

Contents

Preface	xiii
Foreword	xv
1 Generalized Quadrangles	1
1.1 Finite Generalized Quadrangles	1
1.2 Automorphisms	4
1.3 Grids and Dual Grids	4
1.4 The Classical Generalized Quadrangles	4
1.5 The Generalized Quadrangles $T_2(\mathcal{O})$ and $T_3(\mathcal{O})$ of Tits	5
1.6 The Generalized Quadrangles $T_2^*(\mathcal{O})$	7
1.7 Orders of the Known Generalized Quadrangles	8
1.8 Generalized Quadrangles with Small Parameters	8
2 Regularity, Antiregularity and 3-Regularity	11
2.1 Regularity	11
2.2 Regularity and Dual Nets	14
2.3 Antiregularity	16
2.4 Antiregularity and Laguerre Planes	18
2.5 3-Regularity	22
2.6 3-Regularity and Subquadrangles	23
2.7 3-Regularity, Inversive Planes, and Characterizations	24
3 Elation and Translation Generalized Quadrangles	27
3.1 Some Notions from Group Theory	27
3.2 Elation Generalized Quadrangles	29

3.2.1	Elation Generalized Quadrangles: Definition and Two Fundamental Structure Theorems	29
3.2.2	4-Gonal Families and EGQs	30
3.3	Translation Generalized Quadrangles	32
3.3.1	Symmetry	33
3.3.2	Translation Generalized Quadrangles: Definition	34
3.3.3	Symmetries and Translations	34
3.4	The Kernel of a Translation Generalized Quadrangle	37
3.5	$\mathbf{T}(n, m, q)$ s and Translation Generalized Quadrangles	39
3.6	Regular Pseudo-Ovals and Regular Pseudo-Ovoids	42
3.7	Automorphisms of Translation Generalized Quadrangles	44
3.8	Important Properties of $\mathcal{O}(n, m, q)$	47
3.9	Pseudo-Ovals	49
3.10	Eggs	52
3.11	The Stabilizer of the Base-Point of a Translation Generalized Quadrangle	57
3.12	Structure of the Automorphism Group of a Translation Quadrangle	59
4	Generalized Quadrangles and Flocks	61
4.1	Flocks	61
4.2	Flocks and Translation Planes	62
4.3	Flocks of Ovoids and Hyperbolic Quadrics	63
4.4	Flocks of Cones	64
4.5	Semifield Flocks	69
	Known Examples of Semifield Flocks	71
4.6	Generalized Quadrangles and Flocks	72
4.7	Semifield Flocks and Translation Generalized Quadrangles	74
4.7.1	Position of a Translation Line in a Flock Quadrangle	74
4.7.2	Additive q -Clans and Translation Generalized Quadrangles	74
4.7.3	Known Cases	76
4.8	Derivation and BLT-Sets	77
4.9	Constructions	78
4.10	Property (G) for Generalized Quadrangles of Order (s, s^2)	81
4.11	Flocks, Subquadrangles and Ovals	85
	Addendum A: Isomorphisms of Flock Quadrangles and Associated Geometries	87

4.12	The Fundamental Theorem of q -Clan Geometry, and Applications . . .	87
	Addendum B: Basic Questions on Elation Groups	92
4.13	The Standard Conjectures and Questions	92
4.14	Some Results by Payne and K. Thas	96
4.15	Elation Generalized Quadrangles with Nonisomorphic Elation Groups	98
5	Good Eggs	101
5.1	Good Eggs and Good Translation Generalized Quadrangles	101
5.2	Good Eggs and Veronese Surfaces	116
5.3	Coordinatization and Applications	121
5.3.1	Coordinatization of Eggs and Dual Eggs	121
5.3.2	The Known Examples Revisited	123
5.3.3	A Geometric Connection between Semifield Flocks and Good Eggs	124
5.3.4	Further Characterizations of Good Eggs in Odd Characteristic	125
6	Generalized Quadrangles, Nets and the Axiom of Veblen	129
6.1	Generalized Quadrangles and the Axiom of Veblen	129
6.2	Translation Generalized Quadrangles and the Axiom of Veblen . . .	131
6.3	Property (G) and the Axiom of Veblen	132
6.4	Flock Generalized Quadrangles and the Axiom of Veblen	135
6.5	Subquadrangles and the Axiom of Veblen	138
6.6	Nets and Characterizations of Translation Generalized Quadrangles	139
7	Ovoids and Subquadrangles	145
7.1	Ovoids of $\mathbf{Q}(4, q)$	145
7.2	Subquadrangles and Ovoids	146
7.3	Translation Ovoids and Semifield Flocks	147
7.4	Coordinates of the Known Nonclassical Ovoids of $\mathbf{Q}(4, q)$	148
7.5	Subquadrangles of $\mathbf{T}(\mathcal{O})$, with \mathcal{O} Good: the Even Case	150
7.6	Subquadrangles of $\mathbf{T}(\mathcal{O})$, with \mathcal{O} Good: the Odd Case	156
7.7	Subquadrangles: Remaining Cases and Some Applications	157
7.8	Translation Generalized Quadrangles with One Classical Subquad- rangle	160
7.9	Elation Generalized Quadrangles with a Subquadrangle	163

8	Translation Generalized Ovals	169
8.1	Translation Generalized Ovoids and Translation Generalized Ovals	169
8.2	Note on the Definition of Translation Generalized Oval/Ovoid . . .	174
8.3	Characterizations of the $T_2(\mathcal{O})$ of Tits	175
8.4	A Characterization of Translation Generalized Ovals	176
8.5	Classification of 2-Transitive Generalized Ovals in Even Characteristic	177
9	Moufang Sets and Translation Moufang Sets	181
9.1	Definition and General Results	181
9.2	Finite Moufang Sets	186
9.2.1	The Case $PSL_2(q)$	188
9.2.2	The Case $PSU_3(q)$	189
9.2.3	The Case $Sz(q)$	190
9.2.4	The Case $R(q)$	192
9.2.5	Sub Moufang Sets	196
10	Configurations of Translation Points	199
10.1	Span-Symmetric Generalized Quadrangles	200
10.2	Groups Admitting a 4-Gonal Basis	201
10.3	SPGQs and Moufang Sets	203
10.4	Basic Structural Lemmas	204
10.5	Classification of SPGQs of Order (s, t) , $1 < s \leq t < s^2$	208
10.6	SPGQs of Order (s, s^2)	209
10.7	Generalized Quadrangles with a Line of Translation Points	212
10.8	On the Classification of Translation Generalized Quadrangles	216
11	Moufang Quadrangles with a Translation Point	219
11.1	Notation	219
11.2	Some General Elementary Lemmas	220
11.3	The Moufang Property and Analogues	225
11.4	Tits Generalized Quadrangles and Tits Systems	229
11.5	Properties of Moufang Quadrangles	236
11.6	Half 3-Moufang Quadrangles	247
11.7	2-Moufang Quadrangles and Fong-Seitz Quadrangles	252
11.8	Conclusion	259

12 Translation Ovoids in Translation Quadrangles	261
12.1 Ovoids, Elation or Translation with respect to a Flag or a Point . . .	262
12.2 Self-Polar Elation Generalized Quadrangles	264
12.3 Suzuki-Tits Moufang Sets	267
12.3.1 Polarities in the Symplectic Quadrangle $W(q)$	267
12.3.2 The Suzuki-Tits Ovoids	271
12.4 Subtended Elation Ovoids	273
13 Translation Generalized Quadrangles in Projective Space	279
13.1 Generalities about Lax Embeddings	280
13.2 Planar Translation-Homogeneous Embeddings	288
13.3 Exceptional Non-Planar Translation-Homogeneous Embeddings . .	295
13.4 Non-Planarly Embedded Small Translation Generalized Quadrangles	301
13.5 Non-Planarly Embedded Translation Generalized Quadrangles . . .	307
A Open Problems	317
A.1 Chapter 1	317
A.2 Chapter 2	318
A.3 Chapter 3	318
A.4 Chapter 4	320
A.5 Chapter 5	320
A.6 Chapter 6	321
A.7 Chapter 7	321
A.8 Chapter 8	322
A.9 Chapter 9	322
A.10 Chapter 10	322
A.11 Chapter 11	323
A.12 Chapter 12	323
A.13 Chapter 13	324
Bibliography	325
Index	337