Preface

Precision of experimental data in many areas of elementary particle physics is quickly improving. For example, the anomalous magnetic moment of the muon is measured with a fantastically high precision. Results from B factories at SLAC and KEK for many quantities have low systematic errors and very high statistics. Of course, there are many more examples of such progress.

To compare high-precision experimental data with the theory, one has to obtain equally high-precision theoretical expressions for the measured quantities. Preparing physical programs for future colliders also requires high-precision theoretical calculations. In order to be able to search for a new physics, one has to understand standard processes (which can be a source of background) at a highly detailed level.

This means, in particular, calculation of higher radiative corrections. They are described by Feynman diagrams with one or several loops. Calculation of such diagrams is a very non-trivial task. It involves solving deep mathematical problems. Even when a suitable calculation algorithm has been constructed, this is not the end of the story. Often, many thousands of diagrams have to be calculated. This requires an unprecedented level of automation of theoretical research: generation and calculation of the diagrams have to be done systematically, by computer programs, without any interference of a human researcher. Some of the calculations of this kind are among the largest computer-algebraic calculations ever performed. This area of theoretical physics is progressing rapidly. A large number of physicists in many countries are involved in such activities. And this number is increasing. Many of today's students in the area of theoretical high energy physics will be involved in calculations of radiative corrections in the course of their careers.

Quantum field theory textbooks usually don't describe methods of calculation of multiloop Feynman diagrams. Most textbooks discuss quantization of fields (including gauge theories), obtain Feynman rules, and show a few simple examples of one-loop calculations. On the other hand, there is a huge amount of literature for experts in the area of multiloop calculations, usually in the form of original papers and specialized review articles. The purpose of this book is to close the gap between textbooks and the modern research literature. The reader should have a firm grasp of the basics of quantum field theory, including quantization of gauge fields (Faddeev–Popov ghosts, etc.) and Feynman rules. These topics can be found in any modern textbook, e.g., in [Peskin and Schroeder (1995)]. No previous experience in calculating Feynman diagrams with loops is required. Fundamental concepts and methods used for such calculations, as well as a large number of examples, are presented in this book in detail.

The main focus of the book is on quantum electrodynamics (QED) and quantum chromodynamics (QCD). In the area of QED, some extremely high-precision experimental data are available (anomalous magnetic moments, hydrogen atom, positronium). Correspondingly, some groundbreaking theoretical calculations have been done. In the area of QCD, very high precision comparisons of the theory and experiments are never possible, because we still don't know how to take non-perturbative phenomena into account quantitatively and in a model-free way (except by lattice Monte–Carlo simulations, whose accuracy is not very high but is increasing). However, the QCD coupling constant is much larger, and several terms of perturbative series are usually required to obtain the necessary (moderate) precision. Calculations in QED and QCD are usually very similar, but QCD ones are more lengthy — more diagrams, colour factors, etc. Therefore, a large fraction of the text is (technically) devoted to QED, but it should be considered also as a demonstration of methods which are used in QCD.

The first part of this book is based on lectures given to students preparing for the M. Sc. degree at Dubna International Advanced School on Theoretical Physics in 2005 and at Universität Karlsruhe. They were published as hep-ph/0508242. They were revised and extended for this book.

Practically all modern multiloop calculations are performed in the framework of dimensional regularization. It is discussed in Chap. 1, together with simple (but fundamentally important) one-loop examples. In Chaps. 2 and 3, one-loop corrections in QED and QCD are discussed. Here we use the $\overline{\text{MS}}$ renormalization scheme, which is most popular, especially in

QCD. Methods and results of calculation of two-loop corrections in QED and QCD are introduced in Chap. 4, also using $\overline{\text{MS}}$ scheme. Chap. 5 is devoted to the on-shell renormalization scheme, which is most often used in QED at low energies, but also for heavy-quark problems in QCD. Decoupling of heavy quarks is most fundamental in QCD; it is employed practically every time one does any work in QCD. It is presented in Chap. 6, where a simplified QED problem is considered in detail; it makes understanding the problem much easier. This is the first time decoupling in the $\overline{\text{MS}}$ scheme is considered in a textbook, with full calculations presented. Finally, Appendix A is a practical guide on calculating colour factors, which is a necessary (though simple) step in any QCD work. Here I follow an excellent book [Cvitanović (web-book)] available on the Web.

The second part is based on lectures given to Ph. D. students at the International School "Calculations at modern and future colliders", Dubna (2003), and at Universität Karlsruhe. They were published in Int. J. Mod. Phys. A **19** (2004) 473. They are (slightly) revised for this book. This second lecture course forms a natural sequel to the main one. It discusses some advanced methods of multiloop calculations; in cases when the same problem is discussed in both courses, it is solved by different methods. So, studying both of the courses gives a wider perspective and a better toolbox of methods. For a much more comprehensive presentation of modern methods of calculating Feynman integrals, the reader is addressed to a recent book [Smirnov (2006)].

Of course, there are a lot of things which are *not* discussed in this book. It only shows the most simple and fundamental examples. More complicated scattering processes (diagrams with more external legs) and radiative corrections in the electroweak theory (which often involve several kinds of particles with different masses) are not considered here. But the general approaches (dimensional regularization, $\overline{\text{MS}}$ renormalization, integration by parts...) remain the same. After reading this book, the reader should have no problems reading specialized literature about more advanced problems.

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