

OPTIMAL BURN-IN FOR MINIMIZING TOTAL WARRANTY COST

JI HWAN CHA

*Division of Mathematical Sciences,
Pukyong National University,
Busan 608-737, KOREA
E-mail: jhcha@pknu.ac.kr*

Burn-in is a widely used method to improve the quality of products or systems after they have been produced. In this paper, optimal burn-in procedures for a system with two types of failures (i.e., minor and catastrophic failures) are investigated. A new system surviving burn-in time b is put into field operation and the system is used under a warranty policy under which the manufacturer agrees to provide a replacement system for any system that fails to achieve a lifetime of at least w . Upper bounds for optimal burn-in time minimizing the total expected warranty cost are obtained under a more general assumption on the shape of the failure rate function which includes the bathtub shaped failure rate function as a special case.

1. Introduction

ACRONYMS AND ABBREVIATIONS

- CDF cumulative distribution function
- DIB bathtub shape
- FR failure rate (function)
- PDF probability density function
- r.v. random variable
- s- statistical(ly)
- Sf survivor function

NOTATION

- X lifetime of a system, $X \geq 0$; a r.v.
- b burn-in time
- $p(t)$, $1 - p(t)$ the probability of Type II and Type I failure, respectively
- Y_b the time to Type II failure of a burned-in system with fixed burn-in

time b , when only minimal repair is performed for each Type I failure occurring during operation; $Y_b \geq 0$; a r.v.

- $f(t), F(t), \bar{F}(t)$ pdf, Cdf, Sf of X
- $G_b(t), \bar{G}_b(t)$ Cdf, Sf of Y_b
- $r(t) = f(t)/\bar{F}(t)$; FR of X
- t_1, t_2 the first, and second change point of the FR, respectively, when the FR is DIB
- t^*, t^{**} the first, and second wear-out point of the FR, respectively, when the FR is eventually increasing
- $\Lambda(t) = \int_0^t r(u)du$; cumulative FR
- $\Lambda_p(t) = \int_0^t p(u)r(u)du$
- $Z_b(T) = \min\{Y_b, T\}$, where $T > 0$ is a fixed constant; a r.v.
- $N(Z_b(T))$ the total number of minimal repairs of a burned-in component (with fixed burn-in time b) which occur during operation time interval $(0, Z_b(T)]$ when only minimal repair is performed for each Type I failure occurring during operation; a r.v.
- c_0 a constant which is proportional to the total burn-in time
- c_s shop complete repair cost per Type II failure during burn-in process
- c_{sm} shop minimal repair cost per Type I failure during burn-in process
- $h_i(b)$ the random cost incurred until the first component survives burn-in for the Burn-In Procedure I and Procedure II, respectively, $i = 1, 2$
- c_m minimal repair cost per Type I failure during field operation
- c_f replacement cost per Type II failure during field operation
- $w_i(b)$ the total warranty cost when the Burn-In Procedure I or Procedure II is applied, respectively, $i = 1, 2$
- b_i^* optimal burn-in time which minimizes the total warranty cost, respectively, $i = 1, 2$.

The burn-in procedure is a manufacturing technique that is intended to eliminate early failures of the system or product. To burn-in a system means to subject it to a period of use prior to the time when it is to actually be used. Due to the high failure rate in the early stages of system life, burn-in procedures have been widely accepted as a method of screening out failures before systems are actually used in field operations. An introduction to this important area of reliability can be found in Ref.⁹, or Ref.¹⁰. Because too excessive or insufficient burn-in is sometimes harmful to the performance of the burned-in component or system, and also because burn-in is usually costly, one of the major problems is to decide how long the procedure should be. The best time to stop the burn-in procedure for a given criterion is

called the optimal burn-in time. In the literature, certain cost structures have been proposed, and the corresponding problem of finding the optimal burn-in time has been considered.

In this paper, burn-in procedures for a general failure model are considered. In the general failure model, when the unit fails at its age t , Type I failure occurs with probability $1 - p(t)$, and Type II failure occurs with probability $p(t)$, $0 \leq p(t) \leq 1$. It is assumed that Type I failure is a minor one, thus it can be removed by a minimal repair; whereas Type II failure is a catastrophic one or total breakdown of the system, thus it can be removed only by a complete repair (or a replacement). Recently, Ref.⁵-Ref.⁷ studied two types of burn-in procedures (Burn-In Procedure I & II) for the general failure model.

In this paper, under the general failure model, the problem of determining optimal burn-in time which minimizes the total expected warranty cost is investigated. In most burn-in models, the optimal burn-in time cannot be given by an explicit form and must be obtained numerically. In this case, some bounds for the optimal burn-in time may be useful since the numerical search for the optimal burn-in time will be greatly reduced. This paper presents upper bounds for optimal burn-in time under a more general assumption on the shape of failure rate function of products which includes the traditional bathtub-shaped failure rate function as a special case. In Section 2, a more general assumption on the shape of failure rate function is introduced. In Section 3, the detailed burn-in model which is studied in this paper will be described and the upper bounds for optimal burn-in time will be presented. Finally, some concluding remarks are discussed in Section 4.

2. General Assumption

It is widely believed that many products, particularly electronic products or devices such as silicon integrated circuits, exhibit bathtub-shaped failure rate functions. This belief is supported by much experience and extensive data collected by practitioners and researchers in different industries. Hence many researches on burn-in have been done under the assumption of bathtub-shaped failure rate function.

Recently, there have been many researches on the shape of failure rate functions of mixture distributions. For instance, in Ref.³, Ref.⁴ and Ref.¹¹, the shape of failure rate functions of mixture distributions which is neither of the traditional bathtub-shape nor of the modified bathtub-shape are in-

vestigated, where Ref.¹¹ pointed out that the assumption of the traditional bathtub-shaped failure rate function could be rather a restrictive assumption for the researches on burn-in procedures. Especially, Ref.¹² asserts that the bathtub-shaped failure rate function describes only 10% to 15% of applications.

In this paper, more general model for the failure rate function is assumed. It can be seen that this general assumption includes the traditional bathtub-shaped failure rate function and the modified bathtub-shaped failure rate function as special cases.

Definition 1 A failure rate function $r(x)$ is eventually increasing if there exists $0 \leq x_0 < \infty$ such that $r(x)$ strictly increases in $x > x_0$. For an eventually increasing failure rate function $r(x)$ the first and second wear-out points t^* and t^{**} are defined by

$$\begin{aligned} t^* &= \inf\{t \geq 0 : r(x) \text{ is nondecreasing in } x \geq t\} \\ t^{**} &= \inf\{t \geq 0 : r(x) \text{ strictly increases in } x \geq t\}. \end{aligned}$$

Obviously $0 \leq t^* \leq t^{**} \leq x_0 < \infty$ if $r(x)$ is eventually increasing. Note also that if $r(x)$ has a bathtub shape with change points $t_1 \leq t_2 < \infty$ (or $r(x)$ is modified bathtub-shaped failure rate function with $0 \leq t_0 \leq t_1 \leq t_2 < \infty$), then it is eventually increasing with $t^* = t_1$ and $t^{**} = t_2$. Therefore, the eventually increasing failure rate function includes both the traditional bathtub-shaped and the modified bathtub-shaped failure rate function as special cases.

Ref.¹³ considered optimal burn-in under the assumption of eventually increasing failure rate function. In this paper we will derive the upper bounds for optimal burn-in time under the assumption of eventually increasing failure rate function. For more detailed discussions about general assumptions for the shape of failure rate function in burn-in model, see also Ref.⁸.

3. Optimal Burn-in

In the present paper, during the burn-in process, the following two types of burn-in procedures are considered under the general failure model:

- **Burn-In Procedure I:** Consider a fixed burn-in time b , and begin to burn-in a new component. If the component fails before burn-in time b , then repair it completely regardless of the type of failure with shop complete repair cost c_s , and then burn-in the repaired component again (i.e., restart the burn-in procedure), and so on.

• **Burn-In Procedure II:** Consider a fixed burn-in time b , and begin to burn-in a new component. On each Type I failure during burn-in, only minimal repair is done with shop minimal repair cost $0 < c_{sm} \leq c_s$, and continue the burn-in procedure for the repaired component. If a Type II failure occurs before burn-in time b , then a complete repair is performed with shop complete repair cost c_s , and then restart the burn-in procedure for the repaired component.

Note that Procedure I stops when there is no failure during a fixed burn-in time $(0, b]$ at the first time, whereas Procedure II stops when there is no Type II failure during a fixed burn-in time $(0, b]$ at the first time.

In many cases, because of practical limitations, products which fail during the burn-in are just scrapped. In this situation the Burn-In Procedure II can not be applied, and only the Burn-In Procedure I should be used. On the other hand, when the minimal repair method is applicable during a burn-in process, both the Burn-In Procedure I and the Burn-In Procedure II can be applied. In this situation (when both Burn-In Procedure I and Burn-In Procedure II are applicable), the problem of selecting a better (i.e., more economical) burn-in procedure is important to save total expected cost.

Using similar arguments as those given in Ref⁷, $E[h_i(b)]$, $i = 1, 2$, can be obtained by

$$E[h_1(b)] = \frac{c_0 \int_0^b \bar{F}(t) dt}{\bar{F}(b)} + c_s \frac{F(b)}{\bar{F}(b)},$$

and

$$E[h_2(b)] = \frac{c_0 \int_0^b \bar{G}_0(t) dt}{\bar{G}_0(b)} + \frac{c_{sm} [\int_0^b (1 - p(t)) r(t) \bar{G}_0(t) dt]}{\bar{G}_0(b)} + \frac{G_0(b)}{\bar{G}_0(b)} c_s,$$

where $\bar{G}_b(t) \equiv 1 - G_b(t)$ is given by

$$\begin{aligned} \bar{G}_b(t) &= P(Y_b > t) \\ &= \exp\left\{-\int_0^t p(b+u)r(b+u)du\right\} \\ &= \exp\{-[\Lambda_p(b+t) - \Lambda_p(b)]\}, \quad \forall t \geq 0, \end{aligned}$$

In this paper, we consider the following warranty policy.

• **Warranty Policy:** A new system surviving burn-in time b is put into field operation and the system is used under a warranty policy under which the manufacturer agrees to provide a replacement system with burn-in time b for any system that fails to achieve a lifetime (time to total breakdown of the system) of at least w .

During field operation, if a Type I Failure occurs then it can be removed instantly in the field operation by a minimal repair with minimal repair cost c_m . If a total breakdown occurs then it is sent to the repair shop and replaced by another new burned-in system with burn-in time b with replacement cost c_f , and so on. Implicit in the above defined warranty policy is that, under policies of this type, replacement systems are warranted anew. Similar warranty policy is considered in Ref.²(p.184, Policy 6) for an item with only Type II failure(i.e., $p(t) = 1, \forall t \geq 0$).

Now the total expected warranty cost is derived. Let $w_i(b)$ be the total warranty cost when the Burn-In Procedure I or Procedure II is applied, respectively, $i = 1, 2$. Also let Y_{b1} be the time to Type II failure of a new burned-in system with burn-in time b which is firstly put into field operation. Then the following two equations hold :

$$E[w_i(b)|Y_{b1} > w] = E[h_i(b)] + c_m E[N(Z_b(w))|Y_{b1} > w], \quad (1)$$

and

$$E[w_i(b)|Y_{b1} \leq w] = E[h_i(b)] + c_m E[N(Z_b(w))|Y_{b1} \leq w] + c_f + E[w_i(b)], \quad (2)$$

for $i = 1, 2$. Combining (1) and (2), it holds that

$$E[w_i(b)] = E[h_i(b)] + c_m E[N(Z_b(w))] + G_b(w) \cdot c_f + G_b(w) \cdot E[w_i(b)], \quad (3)$$

for $i = 1, 2$. From (3) and the results given in Ref¹, we have

$$\begin{aligned} E[w_i(b)] &= \frac{E[h_i(b)] + c_m E[N(Z_b(w))] + G_b(w) \cdot c_f}{\bar{G}_b(w)} \\ &= \frac{E[h_i(b)] + c_m \cdot \int_0^w r(b+t) \bar{G}_b(t) dt + (c_f - c_m) \cdot G_b(w)}{\bar{G}_b(w)}, \quad (4) \end{aligned}$$

for $i = 1, 2$. Let b_i^* be the optimal burn-in time which minimizes $E[w_i(b)]$ in (4), $i = 1, 2$. The following results give upper bounds for optimal burn-in time.

Theorem 3.1. *Suppose that the FR $r(t)$ is eventually increasing with the first wear-out point t^* and $p(t)$ is strictly increasing function. Let b_i^* be optimal burn-in time which minimizes $E[w_i(b)]$, $i = 1, 2$. Then optimal burn-in time satisfies $0 \leq b_i^* \leq t^*$, $i = 1, 2$.*

proof

It is obvious that $E[h_i(b)]$ is strictly increasing in $b > 0$. Observe that

$$\frac{\partial \bar{G}_b(w)}{\partial b} = -[p(b+w)r(b+w) - p(b)r(b)] \cdot \exp\{-[\Lambda_p(b+w) - \Lambda_p(b)]\}$$

$$< 0, \forall b > t^*.$$

This implies that $\bar{G}_b(w)$ is strictly decreasing in $b > t^*$ and $G_b(w) = 1 - \bar{G}_b(w)$ is strictly increasing in $b > t^*$. Thus both $E[h_i(b)]/\bar{G}_b(w)$ and $(c_f - c_m)G_b(w)/\bar{G}_b(w)$ in (4) are strictly increasing in $b > t^*$. Now we consider

$$\begin{aligned} \eta(b) &\equiv \frac{c_m \cdot \int_0^w r(b+t)\bar{G}_b(t)dt}{\bar{G}_b(w)} \\ &= c_m \exp\{\Lambda_p(b+w)\} \cdot \int_b^{b+w} r(t) \exp\{-\Lambda_p(t)\} dt. \end{aligned}$$

Differentiating $\eta(b)$, we have

$$\begin{aligned} \eta'(b) &= c_m p(b+w)r(b+w) \cdot \exp\{\Lambda_p(b+w)\} \cdot \int_b^{b+w} r(t) \exp\{-\Lambda_p(t)\} dt \\ &\quad + c_m \exp\{\Lambda_p(b+w)\} \cdot [r(b+w) \exp\{-\Lambda_p(b+w)\} - r(b) \exp\{-\Lambda_p(b)\}] \\ &= c_m p(b+w)r(b+w) \cdot \exp\{\Lambda_p(b+w)\} \cdot \int_b^{b+w} r(t) \exp\{-\Lambda_p(t)\} dt \\ &\quad + c_m [r(b+w) - r(b) \exp\{\Lambda_p(b+w) - \Lambda_p(b)\}] \\ &> c_m r(b+w) \exp\{\Lambda_p(b+w)\} \cdot \int_b^{b+w} p(t)r(t) \exp\{-\Lambda_p(t)\} dt \\ &\quad + c_m [r(b+w) - r(b) \exp\{\Lambda_p(b+w) - \Lambda_p(b)\}] \\ &= c_m r(b+w) \exp\{\Lambda_p(b+w)\} \cdot [-\exp\{-\Lambda_p(t)\}]_b^{b+w} \\ &\quad + c_m [r(b+w) - r(b) \exp\{\Lambda_p(b+w) - \Lambda_p(b)\}] \\ &= c_m [r(b+w) - r(b)] \exp\{\Lambda_p(b+w) - \Lambda_p(b)\} > 0, \forall b > t^*. \end{aligned}$$

Accordingly, we have shown that $E[w_i(b)]$ is strictly increasing in $b > t^*$. Therefore, we can conclude that $b^* \leq t^*$. ■

Corollary 3.1. *Suppose that the FR $r(t)$ is DIB with the first change point t_1 and $p(t)$ is strictly increasing function. Let b_i^* be optimal burn-in time which minimizes $E[w_i(b)]$, $i = 1, 2$. Then optimal burn-in time satisfies $0 \leq b_i^* \leq t_1$, $i = 1, 2$.*

Remark 3.1. In this section, two types of burn-in procedures have been considered. By similar arguments as those described in Ref.⁵ and Ref.⁶, when the minimal repair method can be applied during burn-in procedure, it can be shown that the Burn-In Procedure II is always preferable to the Burn-In Procedure I.

4. Concluding Remarks

In the literature, many objective cost or system performance functions related with burn-in have been discussed assuming DIB FR, and it has been

obtained that the optimal burn-in time must be before the first change point t_1 if the underlying lifetime distribution has a DIB FR. In this paper, the problems of determining optimal burn-in time have been considered under a general FR model for burn-in procedures, i.e., eventually increasing FR. This general FR model includes as special cases not only the DIB FR or modified DIB FR but also the FRs of many mixture distributions that have attracted a great attention recently. Assuming the proposed general FR model, we have obtained upper bounds for the optimal burn-in time. As it can be seen from the above, t^* plays the same role as t_1 .

References

1. F. Beichelt, A Unifying Treatment of Replacement Policies with Minimal Repair, *Naval Research Logistics*, 40, 51-67, 1993.
2. W. R. Blischke and D. N. P. Murthy, *Warranty Cost Analysis*. Marcel Dekker, Inc., New York, 1994.
3. H. W. Block, Y. Li and T. H. Savits, Initial and Final Behavior of Failure Rate Functions for Mixtures and Systems, *Journal of Applied Probability*, 40, 721-740, 2003.
4. H. W. Block, Y. Li and T. H. Savits, Preservation of Properties under Mixture, *Probability in the Engineering and Informational Sciences*, 17, 205-212, 2003.
5. J. H. Cha, Burn-in Procedures for a Generalized Model, *Journal of Applied Probability*, 38, 542-553, 2001.
6. J. H. Cha, A Further Extension of the Generalized Burn-in Model, *Journal of Applied Probability*, 40, 264-270, 2003.
7. J. H. Cha, On Optimal Burn-in Procedures-A Generalized Model, *IEEE Transactions on Reliability*, 54, 198-206, 2005.
8. J. H. Cha and J. Mi, Optimal Burn-in Procedures in a Generalized Environment, *International Journal of Reliability, Quality and Safety Engineering*, 12, 189-202, 2005.
9. F. Jensen and N. E. Petersen, *Burn-in*: John Wiley, New York, 1982.
10. W. Kuo and Y. Kuo, Facing the Headaches of Early Failures: A State-of-the-art Review of Burn-in Decisions, *Proc. IEEE*, 71, 1257-1266, 1983.
11. G. Klutke, P. C. Kiessler and M. A. Wortman, A Critical Look at the Bathtub Curve, *IEEE Transactions on Reliability*, 52, 125-129, 2003.
12. D. Kececioglu and F. Sun, *Environmental Stress Screening: Its Qualification, Optimization, and Management*: Prentice Hall, 1995.
13. J. Mi, Optimal Burn-in Time and Eventually IFR, *Journal of the Chinese Institute of Industrial Engineers*, 20, 533-542, 2003.