Chapter 1

Introduction

$E\nu$ $\alpha\rho\chi\eta_\iota$ $\eta\nu$ τo $X\alpha o s$ - In the beginning was Chaos (Isiodos, 850 BC)

1.1 Historical background

Whether or not the entertaining details of the story of Archimedes' discovery (Eureka! Eureka!) pertaining to the hydrodynamic force on a stationary object immersed in a fluid are correct, it is a historical fact that he determined the hydrostatic force acting on an immersed material object of any shape and density as well as the concept of the integrals that make possible such calculations. This force is among the type of forces that are now called "body forces." Archimedes' principle has been widely used not only in the determination of forces on immersed objects but also in the development of pycnometers, instruments for the measurement of the density and composition of fluids. This principle appears to have been misinterpreted by some in the middle ages, who derived implausible and contradictory results about the density of ice. For this reason, Galileo (1612) published a short treatise on the subject, where the correct use of the principle is pointed out and its implications on the dynamics of immersed or partially immersed bodies are emphasized. The next significant advance on the mechanics and dynamics of immersed objects was not accomplished until the late 17th century (1687) with Newton's publication of the monumental *Principia,* a book that changed the world of science and became the cornerstone of modern mechanics.

In this short account of the more recent history of significant advances in our knowledge of the motion and thermal behavior of particles, bubbles and drops we will separate the progress made in two parts: the first is the determination of the hydrodynamic force acting on a bubble, drop or solid particle in a viscous fluid and the development of their equation of motion. The second is the determination of the rate of heat and mass transfer from these immersed objects, and the development of the pertinent theory.

1.1.1 Forces exerted by a fluid and the equation of motion

The attention to the hydrodynamic force acting on a solid sphere carried by an inviscid fluid, first started with the work of Poison (1831) almost twenty years before the publication of what we now call "the Navier-Stokes equations." Poison solved the potential flow equations around a sphere and determined that the transient force exerted by an inviscid fluid on the sphere is equal to:

$$\frac{1}{2} m_f \frac{d}{dt}(v_i - u_i) , \tag{1.1.1}$$

where m_f is the mass of the fluid that has the same volume as the sphere and the term in parenthesis is equal to the velocity of the sphere with respect to the fluid. Thus, Poison correctly deduced that the coefficient of what we now call the "added mass term" is equal to ½, a result that has been confirmed analytically and experimentally by many others. Shortly thereafter, Green (1833) extended this result to the flow around an ellipsoid, again in an inviscid fluid. He deduced that the added mass coefficient of the ellipsoid is also equal to ½. Almost a decade later, Poiseuille (1841) studied experimentally the motion of spheres in a solid conduit in order to understand the flow of blood corpuscles. He derived the first results for the flow of a liquid in a pipe, a type of flow that now bears his name.

Stokes (1845, 1851) was the first to analyze the motion of a solid sphere in a viscous fluid. As one of the first applications of what is now known as the "Navier-Stokes equations," he obtained a solution for the steady-state flow around a sphere moving in an otherwise stagnant, viscous fluid. When the relative velocity of the sphere is very low, he determined that the hydrodynamic resistance force is equal to: $F=6\pi\alpha\mu_f v$. This is now called the "Stokesian drag force," and the dimensionless drag coefficient, which results from this expression, is called "the Stokes drag coefficient." The latter is equal to C_D=24/Re, where Re is what we now call the "Reynolds number" of the sphere. This dimensionless number is based on the diameter (d=2α) and the relative velocity of the sphere with respect to the fluid.

In a little known paper to the Academy of Paris, Boussinesq (1885a) was the first to publish an analytical expression for the transient hydrodynamic force exerted by a viscous fluid on a solid sphere, when the relative velocity of the sphere is very small (creeping flow). Based on a mathematical method outlined in his book on the methods of calculation of the rate of heat transfer, Boussinesq (1885b), he presented an accurate and succinct method for the determination of the transient equation of the viscous fluid motion around a solid sphere at creeping flow conditions (Re→0) and derived an analytical expression for the transient hydrodynamic force, which is composed of three terms: the steady state part, the added-mass or virtual-mass part and the history part. Three years later, Basset (1888a and 1888b), apparently unaware of the work of Boussinesq, presented a similar method and derived an identical expression for the transient hydrodynamic force acting on a solid sphere. While both Boussinesq and Basset derived their expressions for a solid sphere moving in a stagnant fluid, it is rather trivial to extend their theory to a sphere moving in a fluid of uniform velocity, u_i. In this case, the transient hydrodynamic force, $F_i(t)$, may be written as follows:

$$F_i(t) = -6\pi\alpha\mu_f(v_i - u_i) - \frac{1}{2}m_f\frac{d}{dt}(v_i - u_i) - 6\alpha^2\sqrt{\pi\mu_f\rho_f}\int_0^t \frac{\frac{d}{d\tau}(v_i - u_i)}{\sqrt{t-\tau}}d\tau. \tag{1.1.2}$$

The first term of the above expression is the steady-state part exerted by the viscous fluid and is identical to the steady-state Stokes drag. The second term is due to the transient motion of the surrounding fluid. Although the domain of the fluid that is influenced by the transient motion of the sphere may be considerably larger, the net effect of the fluid motion on the transient force exerted on the sphere is equivalent to the simultaneous acceleration of a volume equal to half the volume of the sphere. The third term in the equation is due to the diffusion of the vorticity around the sphere and, for slow transient flows, decays at a rate proportional to $t^{-1/2}$, which is typical of diffusion processes. It is called the "history term" or "history force" and sometimes the "Basset force."

The analytical studies by Boussinesq and Basset are based on an assumption that neglects the inertia terms in the Navier-Stokes equations. For this reason, Eq. (1.1.2) strictly applies to the case of a sphere moving very slowly in the viscous fluid, a condition, which is satisfied only when the Reynolds number of the sphere, Re, approaches the value zero. It must be pointed out that, in the case of a sphere in a fluid, which is itself in motion, there are two pertinent Reynolds numbers: the first is based on the characteristic velocity, U, and the characteristic dimension of the flow, L_f, and the second is based on the local relative velocity of the sphere and the characteristic dimension of the sphere, $d=2\alpha$:

$$Re_f = \frac{L_f \rho_f U}{\mu_f} \quad \text{and} \quad Re = \frac{2\alpha \rho_f |u - v|}{\mu_f}. \tag{1.1.3}$$

It is evident that $Re_f > Re$ and, in most cases $Re_f >> Re$. For the Boussinesq/Basset equation to be valid, the necessary condition is $Re << 1$.

The first attempt to use an asymptotic theory and solve the full Navier-Stokes equations for a sphere and to derive an expression for the transient force on a solid sphere at finite values of Re was made by Whitehead (1889). His attempt was unsuccessful, because of an incorrect matching of the near- and far-field conditions around the sphere. Two decades later, Oseen (1910, 1913) using a correct asymptotic analysis, was able to decompose successfully the inertia terms of the Navier-Stokes equations and used the correct matching conditions. He obtained a zeroth-order expression for the velocity perturbation around the sphere, which enabled him to derive a solution for the hydrodynamic force, valid at finite but small Re (Re<1). The studies by Oseen (1910, 1913) resulted in the extension of the Stokes' drag coefficient to an expression, which is often called "Oseen's drag coefficient:" $C_D=24(1+3/16Re)/Re$. A decade later, Oseen's student, Faxen (1922) studied the flow of spheres close to solid boundaries and extended the theory of the transient flow around a sphere to non-uniform flows. His work resulted in the introduction of new terms in the expression of the hydrodynamic force, which are now called the "Faxen terms." These terms account for the non-uniformity in the flow field surrounding the sphere and are expressed in terms of the Laplacian derivatives of the flow field. A more detailed description and explanation of these terms is given in Ch. 6.

Tchen (1949) derived the creeping flow equation for a solid sphere in a fluid with a time-varying velocity field $u_i(t)$. He included the acceleration/inertia term that results from any pressure gradient far from the sphere. A variant of this expression, with a small correction for the effect of the pressure gradient, was used by Corssin and Lumley (1957) for the study of small particles in a time-dependent turbulent flow field. A few years later, Sy et al. 1970, also derived the transient equation for the motion of a solid sphere at very low Re and coined the term "creeping flow," to denote the vanishingly small relative velocity of the sphere.

Proudman and Pearson (1956) used an asymptotic method of higher order to extend Oseen's result for the steady motion and to derive a higher-order expression for the steady drag coefficient of a sphere at finite but still small values of Re. Twenty-five years later, Sano (1981) using an asymptotic method derived an analytical expression for the transient hydrodynamic force on a stationary sphere, when the fluid around the sphere undergoes a step velocity change and the Re is small but finite. He was first to show that, at finite Re, the vorticity gradients around the sphere are advected far from the sphere and the history terms decay faster than $t^{-1/2}$, which is the consequence of the creeping flow theory.

The study of Maxey and Riley (1983) has been considered by many as the definitive study on the equation of motion of a solid sphere under creeping flow conditions. The resulting form of the transient equation of motion encompasses the unsteady and non-uniform fluid motion as well as body forces. Their final expression is the Lagrangian equation of the motion of the sphere, from which the hydrodynamic force may be easily deduced. In a lesser-known paper, published on the same year, Gatignol (1983) also derived a very similar expression for the Lagrangian equation of motion of a solid sphere. A few years later, Michaelides and Feng (1995) derived an extension to this equation that includes the velocity slip on the surface of the sphere as well as the effects of finite viscosity (for bubbles and drops) always under creeping flow conditions ($Re \ll 1$).

The first part of the 1990's saw a great deal of activity and many excellent studies on the analytical expressions as well as significant computational results on the transient hydrodynamic force that acts on particles, bubbles and drops. Mei et al. (1991) in a semi-analytical study investigated the dependence of the transient hydrodynamic force on the frequency of the flow, for finite values of Re. Mei and Adrian (1992) extended Sano's (1981) result to higher Re and proved that, at high values of Re, the history component of the hydrodynamic force essentially decays as t^{-2} and not as $t^{-1/2}$. Lovalenti and Brady (1993a) used an asymptotic expansion method to obtain a general expression for the transient hydrodynamic force on a rigid particle of any shape, subjected to arbitrary fluid motion. Lovalenti and Brady (1993b) extended this approach to determine the hydrodynamic force on bubbles and drops. In an appendix of the first article, Hinch (1993) gave a physical explanation of the various terms in the analytical expression of the hydrodynamic force, and derived their rates of decay in a simple manner. The last two studies show that the force during the acceleration and deceleration of spheres in viscous fluids at finite Re, are different, because of the influence of the inertia of the fluid wakes that are formed behind the sphere. Thus, in the case of finite Re, the motion of a sphere in a fluid is not invariant with respect to time, while the creeping flow solutions are invariant with respect to time, since the resulting hydrodynamic force does not change by substituting -t for t. This invariance is not to be confused with thermodynamic irreversibility: the steady drag and history parts of the hydrodynamic force are due to fluid friction and, hence, the motion of any object in a viscous fluid is inherently irreversible.

As a result of the recent studies, it has become apparent that exact analytical expressions for the transient hydrodynamic force on a sphere may only be obtained at low values of the Reynolds numbers (Re<1) and that such solutions at moderate to high Re are impossible to obtain analytically. However, several practical applications, actually the vast majority of engineering applications, pertain to flows at moderate to high Re. For this reason, in the last few years, the attention and efforts of the researchers have been focused on numerical studies, which are very specific in their processes and values of parameters, but give accurate and useful results under conditions that cannot be matched by analytical studies. Examples of such numerical or semi-analytical studies are the ones by Chang and Maxey, (1994, 1995) and Magnaudet et al. (1995) which complements the analytical approaches and help determine or elucidate the behavior of the transient terms at different flow conditions.

1.1.2 Heat transfer

It is rather significant that the basic studies on the transient heat transfer from a sphere immersed in a fluid have preceded the studies on the transient motion of a sphere in a viscous fluid. In an attempt to calculate the age of the earth, Jacques Fourier undertook the first study on the transient rate of heat transfer from a solid sphere to a fluid. Fourier published his theory and results on the heat transfer from spheres in a series of articles of the transactions of the Academy of Paris. These articles culminated in the printing of his famous book on the heat transfer (Fourier, 1822). A student of Fourier, Peclet, was among the first to conduct experiments on the heat transfer by convection, which is caused by the motion of a fluid. He confirmed that the advection process enhances the heat transfer coefficient and, also, observed that the rate of heat transfer from the fluid close to a solid boundary is significantly lower than in the bulk of the fluid. Peclet attributed this phenomenon to the slowing and stagnation of the fluid motion at the wall and, thus, stipulated for the first time what is now called the "no-slip condition" on solid boundaries. This is the first application of the subject of thermal velocimetry.

Fourier's book on heat transfer has been considered as one of the most important scientific developments of the nineteenth century, as well as the intellectual stimulus for methods adopted in other scientific fields including the flow of electric currents (Tait, 1885) and the development of irreversible thermodynamics (Prigogine, 1955). Nusselt and others, who worked on the convection mode of heat transfer, basically followed Fourier's closure equation and expressed their results for the rate of heat transferred in terms of a heat transfer coefficient, in analogy with Fourier's expression. Thus, based on $Q=-kA\Delta T$, they proposed the analogous expression: $Q=-hA\Delta T$. In a twentieth-century treatise of the subject of conduction, Carslaw and Jaeger (1947) basically extended Fourier's ideas on the transient conduction from a solid sphere as well as other simple geometrical shapes and presented several analytical solutions on different applications of transient heat conduction. It must be pointed out that, because Carlslaw and Jaeger (1947) dealt strictly with the subject of conduction, their solutions apply to fluids only under the condition of vanishingly small Peclet numbers (Pe=Re*Pr). The latter, is the dimensionless number for the energy transfer processes and is analogous to the Reynolds number of the equation of motion. As with the Reynolds number of Eq. (1.1.3), one may define two Peclet numbers, for the fluid and for the sphere that moves inside the fluid and exchanges energy:

$$Pe_f = \frac{L_f \rho_f c_{pf} U}{k_f} \quad Pe = \frac{2\alpha\rho_f c_{pf} |u - v|}{k_f} . \tag{1.1.4}$$

Although the original work on the steady-state heat transfer from a sphere preceded the studies on the hydrodynamic force, most of the recent studies on the heat or mass transfer from spheres followed the corresponding studies for the determination of the hydrodynamic force and are often based on the same or similar analytical methods. For example, in order to calculate the steady rate of heat transfer from a sphere at finite but small Pe, Acrivos and Taylor (1962) used the asymptotic method of Proudman and Pearson (1956) to derive an expression for the Nusselt number. Several other empirical studies in the 1960's and 1970's resulted in correlations for the convective heat transfer coefficients at transient or steady processes, which are similar to the corresponding expressions for the steady drag coefficients.

An analytical expression for the transient energy equation of a sphere, which would correspond to Eq. (1.1.2), was not known until the later part of the twentieth century. Michaelides and Feng (1994) used an analytical method, and obtained the first complete analytical solution for the unsteady energy equation of a sphere at creeping flow conditions. They showed that the general form of the transient energy equation from a sphere, at $Pe \ll 1$, contains a history term, which emanates from the temporal change of the temperature gradients around the sphere. This history term is similar in its functional form to the history term of the Boussinesq/Basset expression and decays as $t^{-1/2}$. A subsequent study by Feng and Michaelides (1998a) showed that the transient heat transfer from a particle is significantly different at small but finite values of Pe and that the history term decays faster than $t^{-1/2}$ when advection affects the process and Pe is finite.

The main results on the momentum transfer from a sphere to a fluid as well as the energy and mass transfer at wide ranges of Re and Pe may be found in several specialized treatises on these subjects, such as the ones by Leal (1992), Kim and Karila (1991) Crowe et al. (1998) and Sirignano (1999), or in specific review articles, such as the ones by Leal (1980) on the motion of fine particles at low Re, Brady and Bossis (1988) on the Stokesian formulation of suspension systems, Feuillebois (1989) on the asymptotic methods applied to the equation of motion of spheres in viscous liquids, Sirignano (1993) on the formation and flow of drops and sprays, Stock (1996) on particulate dispersion and the effect of crossing trajectories, Michaelides and Feng (1996) and Michaelides (2003a) on the analogies between the heat transfer and motion of particles, Michaelides (1997) on the transient equation of motion of particles, Loth (2000) on the numerical methods for the treatment of the motion of immersed objects, Koch and Hill (2001) on the inertia effects of suspended particles and Sazhin (2006) on droplet heat transfer and combustion. Older monographs by Levich (1962), Clift et al. (1978), Happel and Brenner (1963), Govier and Aziz (1977) and Soo (1990) include useful theoretical and empirical results on the motion and heat or mass transfer processes. In addition, the proceedings of the recent International Conferences on Multiphase Flows (ICMF-98, ICMF-2001, ICMF-2004) and a series of several symposia on Gas-Particle flows (Stock et al. 1993, 1995, 1997, 1999, 2003 and 2005) comprise a variety of analytical, numerical and experimental papers on the equation of motion of particles as well as a multitude of industrial applications on the subject.

1.2 Terminology and nomenclature

The experimental, analytical and numerical results that apply to the motion, heat and mass transfer of particles, bubbles and drops have been derived for specific flow and thermal conditions and, invariably, under restrictive conditions for the ranges of properties of the surrounding fluid as well as the material properties and shapes of the particles, bubbles or drops. These final results are frequently used by scientists and engineers in the design of equipment and processes or in the development of numerical algorithms. Therefore, it is of paramount importance, for all results, to be faithfully quoted in a precise manner that minimizes any confusion and misunderstanding as to their validity and the range of their applications. For this reason, an attempt will be made to quote in a precise and explicit manner all the pertinent conditions, under which results or formulae have been derived and experimental or numerical data have been produced. Details of derivation will not be repeated. The interested reader may find all the details by consulting the appropriate references.

1.2.1 Common terms and definitions

In a book that is dedicated to the thermal and hydrodynamic behavior of particles, bubbles and drops, it becomes repetitive and mundane to keep referring to all three by name. For this reason the term "object" or "immersed object" will be used to denote that the derived results (equations, experimental or numerical data and analytical methods) pertain to all, solid particles, bubbles and drops. A distinction will be made for the results that pertain to spheres exclusively as opposed to results that pertain to other shapes of immersed objects. Although the term "particles" is occasionally used to denote both solid particles and viscous drops, this practice will be avoided and the term "particles" will be reserved for solid particles, which often have irregular shapes. When results apply only to a spherical particle, this will be explicitly denoted. The term "drop" will denote a viscous object of any shape with density higher than the density of the carrier fluid, while the term "bubble" will denote a similar object with density lower than that of the carrier fluid. The term "viscous spheres" will refer to both bubbles and drops with finite viscosity and spherical shapes and the term "viscous objects" to bubbles and drops with non-spherical shapes. The terms "inviscid spheres" or "inviscid objects," refers to bubbles whose viscosity is much less than the viscosity of the surrounding fluid ($\lambda << 1$). The most common symbols that are used throughout this treatise are listed in the following section. Any diversion from this nomenclature is only made because common practice is followed and is explicitly mentioned in the appropriate section.

1.2.2 Nomenclature

1.2.2.1 Latin symbols

a	thermal diffusivity
a, a_i	acceleration (vector)
a, b	dimensions of spheroids
A	area
Ac	acceleration number
Ar	Archimedes number
b, b_i	body force (vector)
B_H, B_m	Blowing coefficients
Bi	Biot number
c	velocity of sound
Ca	capillary number
Co	Corey shape factor
c_p	specific heat
C_D	drag coefficient
C_L	lift coefficient
d	diameter
d_e	equivalent diameter
D	diffusion coefficient
D	pipe diameter
De	deformation number
e	energy
e_r	restitution coefficient
E	aspect ratio
E_E	electric field intensity
E_Y	Young's modulus
Eo	Eötvös number
erf	error function
erfc	complem. error function
f	frequency distribution
f_f	friction factor
f_p	probability distribution
F, F_i	force (vector)
F_F	Faraday constant
g, g_i	acceleration of gravity
G	mass flux
h	heat transfer coefficient
h_{fg}	latent heat
H	height
I	turbulence intensity
i,j,k	unit vectors
J(ε)	lift force function
k	thermal conductivity
k_T	turbulent kinetic energy
k_B	Boltzmann constant
k_s	stiffness coefficient
$k_{c\text{-}o}$	cut-off wave number
K	constant
K_{th}	thermal correction factor
K_p	permeability
Kn	Knudsen number
l, L	length
$l_{c\text{-}o}$	cut-off length
Le	Lewis number
m	mass
$\dot{m}$	mass flow rate
m^*	mass loading
m_c	wetting coefficient
M	molecular weight
Ma	Mach number
Mo	Morton number
n	unit normal vector
n	number density
N	total number
Nu	Nusselt number

p	perimeter
P	pressure
Pe	Peclet number
Pr	Prandtl number
q	heat flux
q_E	electric charge
q_t	porous flow velocity
Q	heat transfer
r, $\mathbf{r_i}$	radius (vector)
r	magnitude of radius
R_g	gas constant
$\tilde{R}$	universal gas constant
$\mathbf{R_U}$, $\mathbf{R_E}$	fluid resistance matrices
Re	Reynolds number
Re_S	Re based on superficial velocity
S	surface area
Sc	Schmidt number
Sh	Sherwood number
Sp	Slip number
St	Stokes number
Sl	Strouhal number
S_r	Radiative source strength
S_s	Solidification parameter
Ste	Stephan number
t	time
T	temperature
T	torque
u, u_i	carrier fluid velocity
U	characteristic velocity
v, v_i	velocity of particle, bubble or drop
V	volume
w, w_i	relative velocity
W_i	weight functions
We	Weber number
x,y,z	Cartesian coordinates
z_E	electric charge/valence
X	mole fraction
Y	mass fraction

1.2.2.2 Greek symbols

α	radius
β	density ratio
β'	heat capacity ratio
γ	rate of shear
γ	ratio of specific heats
δ	film thickness, distance
Δ	difference
ε	eccentricity,
ε	velocity ratio
ε_E	electric permitivity
ε_p	porosity
ε_T	turbulent dissipation
ζ	vorticity
ζ_E	electric potential
ζ_F	friction coefficient
η	tortuosity
η_D	damping factor
θ	angular coordinate
κ	von Karman constant
κ_s	shear rate
λ	ratio of viscosities
λ_D	Debye length
λ_T	Turbulence parameter
μ	dynamic viscosity
ν	kinematic viscosity
ξ	length parameter
Ξ	dimensionless distance
ρ	density

σ	surface tension
σ_{ij}	stress tensor
σ_E	electrical conductance
σ_P	Poison ratio
τ	timescale
τ	variable for time
ϕ	concentration
Φ	potential function
χ	Laplace timescale
χ_{12}	separation distance
ψ	streamfunction
ω	rotational speed
ω	vorticity
Ω	angular integral

1.2.2.3 Subscripts

*	friction velocity
AM	added mass
b	bubble
bk	bulk
br	brownian
c	continuous phase
ch	characteristic
cs	cross-sectional
cr	critical
d	dispersed phase
dr	drop
ed	eddy
ep	electrophoresis
eo	electro-osmosis
F	carrier flow or fuel
f	carrier fluid
g	gas
h	hydrodynamic
H	pertains to history
I	interaction
in	inlet
ip	interparticle
ir	irregular
K	Kolmogorov
l	liquid
LR	rotation lift
LS	shear lift
m	pertains to mass
M	pertains to momentum
max	maximum
min	minimum
mol	molecular
p	solid particle
P	projected (for areas)
R	reference
rad	radial
rel	relative
s	sphere
suf	surface
sat	saturated
t	terminal (for velocity)
T	turbulent
th	thermal
tan	tangential
tp	thermophoretic
v	viscous
w	wall
Λ	integral timescale

1.2.2.4 Superscripts

*	dimensionless
‘	fluctuation
.	time rate
∞	undisturbed

ad	advection
o	degree
T	total
//	parallel
⊥	perpendicular
+	wall coordinate

1.2.3 Common abbreviations

ADE	Advection-Diffusion Equation
CFD	Computational Fluid Dynamics
CHF	Critical Heat Flux
DNS	Direct Numerical Simulation
DRA	Drag Reduction Agent
FBR	Fluidized Bed Reactor
FD	Finite Differences
FE	Finite Elements
FV	Finite Volume
HWA	Hot Wire Anemometry/ Velocimetry
IBM	Immersed Boundary Method
LBM	Lattice Boltzmann Method
LDV	Laser Doppler Velocimetry/ Velocimeter
LES	Large Eddy Simulation
MD	Molecular Dynamics
N-SE	Navier-Stokes Equations
ODE/ PDE	Ordinary/Partial Differential Equation
PDF	Probability Distribution function
RANS	Reynolds Averaged Navier-Stokes equations

1.2.4 Dimensionless numbers (L_{ch}=2α)

$$Ac=\frac{|\vec{u}-\vec{v}|^2}{2\alpha\left|\frac{d\vec{v}}{dt}\right|},\quad Ar=\frac{g\rho_f|\rho_f-\rho_s|d^3}{\mu_f^2}=\frac{Eo^{3/2}}{Mo^{1/2}},\quad B_m=\frac{Y_s-Y_\infty}{1-Y_s},\quad Bo=\frac{4g\alpha^2|\rho_s-\rho_f|}{\sigma},$$

$$Bi=\frac{2\alpha h}{k_s},\quad \beta=\frac{\rho_f}{\rho_s},\quad Ca=\frac{\mu_f|\vec{u}-\vec{v}|}{\sigma}=\frac{We}{Re},\quad Co=\frac{L_{min}}{\sqrt{L_{max}L_{int}}},\quad De=\frac{a-b}{a+b},$$

$$Eo=\frac{g\rho_f d^2}{\sigma},\quad Fo=\frac{3k_s t}{2\alpha^2\rho_s c_s},\quad Fr=\frac{|\vec{u}-\vec{v}|^2}{2g\alpha},\quad Kn=\frac{L_{mol}}{2\alpha}=\frac{1.051k_BT}{2\sqrt{2}\pi\alpha d_{mol}^2P}$$

$$\mathrm{Le}=\frac{k_f}{\rho_f c_{pf} D_f},\quad \mathrm{Ma}=\frac{|\vec{u}|}{c},\quad \mathrm{Mo}=\frac{g\mu_f^4}{\rho_f \sigma^3},\quad \mathrm{Nu}=\frac{2\alpha h}{k_f},$$

$$\mathrm{Pe}=\frac{2\alpha\rho_f c_{pf}|\vec{u}-\vec{v}|}{k_f},\quad \mathrm{Pe}_\gamma=\frac{4\alpha^2\gamma\rho_f c_{pf}}{k_f},\quad \mathrm{Pe}_m=\frac{2\alpha|\vec{u}-\vec{v}|}{D_f},\quad \mathrm{Pr}=\frac{c_{pf}\mu_f}{k_f}$$

$$\mathrm{Re}=\frac{2\alpha\rho_f|\vec{u}-\vec{v}|}{\mu_f},\quad \mathrm{Re}_\gamma=\frac{4\alpha^2\gamma\rho_f}{\mu_f},\quad \mathrm{Re}_R=\frac{\rho_f d^2|\vec{\Omega}|}{\mu_f},\quad \mathrm{Sc}=\frac{\mu_f}{\rho_f D_f}$$

$$\mathrm{Sh}=\frac{2\alpha h_m}{D_f},\quad \mathrm{Sp}=\frac{\mu_f w_{tan}}{\alpha\tau_{tan}},\quad \mathrm{St}_M=\frac{\alpha\rho_s|\vec{u}-\vec{v}|}{9\mu_f},\quad \mathrm{St}_{th}=\frac{\alpha\rho_f c_f|\vec{u}-\vec{v}|}{2k_f}$$

$$\mathrm{Sl}=\frac{2\alpha|\vec{\Omega}|}{|\vec{u}-\vec{v}|}\ \text{ or }\ \mathrm{Sl}=\frac{1}{\mathrm{St}_M},\quad \mathrm{Ste}=\frac{c_P\Delta T}{h_{fg}},\quad \mathrm{We}=\frac{2\alpha\rho_f|\vec{u}-\vec{v}|^2}{\sigma}$$

1.3 Examples of applications in science and technology

Applications of the flow, heat and mass transfer of particles, bubbles and drops are omnipresent in everyday life and in engineering practice. Diverse natural and engineering systems, ranging from nuclear reactors to internal combustion engines, from petroleum refining equipment to sediment and pollutant transport processes in aquatic environments entail carrier fluids that convey dispersed materials of another phase, most often in the form of particles, bubbles and drops. The design and optimization of these systems and even the mere understanding of their operation renders necessary the knowledge of the fundamental processes that pertain to the flow, mass and heat transfer from individual particles, bubbles and drops.

In a flowing mixture of two or more phases, the different phases have distinct physical properties and, in general, move with different velocities. In all cases, the constituents of the flowing mixture exchange linear and angular momentum, oftentimes they exchange mass, and in many cases the constituents of the multiphase mixture exchange energy. For example, in the case of a direct contact heat exchanger, where colder drops are sprayed in the midst of a vapor mass, the drops absorb enthalpy from the vapor, and thus, their temperature increases. The vapor stream slows down and then carries the drops by the action of the hydrodynamic force, which is a manifestation of the momentum exchange process. Because of the contact between the cooler drops and the vapor, some of the vapor condenses on the surface of the drops, thus, increasing their size. As a result of the hydrodynamic interaction between the vapor and the drops, larger drops may break-up in two or more smaller drops.

While it is possible to derive general equations for the exchange of mass, momentum and heat in all dispersed multiphase flow applications, because of the complexity of most practical problems, it is difficult and often impossible to obtain an exact solution of these equations in the most general cases without the use of simplifying assumptions that restrict the generality of the solutions. This does not pose a major problem in most engineering applications, because engineers and scientists are not interested in all the details of the flow and the transport processes, but in specific characteristics and properties of the multiphase system, which may be needed for the design of the system or for the optimization of a process. For this reason, the interest in the solution of the multiphase flow equations, and specifically in flows that include immersed objects, is not for all the properties of the fluids and the characteristics of all the interactions, but in specific aspects of these interactions that help answer a scientific or technical question. The following cases give examples of such applications that include particles, bubbles and drops and the characteristic parameters of interest.

1.3.1 Oil and gas pipelines

In the production of liquid hydrocarbons (oil), lighter gaseous hydrocarbons (natural gas) are oftentimes a byproduct. The presence of gas bubbles in the pipeline assists in the "lifting" of the oil and affects the pressure temperature and viscosity of the flowing mixture. In this case, the pipeline engineer who is mainly interested in the volumetric flow rates of the oil and gas at the wellhead, often has to calculate the viscosity of the oil-gas mixture and the relative or terminal velocity of the bubbles under the local conditions. Also of interest is bubble coalescence, which is the result of higher concentration or evaporation. Coalescence of smaller bubbles causes the formation of elongated "Taylor bubbles" and liquid slugs, which alter the flow regime. This needs to be accounted for in the design of the pipeline, because Taylor bubbles are associated with pipe vibrations and possible damage to the long pipeline and the equipment at the well-head.

1.3.2 Geothermal wells

Hot water and steam are carried upwards in a geothermal well under the action of a pressure gradient. The decrease of the pressure of the fluid along the pipeline results in the continuous production of vapor, which in turn results in several interesting flow patterns/regime for the liquid-vapor mixture, as may be seen in Fig. 1.1. The vapor phase first appears as spherical bubbles (bubbly flow). When the concentration of the bubbles exceeds approximately 5%, the bubbles coalesce to form bigger, irregular bubbles or longer elongated Taylor bubbles that have a bullet shape (slug flow). At higher vapor concentrations, the vapor "pushes" the remaining liquid to the sides of the pipe to form a liquid annulus with droplets entrained in the vapor core (annular flow). Finally, if the fluid enthalpy is high enough and the local pressure low enough, the liquid annulus evaporates and breaks down to drops (droplet flow). In this case, the interest of the geothermal engineer is restricted to the mass flow rates of the vapor and liquid at the well-head, as well as their thermodynamic state, which is defined by the wellhead pressure and temperature. Details of the complex flow regimes are of secondary importance.

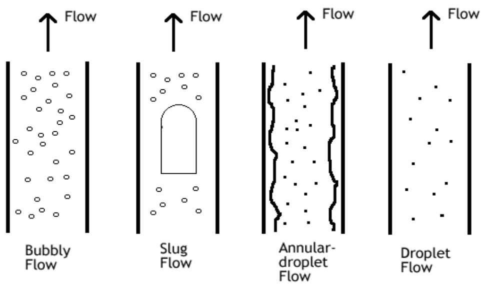

Fig. 1.1 Flow regimes in a geothermal well

1.3.3 Steam generation in boilers and burners

The flow regimes in the production of steam in boilers and burners are very similar to the rising of geothermal fluid. The difference is that evaporation is caused by the addition of heat and spherical bubbles are formed in the early stages of the process. These bubbles coalesce to form Taylor bubbles and then large irregular vapor formations inside the liquid mass. The next stage is annular flow with the liquid flowing at the perimeter of the pipe and the flow of droplets at the core. Finally, both the droplets and liquid film evaporate to vapor. In the case of boiling, the engineer is primarily interested in the rate of heat transfer between the pipe wall and the flowing two-phase mixture. Of interest are also the pressure loss in the equipment, any possible flow-induced vibrations that may affect the structural integrity of the boiler and any flow and thermal instabilities that may affect the temperature of the pipe, such as critical heat flux. Details of the flow regimes are of secondary importance and are usually bypassed in the design calculations.

1.3.4 Sediment flow

Rivers carry a large amount of sedimentary particles that finally deposit in the coastal environment and the oceans. The sedimentary particles by themselves may be carriers of molecules of heavy metals as well as organic and inorganic pollutants that adhere to their surfaces. Environmental engineers are usually interested in the mass and momentum exchange between the sedimentary particles and the carrier fluid. They use "partition coefficients" to determine the amount of materials carried by the liquid stream and by the particles. Thus, they may calculate the transport and effects of pollutants in the aqueous environments. In parallel, their interest may lie in the calculation of transient land erosion and deposition into rivers and lakes or into coastal environments caused by weather events as in the case of rainstorms, hurricanes or tropical storms.

Similarly, hydraulic engineers and scientists may be interested in sediment deposition and resuspension processes. This occurs in works of preservation of a navigation channel by dredging. Also, hydraulic engineers may be interested in wetlands nourishment with new soil, which is accomplished through river diversion and the flooding of wetlands for the deposition of layers of particles that constitute the silt. Scientists and engineers may not be interested in the behavior of the sediment *per se* but only in the transport and fate of pollutants that may be attached to sedimentary particles and are carried downstream. Such an example presents the case of the radionuclides that were released in the aftermath of the Chernobyl accident and were washed off the land by rain runoff. These long-living radionuclides have been deposited and will be present for several years in the sediments of the river system in the countries of Ukraine, Belarus and Russia. The radionuclides are slowly transported with the particles that form the sediment, usually after severe weather events that cause floods, thus spreading small amounts of radioactivity downstream and, eventually into the coast of the Black Sea.

1.3.5 Steam condensation

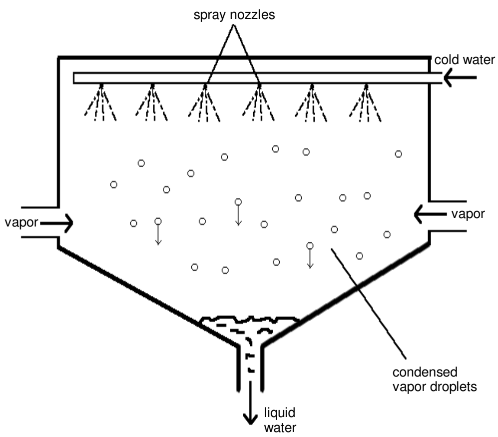

Fig. 1.2 Schematic diagram of a direct contact condenser

The condensation process is the inverse of boiling. Indirect condensation of vapor occurs on the surface of colder tubes while direct condensation occurs at the surface of sprays of cool drops. Fig. 1.2 presents a schematic diagram of direct condensation. Several small spray nozzles introduce colder droplets into the steam and fall towards a liquid well, from where they are pumped to the heat rejection equipment as a stream of liquid before they reenter the condenser. Vapor in contact with the cooler droplets condenses, thus causing an increase of their size. Condensation also releases the latent heat of the drops and warms them. Of primary interest to the engineers of the direct condensation process is the heat transfer to a falling, growing drop. This is determined by the drop size, relative velocity and the hydrodynamic force. The latter determines the time the drop will be in suspension and exchange heat with the vapor. Other details, such as drop coalescence and drop splashing at their contact with the liquid well are of secondary importance.

1.3.6 Petroleum refining

Refining is essentially the combination of evaporation of the crude oil and the subsequent condensation of its constituents at different temperatures. Heat addition by bubbling steam is very common in the evaporation process and, hence, the heat transfer from condensing bubbles is one of the required variables in the design of this equipment. On the condensation side, the several fractions of the oil condense at separate parts or "trays" of the distillation column, each of which is kept at approximately constant temperature. Drop and film formation and growth, the amount of heat provided by the condensing steam bubbles, the amount of heat removed from the condensing petroleum fractions as well as the surface tension properties of the drops on the condensing surface are of interest to the engineers who design this type of equipment and processes.

1.3.7 Spray drying

Figure 1.3 shows the spray drying process, which is used in the production of several types of pharmaceuticals, foodstuffs and chemicals. In such processes, slurry that contains the principal material is introduced at the top of the dryer and is atomized by nozzles into small drops that fall in the dryer. A hot gas, usually hot air, is introduced from the sides and flows upwards towards a vent. The drops of the spray come in contact with this hot gas, exchange heat and cause the evaporation of the liquid in the drops and, hence, the drying of the material. Here, one is interested in the mass of the primary material carried by the drops, the heat transfer from the gas to the drops, the mass exchange that results in the evaporation of the product-laden drops and the quantity of the dried material, which is extracted at the bottom of the dryer.

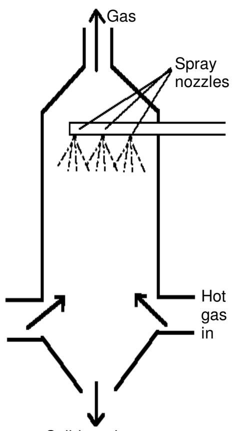

Fig. 1.3 A spray dryer

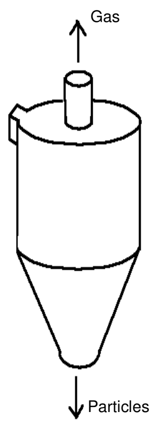

Fig. 1.4 A cyclone separator

1.3.8 Pneumatic conveying

Pneumatic conveying is widely used for the transport of solid particles, such as coal, cement, metal powders and chemicals via a pipeline at distances up to several kilometers. Pressurized air is introduced in the pipeline and, the solid material is introduced downstream through airtight valves. The motion of the air in the pipeline carries the solids to the end. A cyclone separator at the end is used for the separation of solid particles from the carrier air stream. A schematic diagram of such a separator is shown in Fig. 1.4. The air-solids mixture is introduced tangentially at the side of the separator and causes the formation of a large vortex. The particles, under the influence of the vortex, the centrifugal force and the gravitational force, travel tangentially and downwards close to the walls of the separator, and, finally drop in the converging part and exit through the bottom of the equipment. The air mass moves toward the center of the separator and, finally exits through the top. Such cyclones, when well designed, may separate efficiently particles as small as 5 μm from the carrier gas flow. Smaller particles with lower inertia may be carried by the air stream. In the case of a cyclone separator, one is interested in the downward transport of particles, and the sizes that are finally collected at the bottom. Flow turbulence, lift on the particles, particle interactions and collisions with the walls of the separator also play important roles in the process and the design of the equipment (Bohnet et al. 1997).

1.3.9 Fluidized beds

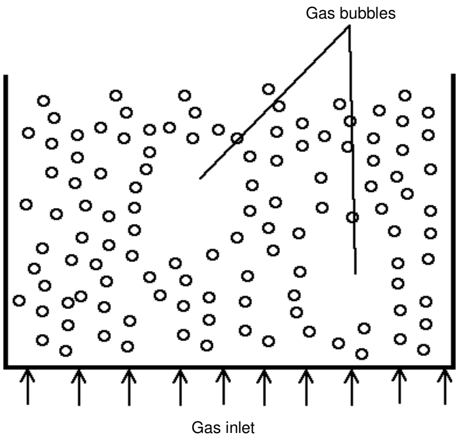

Fig. 1.5 Schematic diagram of a fluidized bed reactor

Fluidized beds are increasingly used for the more efficient combustion of coal and the reduction of pollutants, such as sulfur. Coal particles are burned or gasified in a stream of air that is blown through holes from the bed of particles. Fig. 1.5 shows a schematic diagram of the fluidization process. The coal particles are in suspension supported by the hydrodynamic force of the flowing air stream, something that helps the completion of the combustion and makes the burning process more efficient. Occasionally, “air bubbles” appear in the bed and move upwards through the solid particles. The more efficient combustion and higher temperatures that are achieved in a fluidized bed result in lower percentage of pollution products through the stack. In this case, one is interested in the drag force on the particles that keep them suspended, the interactions of the particles, the rate of heat transfer with the pipes, the rate of coal combustion and the rate of secondary reactions, such as the one of added limestone with the sulfur that is introduced with the coal.