

## PREFACE

The **Second International Conference on Mathematical Analysis in Andalucía**<sup>1</sup> was held in Granada in the period September 20–24, 2004. With the support of both the Spanish National Government and the Andalusian Regional Government,<sup>2</sup> representatives of all Andalusian universities made a concerted effort to organize a meeting that provides a thorough overview of current trends and advances in the field of Mathematical Analysis. The friendly cooperation of the Andalusian research groups in this area has made possible the continuation of a project that originated in Cadiz in 2002, mainly thanks to the initiative of Antonio Aizpuru Tomás and Fernando León Saavedra. Furthermore, it is our pleasure to announce that the next conference in this series will take place in La Rábida (Huelva, Spain) in September 2007. It will be organized by the Universities of Huelva and Seville, under the direction of Tomás Domínguez Benavides and Cándido Piñeiro Gómez.

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The goal of these conferences is to provide an opportunity for distinguished researchers to explore a variety of topics, to initiate the discussion of open problems, and to present tools and techniques that might prove effective in their resolution. The Second Conference consisted of seminars and one-hour plenary talks. While the one-hour talks were intended to provide an overview of a variety of areas of contemporary interest, the seminars extended over several days and permitted in-depth discussion on certain specific topics. Moreover, all participants had the opportunity to present new results of their own research in short communications.

The participation and collaboration of outstanding researchers in the 2004 Conference surpassed all of our expectations and predictions. Approximately 80 lectures were given on topics of current scientific interest, more than meeting our goals. We would like to express our most sincere gratitude to all of those who, through their work, presence, and scientific contributions, made the success of this Conference and the publication of these proceedings possible.

The present book is comprised of works that are closely related to the content of the seminars and plenary talks given at the 2004 Conference. It includes contributions from Richard M. Aron, Fernando Bombal, Javier Duoandikoetxea, Gilles Godefroy, Nigel J. Kalton, Lawrence Narici, Michael M. Neumann, Baldomero Rubio, Manuel Valdivia, Joan Verdera, and Felipe Zó (co-authored with Héctor H. Cuenya).

We would like to take the opportunity of this preface to pay special tribute to the memory of Miguel de Guzmán, who was scheduled to be one of our guest speakers, but passed away shortly before the holding of the Conference. We know we have been deprived of what would surely have been an outstanding contribution. The paper of Baldomero Rubio contains a touching biographical sketch of Miguel de Guzmán, and addresses interesting ideas about the teaching of mathematics, a topic of great importance in which Miguel de Guzmán was the leading authority in Spain.

The principal results of the contribution by Manuel Valdivia, certainly the dean of our main speakers, have not been published before. These results are related to the study of the so-called “lineable” and “spaceable” subspaces of a locally convex space. The article culminates in the construction of a subspace of the space  $H(\Omega)$  (of all holomorphic functions on a domain  $\Omega$  of  $\mathbb{C}^n$ ) which is dense in  $H(\Omega)$  for the compact open topology, is ordered nearly-Baire, and has the property that each of its elements does not extend holomorphically outside  $\Omega$ . The paper also includes a deep general study of nearly-Baire and ordered nearly-Baire locally convex spaces.

The paper by Fernando Bombal reviews much of the literature about the work of Alexander Grothendieck and provides the reader with a perspective of his work on functional analysis together with a glimpse of certain parts of his heritage. The paper focuses mainly on Grothendieck's contributions to the theory of topological tensor products, first in the general setting of locally convex spaces (the subject of his Ph.D. dissertation) and later as a powerful tool to study the structure of Banach spaces (mainly contained in the famous Sao Paolo's *Résumé*). In addition to providing a careful review of these two masterpieces, Bombal comments on Grothendieck's remarkable paper on  $C(K)$ -spaces in the *Canadian Journal of Mathematics*.

The article by Felipe Zó and Héctor Cuenya consists mainly of previously unpublished results. The authors consider a family of monotone function seminorms acting on Lebesgue measurable functions  $F : B \subset \mathbb{R}^n \rightarrow \mathbb{R}^k$ , where  $B := \{x \in \mathbb{R}^n : |x| \leq 1\}$ , as well as the best approximation function  $P_\varepsilon \in A$ , where  $A$  is a given subspace of polynomials. They study the asymptotic behavior of a normalized error function as well as the limit of the net  $\{P_\varepsilon\}$  as  $\varepsilon \rightarrow 0$ . This vector valued approach extends and unifies some classical problems in best local approximation theory.

The contributions of Richard M. Aron, Javier Duoandikoetxea, Gilles Godefroy, Nigel J. Kalton, Lawrence Narici, Michael M. Neumann, and Joan Verdera are survey papers on the respective topics of their specialization.

The paper of Richard M. Aron deals with "lineability", i.e. the problem to decide when vectors having some (perhaps strange) property  $\mathcal{P}$  form an infinite-dimensional space. The author concentrates on three very different instances of lineability: zeros of polynomials, hypercyclic vectors associated to hypercyclic operators, and differentiability of continuous functions. Positive and negative results are given in each instance, and many results (both by the author and by others) are described. The paper concludes with a comment on the relation between lineability and the Baire category argument. A good number of well-chosen examples and many references are provided.

The contribution by Javier Duoandikoetxea about the Hardy-Littlewood maximal function and some related topics represents an excellent overview that will be appreciated not only by the experts, but also by graduate students and other non-specialists who are interested in an introduction to this important and very useful topic of harmonic analysis. The author introduces the basic variations on the classical maximal operator and explains the connection with the differentiation theorem. Some

of the important ideas behind the proofs are sketched. The author also discusses two current research areas (and some of the open problems) that have arisen from the study of this operator: the Kakeya maximal operator and maximal operators defined by sets of directions. An extensive bibliography on the topic is given.

The article by Gilles Godefroy deals with hypercyclic operators, focusing on some important new results introduced by Sophie Grivaux and Frédéric Bayart in a series of papers. The author presents the theorems and their proofs, avoiding technicalities whenever possible. In this way, this paper becomes an invitation to further reading, an introduction to the original articles, and a preparation for the challenging questions which remain open.

The paper by Nigel Kalton surveys recent results (some of them from the author) concerning greedy, quasi-greedy, and almost greedy bases for Banach spaces. The paper provides a gentle introduction to this theory.

The informative article by Lawrence Narici is devoted to presenting the origins, history, and evolution of what is known as the Hahn–Banach theorem, and more precisely, its analytic version. The author reviews some of the direct antecedents of this theorem: the study of infinite systems of linear equations, and the moment problem. The fundamental contributions of F. Riesz and E. Helly, as well as the work of S. Banach and H. Hahn on the subject, are analyzed in detail. Also surveyed are extendible Banach spaces, i.e., those Banach spaces that can play the role of the scalar field in the classical version of the Hahn–Banach theorem for normed spaces. Moreover, the paper gives an interesting account of results about the uniqueness of Hahn–Banach extensions and non-Archimedean versions of the Hahn–Banach theorem.

The contribution by Michael M. Neumann nicely details the history and origins of the classical Cesàro operator, before proceeding to a study of the fine structure of the spectrum and the spectral decomposition properties (specifically, subnormality and hyponormality) of generalized Cesàro operators with rational symbol on Hardy and weighted Bergman spaces. While mainly a survey article, the paper also contains some interesting new contributions.

The paper by Joan Verdera reviews the basics of classical potential theory in such a way that the formal analogy between analytic capacity and classical Wiener capacity becomes evident, and the semi-additivity problem becomes natural. This long-standing open problem in complex analysis has been solved recently using real variable methods.

Most of the aforementioned survey papers will be authoritative refer-

ences for the optimal version of results scattered in the literature through many separate papers. The editors would like to express their gratitude to the authors listed above for their careful preparation of their contributions to this book. The editors are also very grateful to the referees of the papers contained in this book, who must remain anonymous, but who donated their time generously and selflessly.

Last but not least, we would like to thank the representatives of all of the public institutions that supported the Second International Conference on Mathematical Analysis in Andalusia from the beginning. In particular, we would like to thank the Regional Counselor for Innovation, Science and Business, Francisco Vallejo Serrano, and the Chancellor of Granada University, David Aguilar Peña, both of whom granted us the honor of opening what started as a scientific meeting and ends with the publication of the present book.

Granada, Autumn 2006

*The Editors*