

# Contents

<i>Preface</i>	vii
<i>Special Symbols</i>	xi
1. Introduction	1
1.1 Historical notes . . . . .	1
1.2 Preliminaries . . . . .	6
1.3 t-norms and s-norms . . . . .	9
1.4 Copulas . . . . .	17
2. Representation theorems for associative functions	23
2.1 Continuous, Archimedean t-norms . . . . .	23
2.2 Additive and multiplicative generators . . . . .	38
2.3 Extension to arbitrary closed intervals . . . . .	51
2.4 Continuous, non-Archimedean t-norms . . . . .	57
2.5 Non-continuous t-norms . . . . .	64
2.6 Families of t-norms . . . . .	70
2.7 Other representation theorems . . . . .	81
2.8 Related functional equations . . . . .	93
3. Functional equations involving t-norms	99
3.1 Simultaneous associativity . . . . .	99
3.2 $n$ -duality . . . . .	110
3.3 Simple characterizations of Min . . . . .	127
3.4 Homogeneity . . . . .	129
3.5 Distributivity . . . . .	134

3.6	Conical t-norms . . . . .	137
3.7	Rational Archimedean t-norms . . . . .	143
3.8	Extension and sets of uniqueness . . . . .	151
4.	Inequalities involving t-norms . . . . .	173
4.1	Notions of concavity and convexity . . . . .	173
4.2	The dominance relation . . . . .	182
4.3	Uniformly close associative functions . . . . .	189
4.4	Serial iterates and $n$ -copulas . . . . .	194
4.5	Positivity . . . . .	203
Appendix A	Examples and counterexamples . . . . .	209
Appendix B	Open problems . . . . .	219
	<i>Bibliography</i> . . . . .	223
	<i>Index</i> . . . . .	235