1.

Introduction and Overview

"Truth loves its limits, for there it meets the beautiful."
R. Tagore, *Fireflies*

1a. Special relativity is NOT incorrect!

When one encounters a book about relativity theory whose title is not some straightforward variation of the words "special relativity" or "general relativity" and whose purpose is not simply to describe those theories and their applications, there is always the sneaking suspicion that the authors have some kind of vendetta against Einstein's theory and that somewhere along the line, the reader will be treated to some elaborately constructed, but ultimately incorrect, explanation of why Einstein's relativity theories must be wrong. It is the purpose of this first paragraph to assure the reader that this is not the case with the book presently sitting in the reader's hands. For over a century, experiments have been performed to test Einstein's most famous work and none has ever been found to be inconsistent with the predictions of the theory of special relativity. The rigorous testing to which special relativity has been subjected has cemented its place in physics as solidly as Newton's theory of gravitation. The existence of any major flaws in the theory has virtually been ruled out and if corrections to the theory need to be made in the future, the regimes in which those modifications might need to be made (e.g., at very short distances) are well-recognized.[1]

Instead, the purpose of this book is, as its title suggests, to step back and to take a broader view of relativity and in particular, the principle of relativity upon which it is based. In the one hundred years since Einstein formulated the theory of special relativity, our way of thinking about the nature of space and time, especially the units of measurement we use in quantifying them, has undergone a significant conceptual shift. If we were to look back at the principle of relativity now, would we see anything that might have been missed by the great physicists of the early 1900's who, though visionary for their time, were still constrained to some extent by the prevailing notions of space and time? Our answer is yes. In this initial chapter, we give an overview of the content of this book, laying out our ideas and the motivation behind them. Some of the ideas will undoubtedly seem strange, or even absurd,[2] as special relativity must have seemed to many physicists of the early 20th century. While we make no pretentions to our intellectual capacity or insight *vis à vis* that of Einstein or to the effects that our ideas might have compared to the revolution in scientific thought sparked by special relativity, we do ask that the reader keep in mind the ultimate criteria by which the usefulness of theories and ideas are judged in science, namely whether they are

(1) consistent with experimental results, and
(2) useful for predicting and explaining the phenomena we observe in the universe around us.

1b. Idea #1: Einstein's first postulate of relativity (the principle of relativity) is the only necessary ingredient of a viable theory

Part A of this book, chapters 2-6, are devoted to a short history of the theory of special relativity, with particular emphasis on the contributions of those other than Einstein. This is not meant to diminish Einstein's role in the creation of the theory by any means, but instead an attempt to give credit where it is due to some of the other physicists who, though unable to make the great leap that Einstein did, still came close.

In part B of the book, comprised of chapters 7-17, we explore the relationship between the principle of relativity and the units that we use to quantify our measurements of space and time. In particular, we argue that our present definitions of those units, the meter and second in the SI system of units, has restricted the way we think about spacetime[3] and that removing some of those restrictions opens up new ways of thinking about relativity, in addition to putting additional tools in the physics toolbox from which we draw when trying to solve physical problems. This exploration was motivated originally by a careful analysis[4] of some of the precision experimental tests of special relativity and by the realization that "relativistic time, or any particular time system for that matter, is not a necessary ingredient of a theory for it to correctly reproduce all known experimental results. The four-dimensional symmetry (i.e., the Lorentz and Poincaré invariance) of the physical framework is all that matters."[5]

Our argument is in some sense an extension of that made by Taylor and Wheeler[6] in their excellent Spacetime Physics text. In that book, Taylor and Wheeler argue that since special relativity has shown that space and time can be put on an equal footing (though they are not exactly the same thing), it is more logical to express both spatial and temporal intervals using the same units.[7] Though numerous other authors have made such a statement, Taylor and Wheeler actually follow through on this line of thought by quantifying all space and time measurements in their text using the unit meter. They further argue that expressing both the spatial and temporal components of the spacetime interval using the same unit better emphasizes the unified nature of spacetime and that doing so reveals that the value one typically associates with the speed of light c, namely 299792458 m/s, is not a fundamental constant of our universe at all, but merely a conversion constant between the independently developed units of meter and second that humans created before being aware of the four-dimensional nature of spacetime.

Taylor and Wheeler[6] also state (and we show explicitly) that if one expresses both spatial and temporal intervals using the unit meter, the principle of relativity by itself implies that the speed of light is both isotropic and has the same value of one meter per meter in all inertial frames, without any need for a second postulate. Einstein's second postulate is thus effectively a definition for the unit second, making it proportional to the unit meter and insuring that the speed of light expressed in units of meter per second is a universal constant. In Einstein's time, when space and time were thought to be completely independent entities, it was a necessary postulate in the formulation of special relativity. Given that we now view space and time as parts of a unified spacetime however, it is superfluous. Furthermore, this view leads us to see that the unit second is completely superfluous to physics.

We take Taylor and Wheeler's line of thought one step further. If the unit second is superfluous to physics and is a human construct, this implies that we may define it in any way we like. In particular, we should feel free to define it in a way that makes the problems we want to solve more convenient to attack. The second postulate of special relativity in 1905 and the 1983 General

Conference on Weights and Measures established a particular definition for the second.[8] While this definition is convenient in that the meter and second are directly proportional, making the speed of light a universal constant and simplifying calculations for problems involving the propagation of light signals, it does introduce other inconveniences when trying to solve certain other kinds of problems. For example, when studying many particle systems, the use of relativistic time means that one must use the proper time for each particle to express the covariant equations of motion and thus, one cannot derive an invariant Liousville equation in special relativity.[9] In addition, relativistic time presents some difficulties in defining an invariant temperature, leading to problems in developing an invariant Planck law for black-body radiation, and in providing a theoretical framework for calculating the radiative reaction force for accelerating charges.[9]

In chapters 7 through 17, we propose a new definition of the unit second (which we call the common-second to distinguish it from the traditionally defined unit of time) that can overcome these difficulties and in addition, carries other benefits such as the ability to define an invariant quantity we call the "genergy" that allows for a fuzziness of the position operator of a quantum particle at short distances, and for the construction of an invariant Planck's law for black-body radiation.[10] Of course, this new definition carries some inconveniences of its own. The speed of light for example, measured in meters per common-second, is no longer a universal constant. However, as we show, this effect is merely an artifact of a particular human definition of the unit of time and does not affect any of the underlying physics (the speed of light measured in natural units of meter per meter is still a universal constant). Because the present definition of our unit of length, the meter, is dependent on the definition of the second, we redefine that unit as the length of some standard object. Although such a definition is not as convenient for experimental studies requiring high precision, it has no drawbacks in terms of the theoretical discussion.

A good analogy to use in thinking about the relationship between the traditional definition of the second and the common-second is the relationship between the different coordinate systems, Cartesian, spherical, parabolic, etc., we use in physics. All of the systems are "right" in the sense that they can all be used to solve any problem. However, some coordinate systems are clearly more convenient for solving certain problems than others. For example, the use of parabolic coordinates when solving a short-range kinematics problem in a flat-Earth approximation is no more desirable than the use of Cartesian coordinates to solve the Schrödinger equation for a hydrogen atom. In special relativity, there is a simple relationship between the speed of light in different inertial frames $c = c'$ which leads to a complicated relationship between time coordinates (measured in the unit second) $t' = \gamma(t - \beta x/c)$. In the relativity theory using the common-second (which we call common relativity),[11] there is a simple relationship between the time coordinates $t'=t$ which leads to a complicated relationship between the speed of light in different inertial frames. Both theories are consistent with all known experiments and so are equally "correct." However, one might choose to use special relativity when solving problems involving the propagation of light and common relativity when solving problems involving the evolution of a many particle system, in which each particle occupies a different inertial frame and has a different proper time.

In summary, our goal in part B is to show that the principle of relativity, by implying that space and time are on equal footings and thus that only one unit is necessary to quantify both, allows us flexibility in the definition of an additional unit, should we choose to use it, in expressing spatial and temporal quantities. The careful construction of such a definition can lead to an alternative unit system, analogous to an alternative coordinate system, that can be of help in solving certain kinds of physics problems.

1c. Idea #2: The principle of relativity is useful as a limiting principle in the discussion of the physics of accelerated frames

Despite the advances in our knowledge of spacetime brought about by the theory of special relativity, our understanding of spacetime is by no means complete. The principle of relativity applies only to physics in inertial frames, which are only approximations and idealizations of the physically realized reference frames within which the vast majority of phenomena in our universe take place. Because of the infinite range of the gravitational force, virtually all physical frames of reference in our universe are non-inertial.

In inertial frames, the Lorentz and Poincaré transformations, which embody the properties of the Lorentz and Poincaré groups, enable physical theories to be formulated covariantly and tested experimentally. In non-inertial frames, the tensor calculus of general relativity leads to an analogous group consisting of all point transformations of spacetime which leave the differential form $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ invariant. However, this group is too general for quantum field theory and results in few testable predictions.

It would be reasonable to assume that that the future of relativity theories for the spacetime of inertial frames lie in their unification with the absolute theory for spacetime of accelerated frames. Although there is no relativity in accelerated motion, we believe that the concept of "limiting four-dimensional symmetry" or "limiting Lorentz and Poincaré invariance," which states that spacetime coordinate transformations between accelerated frames must reduce to the Lorentz transformations in the limit of zero acceleration, can pave the way to this unification. One generalization of the Lorentz transformation leads to the Wu transformation, which relates the spacetime coordinates of an inertial frame to those in a frame undergoing a constant-linear-acceleration.[12] The Wu transformation includes the better known Møller transformation as a special case and provides a general framework within which to understand the physics of both inertial and non-inertial frames. In this framework of limiting Lorentz and Poincaré invariance, the spacetime of non-inertial frames is characterized by a vanishing Riemann-Christoeffel curvature tensor, implying a flat spacetime that avoids the complications of the curved spacetime of general relativity.

In part C of this book we use the principle of limiting four-dimensional symmetry to explore the physics of non-inertial frames, first developing a spacetime transformation between an inertial frame and one undergoing a constant linear acceleration, then between and inertial frame and one with an arbitrary acceleration along a straight line, and finally between an inertial frame and one undergoing a uniform rotation. In addition to the kinematical transformations among such frames, we discuss the dynamics of classical and quantum particles as well as possible experimental tests of the theories.

Finally, we use our examination of non-inertial frames and the flexibility in the definition of our units of space and time to investigate the status of what are traditionally known as fundamental physical constants. At present, the majority of our physical theories are developed for inertial frames. However, true fundamental physical constants of our universe would be those whose values are the same not only in all inertial frames, but all frames in general, both inertial and non-inertial. For example, the speed of light is of course not a constant in non-inertial frames nor is it even necessarily a constant in all inertial frames when measured in a unit system other than natural units if one chooses a different definition than the conventional one for the unit second. On the other hand, the dimensionless electromagnetic coupling strength $\alpha_e = 1/137.036$ not only is invariant under a change of unit systems, but also appears to be a universal constant in both inertial and non-inertial frames.

References

1. Physics at short distances is in many ways equivalent to physics at large momenta or high energies, according to quantum mechanics. It is safe to say that all known laws of physics can be applied to phenomena occurring at length scales larger than 10^{-17}cm, as indicated by high-energy experiments. In other words, Lorentz and Poincaré invariance (or the 4-dimensional symmetry of the Lorentz and Poincaré groups) has been confirmed experimentally down to distances as small as 10^{-17}cm. It is possible that physical laws such as the Coulomb force or physical principles such as Lorentz and Poincaré invariance may become only approximately true at length scales smaller than 10^{-17}cm. See J. Schwinger, *Quantum Electrodynamics* (Dover Publications, New York, 1958), p. xvi. R. P. Feynman, *Theory of Fundamental Processes* (Benjamin, New York, 1962), p. 145; *Quantum Electrodynamics* (Benjamin, New York, 1962), pp. 138-139. For a related discussion, see S. Weinberg, *The Quantum Theory of Fields*, vol. 1, Foundations (Cambridge Univ. Press, Cambridge, 1995), pp. 31-38. For possible modifications at short distances, see discussions in chapters 14 and 15.
2. Editorial, Nature, **303**, 129 (1983). Discussions and comments were made by the editor on the idea of common time embedded in a 4-dimensional symmetry framework. We elaborate this idea in chapter 8.
3. See Appendix A for a discussion of units and the development of relativity theories.
4. This analysis of the ability of precision experiments to determine all of the parameters of the Lorentz transformation was submitted as a term paper for an undergraduate seminar course in Fall 1990. This term paper is reproduced in Appendix B. The main result was published in Leonardo Hsu and Jong-Ping Hsu, Nuovo Cimento **112B**, 1147 (1997).
5. Jong-Ping Hsu and Leonardo Hsu, Phys. Letters A **196**, 1 (1994).
6. E. F. Taylor and J. A. Wheeler, *Spacetime Physics* (Second Ed., W. H. Freeman and Company, 1992), p. 60.
7. Mathematicians and physicists in the early 20th century wrote the Maxwell equations setting c=1 for simplicity. See J. P. Hsu and T. Kleinschmidt, in *Lorentz and Poincare Invariance* (J. P. Hsu and Y. Z. Zhang, World Scientific 2001) pp. 43-53. In the context of special relativity, Poincare

stated this explicitly in his relativity paper, finished in 1905. (See the discussion in section 5c.)

8. Officially, the second is defined in terms of the oscillations based on the frequency of radiation from cesium atoms, while the meter is defined in terms of the second. However, this is merely for reasons of better measurement precision. Logically, the definitions of meter and second are directly linked and it makes no difference which one is dependent on the other.
9. These ideas are discussed in chapters 13 and 14.
10. J. P. Hsu, Nuovo Cimento **74B**, 67 (1983); **93B**, 178 (1986).
11. J. P. Hsu, Found. Phys. **8**, 371 (1978); **6**, 317 (1976); J. P. Hsu and T. N. Sherry, *ibid* **10**, 57 (1980).
12. Jong-Ping Hsu and Leonardo Hsu, Nuovo Cimento **112**, 575 (1997) and Chin. J. Phys. **35**, 407 (1997). This is called the Wu transformation to honor Ta-You Wu's idea of a kinematic approach to finding an accelerated transformation in a spacetime with vanishing Riemann curvature tensor.