra dimension, but which also turned out to be seminal to the entire large extra dimension program. Firstly, they compactified this one extra dimension with an additional Z_2 orbifold symmetry in which opposite points in the compactified fifth coordinate were to be identified. Secondly, they located a 3-brane at each of the two orbifold fixed points, viz. at the two points at the end of a diameter of a circle which self-identify when points on opposite sides of the diameter are identified with other. And thirdly, the decisive step, they took the bulk geometry to no longer be flat, but to instead be the 5-dimensional anti-de Sitter geometry AdS_5. Given such a model, they then found that in it the effective 4-dimensional Planck mass would be given by

$$bM_{EFF}^2 = M_5^3[1 - e^{-2\pi bR}] \ , \tag{1.2}$$

where M_5 is the fundamental 5-dimensional gravitational mass scale, $-b^2$ is the negative AdS_5 5-curvature, and R is the compactification radius. A choice of M_5 and b both of order M_{EW}, could then through the $[1 - e^{-2\pi bR}]$ suppression factor lead, for an appropriate choice for R, to an M_{EFF} which could still be much smaller then M_{EW}. Thus instead of the suppression being provided by the volume of the extra dimension region as in Eq. (1.1), now the suppression was being supplied by the curvature in that region, with AdS_5 acting as a dynamical refractive type medium which sharply modifies the propagation of gravitational signals in it. Beyond possible millimeter region modifications to Newton's law, an extra feature of this AdS_5/Z_2 model[1] was that it predicted electroweakly coupled KK modes in the TeV region, a search for which could also provide a probe of extra dimensions.

However, quite the most dramatic aspect of Eq. (1.2) is that unlike the situation in Eq. (1.1), M_{EFF} would remain finite even in the event that R were allowed to become infinite. In such a case the 3-brane at the second orbifold fixed point could be discarded (without needing to undo the Z_2 symmetry), with the model turning into a model [Randall and Sundrum (1999b)] of just a single 3-brane embedded in an AdS_5/Z_2 bulk which would not be of finite radius at all. In such a model, even though the extra dimension would not merely be large, but as infinitely large as the three ordinary spatial dimensions, gravity would nonetheless still be localized to the brane, with it being the AdS_5/Z_2 curvature which would confine the gravitational flux to the brane. A large fifth dimension could then exist, with the exponential suppression of the exchange of gravitational information between brane and bulk concealing its presence. With the work of Randall and Sundrum then, it became possible to transit from microscopic extra dimensions to ones of unlimited size, to thus open up the entirely new field of brane-localized gravity.[2] It is the purpose of this monograph to describe some of the general aspects of this program, and we shall begin with a detailed discussion of the two Randall-Sundrum papers themselves.

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[1] The set-up consisting of an AdS_5 geometry and an orbifold symmetry is referred to as AdS_5/Z_2.

[2] While quite interested in the Kaluza theory because of its unification aspects, Einstein nonetheless commented [Einstein (1931)] that "Among the considerations which question this theory stands in the first place: It is anomalous to replace the four-dimensional continuum by a five-dimensional one and then subsequently to tie up artificially one of these five dimensions in order to account for the fact that it does not manifest itself". It is thus of interest to note that in brane-localized gravity theories the higher dimensions do not have to be tied up at all.