Preface

> "Everything should be made as simple as possible, but not simpler."
> [Albert Einstein]

In the language of Philip Anderson,[1] nuclear physics is an *emergent phenomenon*. In other words, no body of theoretical physicists, however smart they might be, could have predicted the existence of a nucleus, let alone its properties, from first principles (even if such first principles were known). Nuclear physics is an experimental science. Nevertheless, the theoretical interpretation of what is observed and the construction of logically consistent predictive theories are essential for guiding the course of experiment and for developing an understanding of the subject.

Physics progresses by systematic experimental observations and the construction of models to interpret them. In the normal progression of physics research, the compilation of experimental data runs in parallel with the development of theoretical models in a mutually constructive way. However, sometimes the natural interpretation of a body of data can be at serious odds with the theoretical perspectives of the day. As more data are accumulated, the interpretation may be adjusted and brought into line with an evolving underlying theory. It can also happen that a body of data continues to conflict with conventional wisdom and is far too pervasive to be ignored. Explaining the data may then result in a *paradigm shift*[2] in the theoretical description of the system concerned.

The modern era of nuclear physics was initiated in the middle of the last century by such a paradigm shift. Because of the strong short-range interactions between nucleons, there had previously been little hope that nuclei could be understood at a microscopic level. Thus, radioactivity and macroscopic properties, such as sizes and masses, were described in terms of a so-called *compound nucleus* model and a *liquid drop* model. However, a simple independent-particle shell model proved to be so extraordinarily successful at interpreting a huge body of experimental data that theorists were obliged to seek an explanation of how a system of strongly-interacting nucleons could behave like weakly-interacting particles in a central field.

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[1] Anderson P.W. (1972), *Science* **177**, 393.

[2] Kuhn T.S. (1996), *The Structure of Scientific Revolutions* (University of Chicago Press, Chicago), third edn.

FUNDAMENTALS OF NUCLEAR MODELS - Foundational Models

The subsequent many-nucleon theory of the nucleus emerged from the need to come to terms with the shell-model perspective.

The development of the shell model and an (albeit incomplete) understanding of how it could apply, rapidly led to its acceptance as the underlying theory of nuclear structure in terms of interacting neutrons and protons. This shell-model example illustrates the importance of pursuing alternative interpretations of observed phenomena in spite of conflicts with generally accepted beliefs. One should be aware that a model can be successful in fitting a body of data for reasons quite different from those envisaged in its formulation. This can happen, for example, because different physical models may have common, or simply related, mathematical structures. Thus, fitting a model to a data set may be testing the mathematical structure of the model but not necessarily its physical interpretation. It should also be remembered that a given set of physical observables will, generally, have many representations in quantum mechanics. And, while the states of different representations may be 100% orthogonal to one another, their properties may be indistinguishable by a finite set of measurements taken with limited accuracy. Thus, to extract a convincing physical interpretation of the many models of nuclear structure, it is important to consider how these models fit into the larger overall perspective of the nucleus as a system of interacting neutrons and protons.

Expressing a successful model in microscopic terms is an essential component of understanding its significance. If all attempts to derive such an expression fail, then one must either suspect the interpretation of the model or the validity of the underlying microscopic theory. Whatever the answer, one will undoubtedly emerge from the exercise of rationalising the seemingly incompatible perspectives with a better understanding of the physics of what is going on. *Discovering that a model is successful is not nearly as important as understanding why it is successful. Identifying the limitations of a model are equally important.* Thus, a primary objective of this book is to discover the extent to which the various insightful models of nuclear structure can be embedded as submodels of the shell model. In the end, *the main criteria for acceptance of a theory of nuclear structure are consistency and elegance.*

Many models of nuclear structure phenomena are formulated as submodels of the shell model. Elliott's SU(3) model of nuclear rotations and the BCS theory of pair coupling are notable examples. In contrast, the collective models were initially formulated as liquid drop models and viewed as continuum limits of a many-nucleon system. However, the observation of rotational bands in light nuclei showed that collective states also occur for relatively small numbers of nucleons. In the unified model, the collective models were given a microscopic interpretation in terms of nucleons moving in a mean field with collective properties.

The unified model largely avoids the necessity of requiring the large nucleon-number limit. It is exceptionally insightful and has been deployed in numerous creative ways, e.g., to determine the kinds of shell model configurations necessary to explain collective phenomena. Its deficiency is that it is limited by the constraints of a mean-field approximation to a highly restricted subspace of shell model states. In parallel with the mean-field methods, an algebraic approach, based on dynamical symmetries, which relates models with shell-model coupling schemes, has been developed. Fortunately, the mean-field and algebraic approaches are complementary and can even be used in combination. Both are discussed in this book.

In the algebraic approach, the observables of a successful model are expressed in terms of a Lie algebra of observables called a *spectrum generating algebra*. One then seeks a microscopic expression of these basic observables in terms of nucleon observables (i.e., position and momentum coordinates and spins). Finding such an expression can be a very non-trivial task. It may be that the model is only valid in a limiting situation, such as a limit in which the nucleon number is large. Or it may be that the model is an approximation to a more general model which does have a microscopic expression. One can then attempt to understand the circumstance under which the approximation is valid. Whatever the relationship should be, it is important to seek it out because only by doing so does the model become a contributor to the overall understanding of nuclear structure theory. If a microscopic expression of a model can be found, one can proceed to determine the representations of its spectrum generating algebras in the Hilbert space of the shell model. The models then correspond to shell-model coupling schemes. Thus, they become submodels of the shell model and acquire relationships with other models corresponding to different coupling schemes.

We start, in Chapter 1, with an overview of the variety of experimental observations that are made on nuclei and the model ways in which they are interpreted. By compiling nuclear data and examining them from many perspectives, it appears that the data often suggest their own interpretation. What emerges from Chapter 1 is a view of the nucleus from many vantage points and a rationale for considering a corresponding set of phenomenological models. Later chapters present the models in some depth. This volume focuses on the four foundational models on which most developments in nuclear physics are based: the collective model, the shell model, the pair-coupling models, and the mean-field (Hartree-Fock) models. A subsequent volume will introduce more-microscopic models and emphasise the relationships between the models and their realisation as submodels of an all-embracing shell model.

It is assumed that the reader has a basic knowledge of quantum mechanics and angular momentum theory, as presented in any of the standard texts, and a familiarity with introductory nuclear physics, e.g., at the level of the book *"Introductory Nuclear Physics"* by K.S. Krane.[3] Some familiarity with elementary concepts in symmetry (group theory) will also be useful. No previous knowledge of nuclear structure is required. This book is intended to serve a wide range of uses. At one extreme, it provides a pedagogical introduction to nuclear physics. And, at the other, it provides a resource book for researchers at the frontiers of the subject.

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[3] Krane K.S. (1988), *Introductory Nuclear Physics* (Wiley, New York).

Some aspects of this book were tackled at a much more elementary level in the book "Nuclear Collective Motion" published in 1970.[4] That book did not include the more recently developed algebraic perspectives and is superseded by the present two-volume text. However, the simple pragmatic description of the basic models given in 1970 is as relevant today as when it was published. Thus, for example, a first course in nuclear structure, could profitably be based on the model descriptions given in this 1970 book (to be republished by World Scientific) together with the overview given in Chapter 1 of this volume of our current, more complete, understanding of nuclear structure in terms of them.

The last 40 years have seen huge advances both in experimental and theoretical technology and, consequently, an explosion in the range of nuclei investigated. Thus, a much deeper understanding has developed of the circumstances under which the different nuclear models are applicable. The perspective for deriving the microscopic foundation of these models has also evolved. It is now time to assess how much of what we have learnt is built on firm foundations and where gaps in our understanding remain. It is by exploring failures of models that physics advances.

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[4] Rowe D.J. (1970), *Nuclear Collective Motion: Models and Theory* (Methuen, London).