Preface

Combinatorics is a branch of mathematics dealing with discretely structured problems. Its scope of study includes selections and arrangements of objects with prescribed conditions, configurations involving a set of nodes interconnected by edges (called graphs), and designs of experimental schemes according to specified rules. Combinatorial problems and their applications can be found not only in various branches of mathematics, but also in other scientific disciplines such as engineering, computer science, operational research, management sciences and the life sciences. Since computers require discrete formulation of problems, combinatorial technique has become an essential and powerful tool for engineers and applied scientists, in particular, in the area of designing and analyzing algorithms for various problems which range from designing the itineraries for a shipping company to sequencing the human genome in the life sciences.

The *counting problem*, which seeks to find out how many arrangements there are in a particular situation, is one of the basic problems in combinatorics. Counting has been used in the social sciences for calculating the Shapley–Shubik power index which measures the power of a player in a decision-making body (such as a shareholders' meeting, a parliament or the UN Security Council). In Chemistry, Cayley used graphs to count the number of isomers of the saturated hydrocarbons while Harary and Read counted the number of certain organic compounds built up from benzene rings by representing them as configurations of hexagons joined together along a common edge. In Genetics, by counting all possibilities for a DNA chain made up of the four bases, scientists arrive at an astoundingly huge number and so are able to understand the tremendous possible variation in genetic makeup. Counting has been used as well to study the primary and secondary structures of RNA.

This book is a companion to our earlier book *Counting* (World Scientific, 2002). The book is divided into two sections. In the first section, we provide supplementary material for the purposes of adding to the reader's knowledge about counting techniques and, in particular, as a resource book for junior college students and teachers reading and teaching the topic Combinatorics at H3 level in the new Singapore mathematics curriculum for junior college. Combinatorics is chosen as one of the four topics (the others are Differential Equations, Graph Theory and Plane Geometry) in H3 mathematics. The emphasis in Combinatorics within the syllabus is for honing basic skills and techniques in general problem solving and logical thinking. This book used together with *Counting* will provide a solid base for Combinatorics at the H3 level.

The second section provides solutions to the exercises in the book *Counting*. Each chapter in this section begins with a recapitulation of some of the important concepts and/or formulae. Some of the problems are reproduced by permission of the University of Cambridge Local Examinations Syndicate and those marked with (AIME) are from the American Invitational Mathematics Examination. We would like to express our gratitude to the above organizations for kindly allowing us to include these problems in the book. There is often more than one method to solve a particular problem and we have included alternative solutions whenever we think that they would be of interest. On the other hand, we have only provided hints to AIME problems in deference to the limited permission granted to us.

The first two chapters cover the Principle of Inclusion and Exclusion (PIE), and its general statement. Many situations of counting are complicated by the possibility of double counting. Sometimes, however, when trying to make provisions for double counting, one may overcompensate and deduct more than the number which was double counted. PIE neatly handles this kind of situation.

The quaintly named Pigeonhole Principle is studied in Chap. 3. Unlike the other principles which we have discussed so far in this book and in *Counting*, the Pigeonhole Principle does not actually count the number of ways for a particular situation. Instead, the Pigeonhole Principle is used to check for the existence of a particular situation. This aspect of "checking existence" together with "counting" and, as yet not discussed in our books, "optimization" are the three main focus areas in Combinatorics. In the Pigeonhole Principle, we try to transform the problem partly into one of distributing a number of objects into a number of boxes. The questions to focus on then become "What are the objects?" and "What are the boxes?"

The last four chapters are on recurrence relations. Some counting problems defy the techniques and principles learnt thus far. Chapter 4 introduces the technique of using recurrence relations. These recurrence relations represent algebraically the situation where the solution of a counting problem of bigger size can be obtained from the solutions of the same problem but of smaller size. It is by writing the recurrence relations and obtaining a number series from them, that the counting problem can be solved. Chapters 5 to 7 study three series of numbers which are derived from special recurrence relations. These are the Stirling numbers of the first kind, the Stirling numbers of the second kind and Catalan numbers.

The supplementary chapters are based on a series of articles on Counting that first appeared in Mathematical Medley, a publication of the Singapore Mathematical Society. Many thanks also to Dong Fengming and Katherine Goh for reading through the draft and checking through the problems — any mistakes that remain are ours alone.

For those who would like to know more about the subject, a recommended list of publications for further reading is provided at the end of this book.

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