

## Contents

About the Authors	xi
Introduction	1
1. The $C_p$ Index	7
1.1 Process precision and the $C_p$ index	7
1.2 Estimating and testing $C_p$ based on a single sample	10
1.2.1 Estimation of $C_p$	10
1.2.2 The r-th moment of $\hat{C}_p$	11
1.2.3 Statistical properties of the estimated $C_p$	11
1.2.4 Confidence interval for $C_p$	12
1.2.5 Sample size determination for estimation of $C_p$	14
1.2.6 Hypothesis testing with $C_p$	15
1.3 Estimating and testing $C_p$ based on multiple samples	15
1.3.1 Estimation of $C_p$ and its properties	15
1.3.2 Lower confidence bound on $C_p$	17
1.3.3 Hypothesis testing with $C_p$	18
1.4 Estimating and testing $C_p$ based on $(\bar{X}, R)$ control chart samples	20
1.4.1 Estimation of $C_p$ based on $(\bar{X}, R)$ samples	21
1.4.2 Hypothesis testing for $C_p$ based on $(\bar{X}, R)$ samples	23
1.5 Estimating and testing $C_p$ based on $(\bar{X}, S)$ control chart samples	25
1.5.1 Estimation of $C_p$ based on $(\bar{X}, S)$ samples	25
1.5.2 Hypotheses testing for $C_p$ based on $(\bar{X}, S)$ samples	27
1.6 A Bayesian approach to assessment of $C_p$	28
2. The $C_a$ Index	31
2.1 Process accuracy and the $C_a$ index	31
2.2 Estimating and testing $C_a$ based on a single sample	33
2.2.1 The first two moments of $\hat{C}_a$	34
2.2.2 Confidence interval on $C_a$	35

2.3	Estimating and testing $C_a$ based on multiple samples	35
2.4	Bayesian-like estimator of $C_a$	38
3.	The $C_{pk}$ Index	40
3.1	Process capability and the $C_{pk}$ index	40
3.2	Estimating and testing $C_{pk}$ based on a single sample	45
3.2.1	The r-th moment of $\hat{C}_{pk}$	46
3.2.2	Distributional properties of $C_{pk}$	47
3.2.3	Confidence intervals for $C_{pk}$	50
3.2.4	Hypothesis testing with $C_{pk}$	55
3.3	Estimating and testing $C_{pk}$ based on multiple samples	58
3.4	The Bayesian approach to $C_{pk}$	61
3.5	The Bayesian-like estimator of $C_{pk}$	64
4.	The $C_{pm}$ Index	67
4.1	Process capability and the $C_{pm}$ index	67
4.2	Estimating and testing $C_{pm}$ based on one single sample	73
4.2.1	Estimation and distributional properties of estimators	73
4.2.2	Confidence intervals for $C_{pm}$	78
4.2.3	Sample size determination for $C_{pm}$	83
4.2.4	Hypothesis testing procedure (using $C_{pm}$ )	86
4.3	Estimating and testing $C_{pm}$ based on multiple samples	87
4.4	The Bayesian approach to $C_{pm}$	90
5.	The Loss Indices	95
5.1	Process loss and the $L_e$ index	95
5.2	Estimation of $L_{pe}$ , $L_{ot}$ , and $L_e$	98
5.2.1	Estimating the process relative inconsistency loss, $L_{pe}$	98
5.2.2	Estimation process relative the off-target loss, $L_{ot}$	101
5.2.3	Estimation of process expected relative loss, $L_e$	103
5.3	Upper confidence bounds of $L_{pe}$ , $L_{ot}$ , and $L_e$	105
5.3.1	An upper confidence bound on $L_{pe}$	105
5.3.2	An upper confidence bound of $L_{ot}$	106
5.3.3	An upper confidence bound of $L_e$	106
5.4	Testing process capability based on the process loss	107
6.	The $C_{pmk}$ Index	110
6.1	Process capability and the $C_{pmk}$ index	110
6.2	Estimating and testing $C_{pmk}$ based on a single sample	117
6.2.1	Estimation and the distribution of the estimated $C_{pmk}$	117
6.2.2	Confidence intervals on $C_{pmk}$	123
6.2.3	Hypothesis testing with $C_{pmk}$	124

6.3	Estimating and testing $C_{pmk}$ based on multiple samples	127
6.4	Bayesian-like estimation of $C_{pmk}$	130
7.	The $S_{pk}$ Index	134
7.1	Process capability and the $S_{pk}$ index	134
7.2	Estimating and testing $S_{pk}$ based on a single sample	136
7.2.1	Estimation of $S_{pk}$	136
7.2.2	Confidence intervals for $S_{pk}$	142
7.3	Hypothesis testing with $S_{pk}$	143
8.	The $C_{PU}/C_{PL}$ Index	146
8.1	Process capability and the index $C_{PU}/C_{PL}$	146
8.2	Estimating and testing $C_{PU}/C_{PL}$ : Based on single sample	148
8.2.1	Estimations of $C_{PU}$ and $C_{PL}$	148
8.2.2	r-th moment	149
8.2.3	Distribution	150
8.2.4	Testing hypothesis with $C_{PU}$ and $C_{PL}$	151
8.2.5	Lower confidence bound for $C_{PU}/C_{PL}$	153
8.3	Estimating and Testing $C_{PU}/C_{PL}$ : Based on multiple samples	154
8.3.1	Estimations of $C_{PU}$ and $C_{PL}$ based on multiple samples	154
8.3.2	Testing $C_{PU}$ and $C_{PL}$ based on multiple samples	157
8.4	Estimating and Testing $C_{PU}/C_{PL}$ : Bayesian approach with single sample	158
8.4.1	Bayesian approach to the assessment of $C_{PU}$ and $C_{PL}$ based on single sample	158
8.4.2	A Bayesian approach to assessment with $C_{PU}$ and $C_{PL}$ based on multiple samples	160
9.	Multi-Process Performance Analysis Chart (MPPAC)	162
9.1	Introduction	162
9.2	The modified $C_{pk}$ MPPAC	165
9.3	The $C_{pm}$ MPPAC	169
9.4	The $S_{pk}$ MPPAC	172
9.5	The $L_e$ MPPAC	176
10.	PCIs with Asymmetric Specification Limits	181
10.1	Introduction	181
10.2	The $C_{pk}$ index for asymmetric tolerances	182
10.3	The $C_{pm}$ index for asymmetric tolerances	195
10.4	The $C_{pn}$ index for asymmetric tolerances	202
10.5	The $C_{pmk}$ index for asymmetric tolerances	205
10.6	The loss index for asymmetric tolerances	210

11. Supplier Selection Based on PCIs	218
11.1 Introduction	218
11.2 Tseng and Wu's MLR selection rule based on $C_p$	219
11.3 Chou's approximate selection rule based on $C_{pu}$ and $C_{pl}$	222
11.4 Huang and Lee's approximate selection rule based on $C_{pm}$	224
12. Acceptance Sampling Plans Based on PCIs	231
12.1 Introduction	231
12.2 Acceptance sampling plans based on $C_{pk}$	237
12.3 Acceptance sampling plans based on $C_{pm}$	240
12.4 Acceptance sampling plans based on $C_{pmk}$	243
12.5 Acceptance sampling plans based on $C_{pu}$ and $C_{pl}$	246
13. Process Capability Measures in Presence of Gauge Measurement Errors	249
13.1 Introduction	249
13.2 Estimating and testing $C_p$ in presence of gauge measurement errors	252
13.3 Estimating and testing $C_{pk}$ in presence of gauge measurement errors	257
13.4 Estimating and testing $C_{pm}$ in presence of gauge measurement errors	263
13.5 Estimating and testing $C_{pmk}$ in presence of gauge measurement errors	269
13.6 Estimating and testing $C_{pu}$ and $C_{pl}$ in presence of gauge measurement errors	272
14. Process Capability Assessment with Tool Wear	279
14.1 Introduction	279
14.2 A review of various approaches	281
15. Process Capability Assessment for Non-normal Processes	293
15.1 Introduction	293
15.2 A brief review of various approaches	297
15.2.1 Probability plotting approach	297
15.2.2 Clements' approach	299
15.2.3 Box-Cox power transformation approach	301
15.2.4 Johnson transformation approach	302
15.2.5 Other quantile transform approaches	303
15.2.6 Distribution-free tolerance intervals approach	304
15.2.7 Flexible index $C_{jhp}$	305
15.2.8 The Wright's $C_S$ index	307

15.2.9	A superstructure capability indices $C_{Np}(u, v)$	309
15.2.10	The $C_{pc}$ index	313
15.2.11	The (general) Weighted Variance (WV) method	315
15.2.12	The Weighted Standard Deviation (WSD) method	320
16.	Multivariate Process Capability Indices	326
16.1	Introduction	326
16.2	Multivariate PCIs	328
16.3	Concluding remarks	342
	Bibliography	345
	Index	375