EVOLUTION OF THE SPIN OF MERCURY AND ITS CAPTURE INTO THE 3/2 SPIN-ORBIT RESONANCE

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The present spin of Mercury is very peculiar and was only discovered in 1965: the planet spins three times around its axis exactly in the same time as it completes two orbital revolutions[1]. The way the planet evolved into this configuration remained a mystery until very recently[2]. In order to understand this phenomena we must take into account the planetary perturbations over Mercury's orbit, that continuously change its eccentricity. As a result, for any initial rotation rate it was found that the chances of capture in the present configuration rise to about 55.4%.

Tidal dissipation and core-mantle friction will drive the obliquity of Mercury close to zero. For zero degree obliquity, the averaged equation for the rotational motion near the p resonance (where p is a half-integer) writes[2,3]:

$$\frac{\dot{x}}{3n} = -6\frac{C_{22}}{\xi}H(p,e)\sin 2(\ell - pM) - 3\frac{k_2}{\xi Q}\left(\frac{R}{a}\right)^3\left(\frac{m_0}{m}\right)\left[\Omega(e)x - N(e)\right] \ , \tag{1}$$

where $x = \dot{\ell}/n$ is the ratio of the rotation rate to the mean motion n, M the mean anomaly, e the eccentricity, ξ a structure constant and $H(p,e)$ Hansen coefficients [3]. $\Omega(e) = (1 + 3e^2 + 3e^4/8)/(1 - e^2)^{9/2}$, $N(e) = (1 + 15e^2/2 + 45e^4/8 + 5e^6/16)/(1 - e^2)^6$, k_2 and Q are the second Love number and quality factor, while a, m, m_0 are the semi major axis, the mass of the planet, and the solar mass. The equilibrium is achieved when $\dot{x} = 0$, that is, for a constant eccentricity e, when $x = x_l(e) = N(e)/\Omega(e)$. In a circular orbit ($e = 0$) this equilibrium coincides with synchronization ($x = 1$), while the equilibrium rotation rate $x = 3/2$ is achieved for $e_{3/2} = 0.284927$.

For the present value of Mercury's eccentricity $e \approx 0.206$ the capture probability in the 3/2 spin-orbit resonance[3] is estimated to be about 7.73%. However, using the present value of the eccentricity of Mercury is questionable, as the eccentricity suffers strong chaotic variations in time, due to planetary secular perturbations[4,5]. Indeed, the eccentricity of Mercury can vary from nearly zero to more than 0.45, and thus reach values higher than the critical value $e_{3/2} = 0.284927$ (Fig.1). Additional capture into resonance can then occur, at any time during the planet's history.

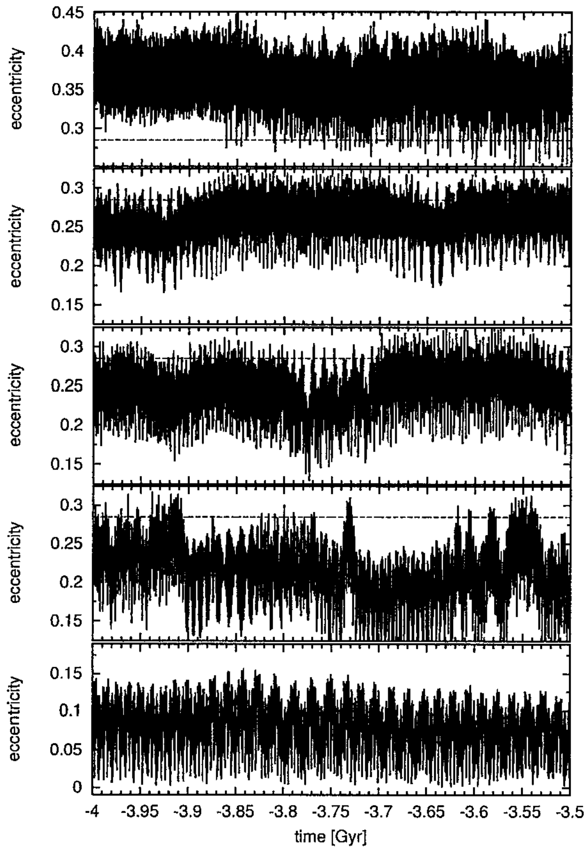

Figure 1. Examples of the possible variations of the eccentricity some 4 Gyr ago. All these solutions converge to the known present evolution of the planet's orbit. Traced horizontal line corresponds to the critical eccentricity $e_{3/2} = 0.2844927$.

In order to check this scenario, it is not possible to use a single orbital solution, as due to its chaotic behavior, the motion cannot be predicted precisely beyond a few tens of millions of years. A statistical study of the past evolutions of Mercury's orbit is then performed, with the integration of 1000 orbits over 4 Gyr in the past, starting with very close initial conditions, within the uncertainty of the present ones (Fig.1). This statistical study was made possible by the use of the averaged equations for the motion of the Solar System[4,5]. For each of these 1000 orbital motion of Mercury, the rotational motion (Eq.1) was integrated numerically with planetary perturbations, for $p = k/2; k = 1, \dots, 10$. Simulations were started at $t_0 = -4$ Gyr, with a rotation period of 20 days ($x \approx 4.4$), using $\xi = 0.3333$, $k_2 = 0.4$ and $Q = 50$. As e is not constant, $x(t)$ will tend towards a limit value $\tilde{x}(t)$ that is similar to an averaged value of $x_l(t)$ and capture into resonance can now occur more often (Fig.2).

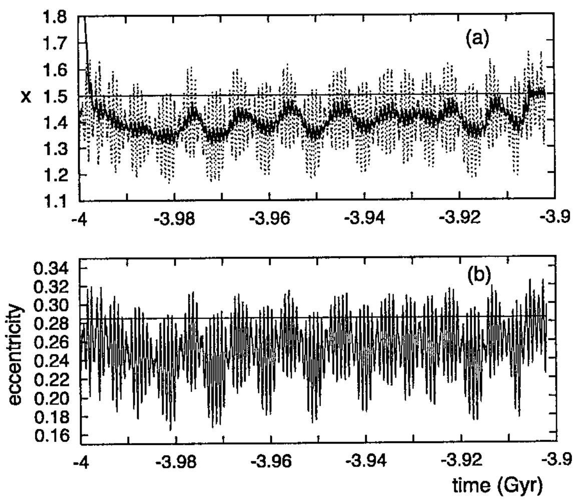

Figure 2. Rotation rate for a non constant eccentricity (b). The limit solution of equation (1) is no longer $x_l = N(e)/\Omega(e)$ [(a) dotted line], but now given by: $\tilde{x}(t) = \left(x(0) + K \int_0^t N(e(\tau))g(\tau)d\tau\right) /g(t)$, where $g(t) = \exp(K \int_0^t \Omega(e(\tau))\, d\tau)$ [(a), filled line]. In this example, there is no capture at the first crossing of the 3/2 resonance (at $t \approx -3.9974$ Gyr). About 100 Myr later, as the mean eccentricity increases, additional crossing of the 3/2 resonance occurs, leading to capture with damping of the libration.

All the 1000 solutions were followed, starting from −4 Gyr, until they reached the present date or get captured into the 2/1, 3/2, or 1/1 resonances. Contrarily to previous studies, it was found that capture into the 1/1 resonance is possible, as the eccentricity of Mercury may decrease to very low values, where the capture can occur, and the resonance remains then stable. The 3/2 remains stable, except for extremely small values of the eccentricity[2]. Indeed, over 554 solutions that were captured into the 3/2 resonance, a single solution, initially captured at −3.995 Gyr, escaped from resonance at about −2.396 Gyr. The solution then got trapped into the 1/1 resonance at −2.290 Gyr, capture that was favored by the low eccentricity required to destabilize the 3/2 resonance. Out of the 56 solutions initially trapped into the 2/1 resonance, 10 were destabilized and only 2 of them were further captured, one into 3/2 resonance, and one into 1/1 resonance. Globally, only 38.8% of the solutions did not end into resonance, and the final capture probability distribution was[2]:

$$P_{1/1} = 2.2\%, \quad P_{3/2} = 55.4\%, \quad P_{2/1} = 3.6\% \ .$$

With the consideration of the chaotic evolution of the eccentricity of Mercury, it is then shown that with a realistic tidal dissipative model that properly accounts for the damping of the libration of the planet, the present 3/2 resonant state is the most probable outcome for this planet. The largest unknown remains the dissipation factor k_2/Q in (Eq.1). A stronger dissipation would increase the probability of capture into the 3/2 resonance, as $x(t)$ would follow more closely $x_l(e(t))$ (Fig.2), while lower dissipation will slightly decrease the capture probability.

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