

## Chapter 1

# Introduction

Strongly interacting elementary particles such as the proton and neutron are not fundamental particles, but are bound states of spin  $-1/2$  fermions, the quarks, and spin-1 gauge bosons, the gluons. The strong interactions are described by a quantum field theory called Quantum Chromodynamics (QCD). It is believed that quarks and gluons are permanently confined, *i.e.*, it is not possible to separate an isolated quark a macroscopic distance from an antiquark or another quark pair. At short distances the effective coupling constant of QCD becomes small, and it is possible to compare results from experiments with theoretical predictions based on perturbation theory. However, at long distances the strong interactions are truly strong, and perturbation theory breaks down. Some nonperturbative technique is necessary to perform calculations of quantities that are sensitive to the long distance behavior of QCD. One of those techniques is the subject of this book: it is called “lattice QCD.”

The approach involves regularizing QCD by introducing a space time lattice. Quarks are restricted to occupy the sites of the lattice; gluons, the links joining the sites. Introducing the lattice spacing converts the Feynman path integral for QCD into an ordinary integral of very large dimensionality. The path integral expectation value of any observable can be computed by evaluating the integral on a computer, typically using importance sampling techniques. Of course, the lattice spacing is itself an unphysical quantity; it was introduced in order to perform the calculation. Physical predictions require an extrapolation from nonzero lattice spacing to zero lattice spacing.

The idea of replacing a field variable, defined at every space-time point, by a variable defined at some location on a lattice is probably older than the idea of a field variable itself. In some physical problems (such as in the description of electrons in a crystal) the lattice is a physical quantity.

However, the particular replacement of a continuum gauge field theory by a lattice theory, from which this field grew, dates only to Wilson (1974). [Lattice gauge theory was in the air in the early 70's: other people working on lattice theories with local gauge invariance included Wegner (1971), J. Smit, and Polyakov (1975)].

Wilson showed that a lattice formulation of QCD exhibited confinement in the strong coupling limit. His work led to an explosive growth of research on the properties of strongly-coupled QCD. At first, all of this research was analytic. The use of Monte Carlo methods for studying statistical systems was already very old [as early as Metropolis *et al.* (1953)], and in 1979 Creutz, Jacobs, and Rebbi (Creutz *et al.*, 1979; Creutz, 1979) adapted these techniques to the study of lattice gauge theories. The first computer simulations involving quarks began two years later (Weingarten and Petcher, 1981; Marinari *et al.*, 1981; Hamber and Parisi, 1981), and by the mid-1980's the field of lattice QCD had been transformed into something like its present form, a mix of analytic calculations and increasingly larger scale numerical simulation. As techniques have advanced, the lattice has become an ever more accurate and important source of calculations of hadronic spectroscopy and matrix elements used in studies of QCD, the Standard Model of elementary particle physics, and beyond.

This book is an introduction to, and motivation for, modern techniques of lattice gauge theory. As we wrote this book, we had several different audiences in mind:

- Graduate students or other scientists beginning to do research in lattice gauge theory. They presumably want to know how quantum field theories are formulated on the lattice, and how to perform calculations in lattice QCD.
- Researchers studying the strong interactions, who might be interested in using lattice calculations of reduced matrix elements in their (non-lattice) calculations. They presumably want to know what lattice people can do, how they do it, and why.
- Experimentalists using results of lattice calculations to interpret measurements. They may want to understand the source of uncertainty in lattice calculations.
- Researchers in computational physics, who might be interested in numerical simulation, and who want to know the physical context in which the calculations are performed.

When we talk about lattice methods or results with a group of particle

theorists who do more conventional (non-lattice) calculations, we are struck by how different lattice techniques seem to be from what our colleagues know or do. Certainly, our day to day activities of writing programs, submitting them to remote supercomputers, and analyzing our results, seem very different from what a weak-interaction model builder (for example) does.

And yet, at the bottom, lattice QCD is merely a corner of quantum field theory, and the techniques lattice theorists use are simply decorated versions of techniques used across the board by physicists studying problems with many degrees of freedom in particle, condensed matter, and nuclear physics.

Many of the recent advances in lattice techniques have come when observant researchers recognized a connection between a lattice problem and some other branch of theoretical physics associated with strong interactions. The modern theory of critical phenomena is the source of many important ideas for lattice QCD, followed by the related topic of effective field theories.

While a number of books about lattice gauge theory already exist [two excellent early examples are Creutz (1983) and Montvay and Muenster (1994)], the last five to ten years have seen enormous progress in the development of algorithms and techniques for constructing and simulating lattice discretizations of QCD. Most of these advances are described only in research articles in journals or in occasional summer school lectures. Some of them are, in fact, of a rather fundamental nature. Our goal in writing this book is to bridge the gap between what we think a conventionally-trained theoretical physicist knows and what is written in modern journal articles about topics in lattice QCD for those “who already know it” .

This means that we must describe the rather specialized techniques associated with formulating field theories (mostly QCD) on a lattice, performing numerical simulations, and interpreting their results. It also means that we should attempt to make connections between lattice QCD and other research areas of strong interaction theory and phenomenology.

We have not made any attempt to include results from lattice simulations. To do so would be to invite obsolescence. While the mean values of results from lattice calculations do not vary too rapidly with time, there is a steady decrease in the size of quoted uncertainties as techniques get refined and as data sets get bigger. And then there are the occasional big jumps when a new technique is invented or exploited. We hope we will give our readers enough background, that they can go directly from this

little book to the current research literature and understand it. Readers interested in the latest results, and are willing to take those results from very brief articles, should consult the Proceedings of the annual conferences in this research field, “Lattice (year).” These volumes exist as real books through the 2004 conference, and have since been published electronically.