

Lifetime and Mass α -Quantization: Physics Beyond the Paradigm

1.1 The Missing Elementary Particle Ground State and Its Mass Generator

This book is about the fine structure coupling constant $\alpha = e^2/\hbar c \cong 1/137$ and about the ground state on which it operates. We present evidence which indicates that the coupling constant α is the most powerful — and most misunderstood — operator in elementary particle physics. More specifically, this book contains analyses of two sets of elementary particle data that, independently of theory, exhibit clear-cut periodicities in powers of the numerical factor 137 or its reciprocal value $1/137$. These α -based periodicities occur in the long-lived, low-mass, threshold-state particles, which serve as the foundation states for the higher-mass excitations, and which are the most revealing in their structure. The first set of data — particle lifetimes — is analyzed in Chapter 2. The second set of data — particle masses — is analyzed in Chapter 3. The accuracy and completeness of these α -quantized data patterns furnishes strong phenomenological evidence for the conclusion that the fine structure constant α , which we know serves as the coupling constant in quantum electrodynamics (QED), also serves as a coupling constant in quantum chromodynamics (QCD). One practical benefit we obtain from this approach is that we can finally handle the oldest unsolved problem in elementary particle physics — the proton-to-electron mass ratio. Guided by the phenomenology of α -dependent threshold-state lifetimes and masses, we obtain a reasonably accurate value for this important ratio without having to resort to the use of adjustable parameters.

In the current paradigm of elementary particle physics, as embodied in the Standard Model, the leptons of QED (the weakly-interacting particles)

and the hadrons of QCD (the strongly-interacting particles) are treated theoretically as separate entities. However, the particle *lifetime* data displays mentioned above show leptons and hadrons combined together on a common α -spaced lifetime grid. Even more compellingly, the particle *mass* data displays reveal that the leptons and the proton occupy a common α -dependent mass excitation tower. The lepton and proton masses are interleaved together, and the overall pattern can only be understood by considering all of these masses in one unified picture. What does this tell us about the similarities and differences between leptons and hadrons? The answer, broadly stated, is that leptons and hadrons share a common set of mass building blocks, and the essential difference between leptons and hadrons arises mainly from the fact that the electric charges on leptons are not fractionated, whereas the electric charges on hadrons are fractionated and spread among the quark substates. This charge fractionization process produces the gluon fields and the distinctive hadron interactions. The forces between elementary particles are created almost exclusively by the charges. The masses are passive entities which carry the bulk energy content and inertial properties of the particles, and also the particle–antiparticle and lepton family quantum numbers, but otherwise are essentially non-interacting. These results follow not only from the lifetime and mass systematics of the elementary particles, which are presented in Chapters 2 and 3, but also from information we can extract from the spectroscopic properties of these particles [1], which are discussed in Chapter 4.

What are the implications of this extensive particle α -quantization with respect to the well-established systematics of the Standard Model? The essential point here, which is in line with the above discussion about leptons and hadrons, is that the distinctive features of QCD — the accurate predictions of isotopic spins, the unique asymptotically-free quark binding energies, the calculations of quark and gluon jets — all involve the *charges* on the elementary particles. The Standard Model handling of elementary particle *masses* has not been nearly as successful. The lifetime of an elementary particle is essentially a measure of the stability of its mass structure, and the mass of the particle reflects the structure itself. Thus the present α -quantization studies pertain to the masses, and have very little to do (at least on this phenomenological level) with the charge states. The α -spaced lifetime groups we show here accurately reflect the Standard Model quark structure, and thus reinforce the reality of the quarks in these particles. But the α -quantized masses we obtain, which are constituent-quark masses, are quite different from the current-quark masses of the Standard

Model, especially in the lower-mass states. QCD is primarily a theory of elementary particle charges, not masses. This conclusion was also set forth by Gottfried and Weisskopf in their classic two-volume *Concepts of Particle Physics* [2], where they commented as follows:

Unfortunately, QCD has nothing whatsoever to say about the quark mass spectrum, nor, for that matter, does any other existing theory.
[3]

The determination of the particle mass spectrum is thus thrown back to the experimental data themselves, which fortunately tell their own story when properly arrayed.

In the remainder of Chapter 1 we briefly discuss some of the problems facing particle physics. Then we move on in Chapter 2 to the lifetime systematics, and in Chapter 3 to the mass systematics. These two chapters involve the phenomenology of the experimental data. In Chapter 4 we introduce some spectroscopic information, which involves mathematical phenomenology (mathology). Finally, in Chapters 5–7 we discuss some of the ramifications that ensue from this pursuit of the coupling constant α .

1.2 The Particle Mass Mystery: Physics from the Higgs Down or the Bottom Up?

The main reason we study elementary particles is to learn something about the make-up of our universe. Leon Lederman expressed this viewpoint very succinctly in his book *The God Particle* [4]:

This book is devoted to one problem, a problem that has confounded science since antiquity. What are the ultimate building blocks of matter? [5]

The present book is focused on this same problem, although with a different approach, and with a different conclusion. An *elementary particle*, by implicit definition, is a fundamental building block out of which everything else is constructed. If we start with a macroscopic object and then divide it into smaller and smaller pieces, we presumably finally reach bed-rock — the smallest pieces there are. These are the elementary particles. And perhaps their most fundamental attribute is their mass — their total energy content. Thus what we want most to learn about these particles is their mass structure. However, this has turned out to be unexpectedly difficult

to accomplish. The total mass of a particle can be accurately measured, but that does not explain why it has this particular value. In the area of atomic physics, by way of contrast, the mass structure of the basic elements is relatively simple. The mass of an atom is mainly in its nucleus, and nuclei are composed of protons and neutrons, which have about the same mass. So the mass of an atom is accurately obtained by simply counting up the number of protons and neutrons in the nucleus. But this simplicity does not carry over to the observed elementary particle states. A good example is provided by the massive *leptons*, of which there are three: the electron, muon, and tau, together with their antiparticles. Using mass units of MeV (million electron volts), with the velocity of light c set equal to unity, as is customary in particle physics, we have:

electron (0.51 MeV), muon (105.66 MeV), tau (1776.99 MeV).

These lepton mass values do not seem to form any kind of logical progression. The leptons are weakly-interacting particles, whose interactions are purely electromagnetic. The strongly-interacting particles, denoted as *hadrons*, also have mysterious masses. Hadrons occur as *mesons* (with integral spins) and *baryons* (with half-integral spins). The lightest meson, the neutral pion, has a mass of 135 MeV, and the lightest baryon, the proton, has a mass of 938 MeV. None of these mass values emerge directly from the existing elementary particle theories.

The entire problem of elementary particle mass generation has remained obscure. In his classic book *QED* [6], Richard Feynman devoted the first three chapters to the interactions of electrons and photons. Then in the fourth and final chapter he took this information over and discussed how much of it applies to hadron interactions. At the very end of the book, in his final observation, Feynman commented on the mass problem as follows:

Throughout this entire story there remains one especially unsatisfactory feature: the observed masses of the particles, m . There is no theory that adequately explains these numbers. We use the numbers in all our theories, but we don't understand them — what they are, or where they come from. I believe that from a fundamental point of view, this is a very interesting and serious problem. [7]

Present-day theories attribute elementary particle masses to the operation of the hypothetical “Higgs particle,” which seems required in order to have a renormalizable particle theory [4]. In most fields of physics, masses are

determined by the lightest members of the field, whose masses then compound or excite to form the higher-mass states. But the Higgs particle, by way of contrast, is a very heavy particle — so heavy that it has not yet been found. In his book *Facts and Mysteries in Elementary Particle Physics* [8], Martinus Veltman describes its salient features in the following way:

*In short, it [the Higgs] must be coupled to **any** particle having a mass. Moreover, the coupling must always be proportional to the mass of the particle to which it is coupled.*

To date the Higgs particle has not been observed experimentally. Unfortunately the theory has nothing to say about its mass, except that it should not be too high (less than, say, 1000 GeV)

Because this Higgs particle seems so intimately connected to the masses of all elementary particles, it is tempting to think that somehow the Higgs particle is responsible for these masses. Up to now we have no clue as to where masses come from: they are just free parameters fixed by experiment.

There is clearly so much that we do not know! . . . [9]

A basic difficulty in trying to understand elementary particle masses is that we need to know more than just the total mass of the particle. We need to know the substructure of the mass. We have discovered so many elementary particles that we now believe they are not truly *elementary*. It seems logical that these elementary particles must be composed of even-more-elementary substates. Before the age of high-energy accelerators, only a handful of elementary particles were known to exist. However, we now have about 200 measured particle states, which suggests that some of these must be more “elementary” than others. A significant step towards simplifying this plethora of particles was the recognition that they can be constructed from a much smaller set of “quark” substates. The so-called Standard Model (SM) of physics, the theoretical model that dominates present-day particle physics, features six spin-1/2 quarks (d , u , s , c , b , t), together with their corresponding antiquarks [10]. The names for these quarks are *down*, *up*, *strange*, *charm*, *bottom* and *top*. These names are denoted collectively as the quark “flavors.” The quarks carry the fractional charge states $(-1/3, +2/3, -1/3, +2/3, -1/3, +2/3) e$, respectively, with opposite signs for the antiquarks. The down and up quarks d and u reproduce the proton and neutron (which are referred to collectively as “nucleons”). The d and u quarks also appear as components in most of

the other hadron states. The s , c , b and t quarks carry the strangeness, charm, bottom and top flavor quantum numbers, respectively. The quark charge assignments, together with the specified quark combinations, accurately reproduce the measured isotopic spins (charge states) of the various particles. This has been one of the major successes of the Standard Model, and has led to many predictive accomplishments.

The difficulties with the quark model arise when attempts are made to separate a particle into its quark substates. We logically expect an integrally charged particle in a high energy collision to break apart into its fractionally charged u , d , s , c , b quark and antiquark components, so that we can measure the masses of the individual quarks. But experimentally this does not occur. (The top quark t is a special case, and is discussed separately in Sec. 3.20.) Specifically, what has been established is that particle break-ups or decays into *fractionally-charged* particles do not occur. If particles break apart into quarks, the quark fragments combine with other created quark states to form combinations that carry integer electric charges. The fragmented *quark charges*, which seem to exist as $1/3$ and $2/3$ entities on the quarks inside of a particle, cannot be extracted as fractional charges outside of the particle: only integrally-charged particles are found in the decay products. Thus if the electric charges do actually separate into $1/3$ and $2/3$ fractions on the quark states, as is indicated by the isotopic spin rules, they are evidently still tied together by unbreakable confining forces. The question then remains as to whether the *quark masses*, apart from the quark charges, can be separated and dislodged in collision or decay processes. Empirically, the masses of the final-state particles are not those of the initial-state u , d , s , c , b quarks. Thus Standard Model quarks are not observable as isolated entities, so that it is difficult to ascertain their intrinsic masses experimentally.

There are two different reasons we can advance to explain why single-quark u , d , s , c , b masses are not observed in particle annihilations: either (1) the quark *masses* are bound so *strongly* that quarks cannot be pulled apart, or (2) the quarks have *lightly-bound* mass *substructures*, so that when a particle is shattered or decays, the quarks also shatter into their mass substates (but not with the fractional quark charges). The first reason (which seems to apply to the quark *charges*) requires a rubber-band type of mass binding energy that increases with separation distance, and is generally the favored explanation. This type of binding is denoted in the Standard Model as “asymptotic freedom.” However, the second reason is the one that seems to emerge from the present studies, which indicate that the Standard Model

quarks have weakly-bound (a few percent) mass substructures. In either case, we have no direct way to determine the intrinsic u , d , s , c , b quark masses (but magnetic moments [11] provide interesting clues). Without intrinsic quark masses, we cannot deduce the masses of the pion and proton, for example, without actually measuring them.

In his analysis of *Facts and Mysteries in Elementary Particle Physics* [8], Veltman expresses his feelings about the mass problem as follows:

*Here is another major problem of elementary particle physics.
Where do all these masses come from?* [12]

In the current paradigm of elementary particle physics [4, 8, 10], the assumption is made that the masses are created from the Higgs downward. However, the phenomenology of α -quantized lifetimes and masses suggests that the coupling constant α acting on the electron creates these masses from the bottom up. In the present book, experimental evidence is displayed to substantiate this “power of α ” viewpoint.

1.3 The Double Mystery of the Fine Structure Constant $\alpha = e^2/\hbar c$

In addition to the mystery of the origin of elementary particle masses, there is the mystery of the dimensionless fine structure constant $\alpha = e^2/\hbar c \cong 1/137$. In his book *Elementary Particles: Building Blocks of Matter* [13], Harald Fritzsch describes α as follows:

Let us emphasize that this number is the most prominent number in all of the natural sciences. Ever since its first introduction it has caused a lot of speculation. After all, α gives the strength of the electromagnetic interaction, which gives it fundamental importance for all the natural sciences and for all technology. [14]

The mystery about α is actually a double mystery. The first mystery — the origin of its numerical value $\alpha \cong 1/137$ — has been recognized and discussed for decades. The second mystery — the range of its domain — is generally unrecognized. One of the most important successes in modern physics has been the theory of QED. This is the theory of the interactions of electrons and muons with photons [6]. The QED electron–photon interaction strength is expressed in terms of the dimensionless coupling constant

$\alpha = e^2/\hbar c$. In the QED calculation of the anomalous magnetic moment of the electron, for example, a perturbation expansion is made in powers of α , and the calculated value matches the experimental value to an accuracy of one part in 10^{10} ! A calculation of comparable accuracy occurs for the anomalous magnetic moment of the muon, another lepton. The theoretical framework for the much stronger hadron interactions is QCD, [4, 8, 10], which was patterned after QED, and which has its own coupling constant. The leptons, including the fine structure constant α , have not been incorporated into QCD.

In QED the coupling constant α gives the exact interaction strength for an electron to produce a photon, including all of the various pathways along which the electron can travel in generating the photon [15]. The calculation of these pathways to high order in perturbation theory has taken years of work. The physical basis for the numerical value of this coupling constant, $\alpha \cong 1/137$ is unknown. Feynman has summarized this situation very vividly in his book *QED*, where he uses the symbol e to stand for α :

*There is a most profound and beautiful question associated with the observed coupling constant, e — the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to -0.08542455 . (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than 50 years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to π , or perhaps to the base of natural logarithms? Nobody knows. It's one of the **greatest** damn mysteries of physics: a **magic number** that comes to us with no understanding by man. You might say the 'hand of God' wrote that number, and 'we don't know how He pushed His pencil.' We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of a dance to do on a computer to make this number come out — without putting it in secretly!*

A good theory would say that e is 1 over 2 pi times the square root of 3, or something. There have been, from time to time, suggestions as to what e is, but none of them has been useful. [16]

The latest value for the numerical value of α is $1/137.03599911$ [10]. In spite of its extreme accuracy, this numerical value does not convey much physical insight as to its phenomenological significance.

Feynman was not the only physicist fascinated by the number 137. Lederman points out [4] that many physicists have pondered where it came from. As he states:

Werner Heisenberg once proclaimed that all the quandaries of quantum mechanics would shrivel up when 137 was finally explained. [17]

Also:

Pauli was in fact obsessed with 137, and spent countless hours pondering its significance. [18]

Lederman himself, while he was director at Fermilab, lived in a 150-year-old farm house situated on Eola Road on the laboratory grounds, to which he assigned the postal address 137 Eola Road.

The puzzlement over the derivation of the “pure number” 137, or $1/137$, has persisted for more than half a century. Eddington’s attempts to explain it are legendary [16]. In the present studies we argue the case that, in addition to its numerical value, there is in fact a second mystery involved with the constant α , which is the question:

What is the extent of the domain in which α operates as a coupling constant?

According to the Standard Model and its associated Grand Unified Theories, the domain of α is restricted to the leptonic particles that are considered in QED, where for example it generates photons in electron interactions. But the experimental data seem to point to a different conclusion. As we will demonstrate in detail in Chapter 2, the long-lived threshold-state elementary particles have lifetimes which exhibit a clear-cut periodicity in powers of α . Due to the conjugate nature of lifetimes and mass widths, this experimental α -dependence also applies to the resonance widths of these threshold-states. Since it applies to the “mass widths,” then it should affect the “masses” themselves. Specifically, there must logically be some kind of “ α -structure” in the masses that accounts for the observed width and lifetime α -quantizations, since masses are “primary” physics quantities, whereas widths and lifetimes merely express the stability of the masses, and

hence are in a sense “secondary” physics quantities. Following this line of reasoning to its conclusion, we arrive at the result that the electron and its coupling constant α generate not only the photon, but also the spectrum of leptons and hadrons. Thus the domain of the fine structure constant $\alpha = e^2/\hbar c$ seems phenomenologically to be larger than currently believed. This viewpoint has in fact long been suggested by the experimental data, and was pointed out in the early literature [19].

1.4 The Dichotomy of Leptons and Hadrons: Interactive Charges and Passive Masses

In the Standard Model classification of particles [10], the *massive* elementary particles are divided into two major classifications — leptons and hadrons. The massive particles we consider here start with the electron at 0.5 MeV and extend up through the Upsilon mesons in the 9 to 11 GeV (billion electron volt) range. The superheavy W and Z vector mesons at 80 and 91 GeV are placed in a different SM category, that of gauge bosons. (The mass systematics developed in the present work shows an interesting mass extrapolation between the W and Z vector bosons and the much lighter ϕ , J/ψ and Υ vector mesons, as we discuss in Sec. 3.19.)

There are only three massive *leptons* — the electron (e^-), muon (μ^-), and tau (τ^-) — and three massive antileptons — the positron (e^+), antimuon (μ^+), and antitau (τ^+). The leptons have negative integer electric charges, and the antileptons have positive integer electric charges. Leptons are weakly-interacting particles, whose interactions with other particles and with each other are purely electromagnetic. By way of contrast, there are about 200 massive *hadrons* (plus associated antihadrons), which are divided into two general categories according to spin — the integral-spin (boson) *mesons* and the half-integral-spin (fermion) *baryons*. Hadrons occur in a variety of integer charge states. They are strongly-interacting particles (Greek *hadros* = “stout”), and their interactions with other hadrons are dominated by the short-range “strong force,” which is much greater than the long-range electromagnetic force.

A striking difference between leptons and hadrons is their measured sizes. Since lepton interactions such as Møller (e^-, e^-) and Bhabha (e^-, e^+) scattering [20] are known to be accurately electromagnetic, measurements of their “sizes” are in actuality determinations of their “charge sizes,” and these measurements show *no size at all*. To be more precise, the exper-

iments show no measurable sizes down to the experimental limit of about 10^{-17} cm. By way of contrast, comparable measurements on hadrons show roughly the Compton sizes, $R_{\text{hadron}} \sim \hbar/mc$ that we expect to find on the basis of their masses m and their spins and magnetic moments [21]. This brings up an interesting question:

*Is the electron a point particle, or is just the **charge** on the electron point-like?*

This is in fact a crucial question to answer, and the prevailing opinion seems to be that the electron is truly point-like. However, if this is so, then both the spin angular momentum and the magnetic moment of the electron are entities that we cannot explain on the basis of any known physics involving mechanics and electrodynamics, and the electron is consigned to the unreachable depths of string theory. The thrust in the present studies is to establish the fact that particle lifetimes and masses are in fact in accord with known physical principles, and we would expect the electron to fall into this same category. This matter is discussed further in Chapter 4. But even if the *mass* of the electron is of finite extent, as seems required for its spin angular momentum, the *charge* on the electron *must* effectively be a point. Otherwise the accurate agreement between theory and experiment for Møller and Bhabha scattering would not be obtained.

Another striking difference between leptons and hadrons involves their charge distributions. All of these particles have integer electric charges in units of the fundamental electron charge e . But the leptons have no discernable substructure to the charge, whereas the hadrons have a quark charge substructure that is indicated both by the isotopic spins (charge states) of the particles, and also by the results of deep inelastic scattering experiments. It appears as if the integer charge e on a hadron is broken into $\frac{1}{3}e$ and $\frac{2}{3}e$ fractions, and these fractions are located on individual *quarks* within the hadron. Unfortunately this is still a theoretical concept, because very determined experimental efforts have failed to produce any evidence of fractional charge states in the decay products of hadrons that have been shattered in high-energy collisions (or from any other decay modes). By assigning fractional charge states to quarks it is possible to reproduce the observed integer charge states of the families of hadrons so accurately that the reality (or at least the usefulness) of this procedure seems firmly established. The assumption of fractional quark charges has led to many predictive successes, and is a core feature of the Standard Model [10].

In addition to the isotopic spin successes, confirmatory evidence for a quark substructure in hadrons comes from deep inelastic scattering experiments [22]. At the low energies of early experiments, electron scattering off protons followed the results expected from simple Coulomb scattering. However, at the much higher energies later reached at the Stanford Linear Accelerator Center (SLAC), a different kind of scattering was observed. Bjorken and Feynman pointed out that this scattering could be explained by assuming that the charge inside the proton is not smoothly distributed, which would lead to Coulomb-like scattering, but instead is in the form of localized charge centers on each of three quarks inside the nucleon, with fractional charges on the quarks. At very high energies the quarks scatter essentially independently and in a quasi-free manner (they are “asymptotically free”). These experimental results furnished the first direct evidence of an actual quark structure inside the proton. As a result of the fact that the electric charge is spread out over three quarks, measurements of the overall electromagnetic size of the proton (and the neutron) give a finite result [21].

Leptons have integer point-like electric charges that show no evidence of substructure. Hadrons have integer charges that are separated into $1/3$ and $2/3$ fractions on quarks within the hadron. Leptons interact only weakly (electromagnetically). Hadrons interact both weakly (electromagnetically) and strongly (with the short-ranged strong force). Thus it seems logical to conclude that the origin of the strong force resides in the fractionization of the electric charge.

The measured size of the charge e on the electron is less than 10^{-17} cm. The electrostatic self-energy of this charge, if evaluated in conventional electromagnetic theory, is much larger than the observed mass of the electron. Since this cannot be correct, conventional theory does not apply, and we must assume that the point electron charge e does not have a conventional self-energy. Spectroscopic calculations [23] in fact indicate that the charge self-energy must be assumed to be zero. The magnetic energy that corresponds to the magnetic moment of the electron can be estimated, and it is roughly 0.1% of the total electron energy [24]. Thus the electron must possess a “mechanical” mass that represents the remaining 99.9% of the energy of the electron [25]. Since the electron interactions are purely electromagnetic, it is apparent that this “mechanical” mass is essentially non-interacting. If we were to strip the charge off an electron, we would be left with a non-interacting spin- $1/2$ mechanical mass that carries the electron lepton quantum number; this is the property we now ascribe to the

electron neutrino (albeit at a vastly smaller mass value). If we carry this result over and apply it to hadrons, so that the hadron strong force comes from its (fractured) charge and associated gluon field, then the residual “mechanical” mass of the hadron is plausibly also more-or-less non-interacting. Thus we have the result that particle charges are interactive, and particle mechanical masses are passive.

This discussion of leptons and hadrons is aimed at establishing one main point: the distinctions we can draw about the differences between leptons and hadrons center mainly on their different charge structures. We actually know very little about their mass structures, apart from the fact that lepton masses show no evidence of a substructure whereas hadron masses indicate a division into quarks. This raises two questions.

- (a) *Do leptons and/or hadronic quarks have fundamental mass “building blocks”?*
- (b) *If so, do the lepton and hadron “building blocks” have anything in common?*

The assumption that is implicitly made in the Standard Model is that the answers to both of these questions would be *no*. Thus the SM treatments of the lepton and hadron masses are not related, and the proton-to-electron mass ratio, which straddles this divide, remains unexplained. Even within the separate domains of leptons and hadrons the masses remain a mystery. The muon-to-electron mass ratio of 206.8 and the tau-to-muon mass ratio of 16.8 do not lend themselves to ready explanations, and the pion mass, which is the lowest hadron state, is unaccounted-for. One way out of this impasse is to examine elementary particle *lifetimes*, which represent the stability of the mass structures, and see if the lifetimes convey any information about the mass structures themselves. As we will demonstrate in our analyses in Chapters 2 and 3, this turns out to be a fruitful line of endeavor.

There is one final question we can ask about the dichotomy of leptons and hadrons, which follows from the questions raised above.

If leptons and hadrons are in fact two completely separate types of particles, then do they share a common ground state, or do they each have a separate ground state?

In the case of leptons, it makes sense to denote the electron and positron as the particle and antiparticle ground states, respectively. They are the

lowest-mass states, and they are stable. Electron–positron pairs can be excited into muon and tau particle–antiparticle pairs, which eventually decay back down to electrons and positrons. But hadrons have the proton and antiproton as the only stable states, and these are not the lowest-mass hadron states. This leads to our final question here.

Where is the hadron ground state?

The spinless pion is the lowest-mass hadron, but it is unstable, and it has balanced particle–antiparticle symmetry. Several other mesons have lower masses than the proton, but they are also unstable. If we do not have an obvious candidate for the hadron “ground state,” which we can define as *the state that is used to generate all of the other hadron states*, then we are free to look anywhere, including at very high masses. This makes the ultramassive Higgs particle a viable candidate. But the Higgs would be an unprecedented ground state in the annals of physics. This ground-state situation is examined in detail in Chapter 3.

1.5 Experiment, Phenomenology, Theory: The Three Steps to Success

The three fundamental steps in the development of theories in physics, and in science in general, are to see *what* is there (experiment), to determine *how* it is arranged (phenomenology), and finally to explain *why* it happens that way (theory). As historians of science point out, our theories are in general forced on us step by step by the experimental data and accompanying phenomenology, and they are not usually theories that we would plausibly arrive at in the absence of data.

One danger in constructing theories is that, given the experimental database, we accidentally overlook some of the phenomenology, and thus end up developing a theory which cannot account for that phenomenology. The resulting theory may not be comprehensive enough to answer questions that we really need to know about. Historically, the need for an elementary particle theory dates back to the discovery of the electron in 1897 and the delineation of the geometry of the proton in 1911. These two particles carry precisely the same magnitude of electric charge with opposite polarities, but they have radically different masses. The main challenge to particle theorists for the past century has been to account for this surprising electron-to-proton mass ratio. Unfortunately, the Standard Model,

the currently-prevailing elementary particle theory [10], puts electrons and protons into two apparently-unrelated categories — leptons and hadrons, as we discussed in Sec. 1.4, and thus provides no clues as to the nature of this mass ratio.

In searching for a way out of this dilemma, we need to re-examine the elementary particle data base to see if there are any regularities that have not been recognized, and which can be used to confront the particle theorists. One area that leaps to the forefront is the collection of elementary particle lifetimes. The long lifetimes of the low-mass threshold states, right from the early days of accelerator particle physics, have exhibited a lifetime separation into distinct groups, with the groups separated by fairly accurate factors of $1/137$, where $\alpha = e^2/\hbar c \cong 1/137$ is the fine structure constant. These results were first published in 1970 [26], using an early Particle Data Group compilation [27]. The α -spaced lifetime groups contained 13 measured lifetimes (the Σ^0 , η and η' lifetimes, for example, had not yet been measured), and the calculations for the lifetime table were performed on a Marchand mechanical desk calculator. Over the past 35 years an additional 23 long-lived particle lifetimes have been measured and documented (Sec. 2.11), and have accurately fitted into this α -spaced systematics, as described in Chapter 2. However, the Review of Particle Physics [10], which summarizes not only the elementary particle data, but also the *physics* that presumably applies to these data, makes no mention of this phenomenology, which has not been incorporated into the Standard Model. The significance of the α -dependence of the lifetimes lies not so much in the lifetimes themselves, although that is certainly of interest, but rather in the fact that since they represent the stability of the masses, the observed α -dependence logically carries over to the masses themselves.

In Sec. 1.6 we describe the elementary particle database. Then in Sec. 1.7 we discuss the nature of the linkage between lifetimes and masses. In Sec. 1.8 we display the numerics of the proton-to-electron mass ratio. After these preliminaries, we move on to Chapters 2 and 3, the pivotal chapters in the book, where we use a series of graphical data displays to lay out the overall systematics of elementary particle lifetimes and masses. Since this is unfamiliar territory, we need to have a good idea of the forest before we spend a lot of time examining the trees. This global overview was already provided in the Introduction, where we used color to bring out some of the systematics more clearly. What we discovered in the Introduction was that threshold-state particle lifetimes reveal an experimental

α -structure which is independent of theory, and this α -structure carries over to the masses in a way that we could not untangle without the empirical guidance of a comprehensive collection of experimental data [10]. Interesting, the global views of the α -spaced lifetime groups in the Introduction also demonstrate how clearly these lifetime groupings reflect their Standard Model s , c , b quark flavor structures, which were deduced quite independently of the lifetime data displays.

Benoit Mandelbrot, known for his pioneering book *The Fractal Geometry of Nature* [28], utilized the power of the computer to produce graphic displays of fractal patterns that would otherwise be impossible to display. Many self-similar fractal patterns closely resemble chaotic shapes that we observe in nature, and they are an aid in developing mathematical descriptions for these shapes. As Mandelbrot commented [29]:

Graphics is wonderful for matching models with reality.

Later in the book he applied this comment to a specific experimental situation [30]:

Feynman 1979 reports that fractal trees made it possible for him to visualize and model the ‘jets’ that arise when particles collide head on at very high energy.

In the present studies we are not dealing with fractal shapes, but we are using graphics as the best way to describe unfamiliar experimental results. These pictorial representations display the *phenomenology* in a way that we hope will help to link *experiment* and *theory* together in a comprehensive triad of results.

1.6 The Review of Particle Physics (RPP) Elementary Particle Data Base

Research in elementary particle physics has led to the identification of roughly 200 different massive particle states (particles with measured rest masses) that can be produced in energetic particle collisions. Most of these states are very short-lived, persisting for no more than a couple of orders of magnitude longer than the collision transit time. However, a few are long-lived metastable threshold states that signal the onset of quark excitations within the particles. In the past half century, after the advent of

high-energy accelerators, the proliferation of these particle states, together with the expensive equipment required to produce and analyze them, has led to worldwide collaborations in experimental work and in the compilation of the experimental data and its systematic analysis. The Particle Data Group (PDG) summarizes these results biennially in the Review of Particle Physics (RPP).

The precursor to the RPP was a data compilation put together annually at UC Berkeley by Arthur Rosenfeld, and known informally among particle physicists as the “Rosenfeld Tables.” Figure 1.6.1 shows Table VI of a Rosenfeld compilation dated September 1962. This single-page table, which was thumb-tacked on the present author’s bulletin board for a long time, summarizes the data on the strongly interacting particles (hadrons). It lists 20 particle states (some with multiple isotopic spins — different charge states) that are still recognized today. This is roughly 10% of the present members of the hadron “elementary particle zoo.” The mass values shown in Table VI are quite accurate, but the lifetime (mass width) measurements are more rudimentary, with all resonance widths less than 15 MeV listed either as upper limits or as effectively zero. The great improvements in the measurements of particle lifetimes or resonance widths represent one of the most striking achievements of particle physics in the past half century, and they reflect the increasing innovation and sophistication in the design of massive particle detectors.

The database for the present work is RPP2006 [10]. Of the approximately 200 elementary particles and resonances listed in RPP2006, 157 have reasonably-well-established experimental lifetime values. These constitute the lifetime database for the present studies, which is listed in Appendix A. In this database, a few charge multiplets with very similar and rather short lifetimes are characterized as single states. Also, some very-broad-width S-state resonances with poorly-determined widths have been excluded, as well as a few narrow-width states whose lifetimes are presently listed in RPP2006 only as upper limits. The corresponding mass database for these same particles is listed in Appendix B.

In addition to the elementary particle data on spectroscopic properties and interactions, RPP2006 also provides summaries of the relationships between these particles, and it describes how they fit in with the general formalism of the Standard Model.

Table VI.
TENTATIVE DATA ON STRONGLY INTERACTING PARTICLES Sept. 1962 A. H. Rosenfeld

Particle	Established Quantum No. $I(J^{PC})$	Possible Assignment		Mass (MeV)	$\Gamma^{[2]}$ (MeV)	Mass ² (BeV) ²	Dominant Decays			
		Quantum No. $I(J^{PC})$	Regge ^[1] Trajectory				Mode	%	$\Omega^{[4]}$ (MeV)	p or Γ_{max} (MeV/c)
Vacuum ?	-	$0(2^{++})$	$+\omega_a$	-	-	-	{even no. pions} [5] RK(K ₁ K ₁ etc.)			
η	$0(0^{-+})$		$+\omega_\beta$	548	< 10	.30	neutrals [3] $\pi^+\pi^-\pi^0$ 75 - - 25±4 136 175			
ω	$0(1^{-+})$		$-\omega_\gamma$	782	< 15	.62	$\pi^+\pi^-\pi^0$ [3, 5] $\pi^+\pi^-\pi^0$ 86 368 326 $\pi^+\pi^-\pi^0$ 14±4 647 379			
π $\begin{cases} \pi^0 \\ \pi^\pm \end{cases}$	$1(0^{-+})$		$-\pi_\beta$	π^0 135 π^\pm 140	0 0	.018 .02	$\pi^+\pi^-\pi^0$ [6] $\pi^+\pi^-\pi^0$ 100 135 67 $\pi^+\pi^-\pi^0$ 58 34 30			
ρ	$1(1^{-+})$		$+\pi_\gamma$	750	100	.56	$\pi\pi$ [3] (p-wave) 100 471 348			
ζ (?)	$1(?)$	$1(0^{+-})$	$-\pi_a$	560	< 15	.31	$\pi\pi$? 290 245			
K $\begin{cases} K^0 \\ K^\pm \end{cases}$	$\frac{1}{2}(0^{-})$		κ_β	K^0 498 K^\pm 494	0 0	.24	$K\pi$ $\rightarrow \pi^+\pi^-$ [6] 2/3K ₁ 219 206 $K\pi$ $\rightarrow \mu\nu$ 58 38 236			
$K_{1/2}^*$ (888)	$\frac{1}{2}(1^{-})$		κ_γ	888	50	.78	$K\pi$ (p-wave) 100 251(K ⁰ π^-) 283			
$K_{1/2}^*$ (730)?	$\frac{1}{2}(?)$?	?	730	< 20	.53	$K\pi$? 101(K ⁻ π^0) 161			
N $\begin{cases} n \\ p \end{cases}$	$\frac{1}{2}(1/2^{+})$		N_a	n 940 p 938	0	.88	$e^- \bar{\nu}_e p$ [6] 100 - .78 1.2			
$N_{1/2}^*$ (1688) = "900 MeV πp "	$\frac{1}{2}(5/2^{+})$		N_a	1688	100	2.84	$N\pi$ (f-wave) ? 610 572			
$N_{1/2}^*$ (1512) = "600 MeV πp "	$\frac{1}{2}(3/2^{-})$		N_γ	1512	150	2.28	$N\pi$ (d-wave) ? 434(π^+p) 450			
$N_{3/2}^*$ (1238) = "isobar"	$\frac{3}{2}(3/2^{+})$		Δ_δ	1238	100	1.53	$N\pi$ (p-wave) 100 160(π^-p) 233			
$N_{3/2}^*$ (1920)	$\frac{3}{2}(J > 5/2)$	$\frac{3}{2}(3/2^{+})$	Δ_δ	1920	200	3.69	$N\pi$ + other ? 842(π^+p) 722			
Λ	$0(1/2^{+})$		Λ_a	1115	0	1.24	π^-p [6] 67 38 100			
Y_0^* (1815)	$0(J > 5/2)$	$0(5/2^{+})$	Λ_a	1815	120	3.29	(KN+other) ? 383(K ⁻ π^0) 541			
Y_0^* (1405)	$0(?)$	$0(1/2^{-})$	Λ_β	1405	$5d^{[5]}$	1.97	$\Sigma\pi$ $\Lambda 2\pi$ {100} 69($\Sigma^-\pi^+$) 144 10($\Lambda\pi^+\pi^-$) 69			
Y_0^* (1520)	$0(3/2^{-})$		Λ_γ	1520	15	2.31	$\Sigma\pi$ (d-wave) 60 194($\Sigma^0\pi^0$) 267 $\bar{K}N$ (d-wave) 30 88(K ⁻ π^0) 244 $\Lambda 2\pi$ 10 125($\Lambda\pi^+\pi^-$) 253			
Σ $\begin{cases} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{cases}$	$1(1/2^{+})$		Σ_a	1189 1191 1196	0 0 0	1.42 1.42 1.42	$n\pi^+$ [6] 50 110 185 $\Lambda\gamma$ 100 76 74 $n\pi^-$ 100 117 192			
Y_1^* (1385)	$1(J > 3/2)$	$1(3/2^{+})$	Σ_δ	1385	50	1.92	$\Lambda\pi$ $\Sigma\pi$ 98 135($\Lambda\pi^0$) 210 2±2 49($\Sigma^-\pi^+$) 119			
Y_1^* (1685)?	$1(?)$?	?	1685?	?	2.857	$(\Lambda\pi+others)$? 435 459			
Ξ $\begin{cases} \Xi^0 \\ \Xi^- \end{cases}$	$\frac{1}{2}(?)$	$\frac{1}{2}(1/2^{+})$	Ξ_a	1311 1321	0	1.72	$\Lambda\pi^0$ [6] - 61 131 $\Lambda\pi^-$ - 66 138			
Ξ^* (1530)	$\frac{1}{2}(?)$?	?	1530	< 7	2.34	$\Xi\pi$ 100 74($\Xi^-\pi^0$) 148			

Fig. 1.6.1 The "Rosenfeld Tables," the predecessor to the Review of Particle Physics. Shown here is Table VI of a data compilation by Arthur Rosenfeld of UC Berkeley in September 1962, which is labeled as "tentative." It summarizes the experimentally identified hadron resonances in a single page. The modern RPP2006 [10] labels for these states, in the order listed in Table VI (below the vacuum state) are: η , ω , π , ρ , (out), K , K^* (892), (out), N , $N(1680)F_{15}$, $N(1520)D_{13}$, $\Delta(1232)P_{33}$, $\Delta(1950)F_{37}$, Λ , $\Lambda(1820)F_{05}$, $\Lambda(1405)S_{01}$, $\Lambda(1520)D_{03}$, Σ , $\Sigma(1385)P_{13}$, $\Sigma(1670)D_{13}$, Ξ , $\Xi(1530)P_{13}$. The mass values shown in Table VI agree well with the modern values. However, the lifetime resonance widths in Table VI that are narrower than 15 MeV are either listed as upper limits or are assigned "zero" widths.

1.7 The Linkage Between Particle Lifetimes/Widths (Stability) and Particle Masses (Structure)

A direct method for calculating elementary particle masses has thus eluded physicists, as we have described above. However, there is an indirect way of approaching elementary particle masses which has not been exploited in devising theories of these particles. This involves making use of the following two facts.

- (1) *Lifetimes and mass widths are conjugate quantum mechanical variables which have a reciprocal relationship to one another via the Heisenberg uncertainty principle.*
- (2) *Mass widths, which represent the stability in the mass structure of the particle, are plausibly linked to the mass structure itself, since the explanations for observable patterns in particle lifetimes must reside in “reciprocal patterns” in particle mass structures.*

In the case of atomic physics, the use of conjugate coordinates was instrumental in unraveling the secrets of the atom. One of the major challenges in physics during the *first half* of the twentieth century was to understand the structure of the atom. A key to this understanding was the discovery of the conjugate relationship between *coordinates* and *momenta*, a discovery which has become a cornerstone of quantum mechanics, and is embodied in Heisenberg’s equations. This led to a comprehensive quantum mechanical model of the atom. A major challenge during the *last half* of the twentieth century was to understand the structure of the elementary particle. A key to this understanding, which has yet to be fully implemented, is the conjugate relationship between *lifetimes* and *mass widths*, with a subsequent extension to include the *masses*.

Quantum mechanics is central to both of these endeavors, and its use of conjugate coordinates is important in each. Historically, the effort to understand atomic structure was a crucial factor in the development of quantum mechanics, and it led to the discovery of the role played by noncommuting observables. The first decisive step in unraveling atomic structure was the demonstration by Rutherford that the positive charge of the atom is concentrated at the center. The next step was the development of the Bohr orbitals, with electrons placed in planetary orbits around the positively-charged nucleus. These orbits were quantized in units of angular momentum, so that velocities combined with spatial positions (to give angular momenta) became the variables of interest. The subsequent discovery of

the de Broglie wavelength of the electron explained the orbital angular momentum quantization in terms of the coordinate wavelength quantization, which led directly to the creation of wave mechanics. Heisenberg clarified the noncommuting nature of conjugate coordinate (x) and momentum (p) variables (operators) in studies that led to the formulation of the Uncertainty Principle, $\Delta x \cdot \Delta p \geq \hbar$. Thus the knowledge obtained initially from the *angular momentum* spectra in the atom was used to obtain important information about the conjugate *position coordinates* that describe the geometry of the atom.

It is interesting to note that Planck's constant h , which was initially introduced in the quantization of black body radiation, subsequently appeared in the quantization of atomic orbitals and in the quantification of the uncertainty that arises from the use of conjugate variables. The constant h also appears in the fine structure constant $\alpha = e^2/\hbar c$. Perhaps most strikingly, h occurs as a direct factor in the macroscopic force that operates in the Casimir effect [31].

In atomic physics the conjugate nature of *coordinate* and *momentum* representations is utilized. Relevant conjugate quantities in elementary particle physics are *mass widths* and *lifetimes* (energy uncertainty and time uncertainty), which are also represented by noncommuting variables. Specifically, the uncertainty in the lifetime of a particle (Δt), which is expressed in the form of its *mean life* $\tau \equiv \Delta t$ [10], is related to the uncertainty in the mass of the particle (Δm), which is conventionally expressed as the full width Γ [10] of the measured resonant state, and these are quantitatively tied together by the Heisenberg uncertainty principle

$$\Delta t \cdot \Delta m c^2 \geq \hbar, \quad (1.7.1)$$

where the numerical factor in front of \hbar depends on the choice of variables (e.g., full width [10] or half width at half maximum for Gaussian distributions), and where the equality that represents the lower limit of the uncertainty in Eq. (1.7.1) is used to define the conjugate transformation of variables. In the present paper we use the appellations *mean life* and *lifetime* interchangeably. Experimentally, the lifetimes τ of the long-lived particles are measured directly, but the lifetimes of the short-lived particles, which do not have measurable flight paths, have to be deduced from the resonance (full) widths Δm , using the equality in Eq. (1.7.1), which has the form

$$\tau \cdot \Gamma = \hbar. \quad (1.7.2)$$

This result is used [10] to transform from mass resonance widths to lifetimes. It thus seems logical that we can invert this conjugate mass width and time relationship and, by linking the mass widths to the masses, use lifetimes as a tool to investigate particle masses. Specifically, if there is a discernable periodicity $f(\tau \equiv \Delta t)$ in the particle lifetime structure, then the explanation for this periodicity must involve a (reciprocal) periodicity $g(\Delta m)$ in the corresponding mass width structure, and plausibly also in the mass structure. Mathematically, we have $f(\tau) \cdot g(\Delta m) = \hbar$ as the limiting equality in Eq. (1.7.1), so that any periodicity f in τ calls for a reciprocal conjugate periodicity $g \propto \hbar/f$ in Δm , and possibly also in m . After making this hypothesis, we can test it by studying the systematics of the particle mass structure and ascertaining whether a periodicity $g(m)$ actually exists.

Lifetime structures are in some respects easier to measure than mass structures, since elementary particles appear to have a (quark) mass substructure that is not amenable to direct observation (single quarks are not observed in collision or decay events). Thus to investigate the “periodicity” g in the quark mass structure, we must determine the observable periodicity f in the global lifetime structure, and then use $f(\tau) \cdot g(\Delta m) = \hbar$ to deduce the mass quantization $g(\Delta m)$ and look for $g(m)$. In particular, if $f = f(\alpha)$, then we expect to find $g = g(\alpha^{-1})$. In Chapter 2 we demonstrate in a series of graphical data plots that an α -quantized periodicity $f(\tau) = f(\alpha)$ is observed in the lifetimes of the long-lived threshold-state elementary particles, and in Chapter 3 we show that a corresponding periodicity $g(m) = g(\alpha^{-1})$ is in evidence in the masses of these same elementary particles. This α -dependence draws lepton and hadron masses together in one unified picture that reveals important contributions from each of these particle families.

1.8 The Numerical Challenge of the Proton-to-Electron Mass Ratio

The field of elementary particle physics was launched with the discovery of the electron by J. J. Thomson in 1897, and with the identification of the point-like nature of the proton by E. Rutherford in 1911. These two particles have electric charges that are identical in magnitude but of opposite signs. This result is in agreement with the general properties of Maxwellian electromagnetism, which features charge-symmetric behavior for positive and negative electric charges. Thus the electron and proton emerged as the

logical negative-charge and positive-charge entities for Maxwellian electrodynamics. However, the masses of the electron and proton turned out to be radically different: the proton is 1836 times as massive as the electron. This result was not anticipated in classical electromagnetic theory, and it posed a problem for elementary particle physicists:

Explain the 1/1836 electron-to-proton mass ratio.

This problem has never been solved. The field of elementary particles sustained a slow steady growth during the first half of the 20th century, and then an explosive growth during the second half, fueled by the development of high energy accelerators for producing these particles. Starting with just the electron and proton, there are now roughly 200 identified elementary particle states, and comprehensive theories have been devised to account for very detailed properties of their interactions and decay modes. However, the challenge of explaining the electron-to-proton mass ratio has never been met. Almost a century later it still remains a mystery. This suggests that something is lacking in our theoretical program. Harald Fritzsch, in his book *Elementary Particles* [13], summarizes this situation very succinctly:

We know the Standard Model does not address a number of decisive problems, most urgently the question for the origin of particle masses. The most telling of these, as far as the structure of matter is concerned, are the electron mass of 0.511 MeV and the proton mass of 938 MeV. [32]

When studying an unfamiliar area in physics, it is helpful to obtain an overall picture of where you are headed, and why it is useful for you to even go there. In order to provide motivation for a phenomenological examination of elementary particle lifetimes and masses, we set ourselves the goal of obtaining a reasonable estimate of the electron-to-proton mass ratio directly from the observed patterns in the experimental data, without recourse to specific elementary particle models, and without employing adjustable parameters. In order to yield a meaningful result, this procedure should also yield information about the masses of the particles in the neighborhood of the electron and proton, which it does in a comprehensive and somewhat surprising manner.

Well before any elementary particles had been identified, Maxwell electromagnetic theory was developed to account for the observed electric and magnetic fields, and to serve as a framework for charged particle interac-

tions. The Maxwell equations feature the negative and positive electric charges that had long ago been identified by Benjamin Franklin, and they use these charges in a completely symmetric manner. When the negatively-charged electron was discovered by Thomson in 1897, it was naturally identified with the negative charges in the Maxwell fields. The only question was whether the electron just carries the negative electric charge, or whether the charge itself *is* the whole electron. This question still persists to some extent at the present time, fueled by the fact that the electron seemingly has no measurable size. The concept of a point electron rules out much of what we know about Compton-sized particles, and it opens the door to speculations that are not amenable to direct experiment, and thus not bounded by experiment.

The discovery of localized protons at the center of the atom provided a carrier for the positive charge that appears in Maxwell's equations. The absolute values of the charges on the proton and electron are equal to within experimental error:

$$\begin{aligned} \text{Proton} &: +4.80320441 \times 10^{-10} \text{ esu}, \\ \text{Electron} &: -4.80320441 \times 10^{-10} \text{ esu}. \end{aligned}$$

The most accurate measurement of this charge ratio is actually between protons and antiprotons, where we have [10]

$$\text{Charge ratio: } \bar{p}/p = 0.9999999991 \pm 0.0000000009.$$

However, this symmetry between the proton and electron absolute electric charges does not carry over to the masses. The proton and electron masses are [10]

$$\begin{aligned} \text{Proton} &: 938.272029 \text{ MeV}, \\ \text{Electron} &: 0.510998918 \text{ MeV}. \end{aligned}$$

Thus their mass ratio is

$$\text{Mass ratio: } p/e = 1836.1527.$$

A challenge to physicists is to be able to deduce this number, or something close to it, from the systematics of the elementary particle data. Unfortunately, our current elementary particle theories [10] place protons and electrons into two unrelated categories of particles — hadrons and leptons, respectively — and hence offer no way of calculating their mass ratio. In lieu of a theory, particle physicists have come up with a handy mnemonic:

$$6\pi^5 = 1836.1181.$$

This illustrates a fact that was noted by Feynman in commenting on attempts to “explain” the numerical value 137 that appears in the coupling constant α :

*Every once in a while, someone notices that a certain combination of pi's and e's (the base of the natural logarithms), and 2's and 5's produces the mysterious coupling constant, but it is a fact not fully appreciated by people who play with arithmetic that you would be surprised how **many** numbers you can make out of pi's and e's and so on. [16]*

In the present studies we attempt to move beyond this mnemonic. A major accomplishment of elementary particle physicists during the past century has been the accumulation of a comprehensive body of highly accurate experimental data. One would think, or at least hope, that these data would tell their own story, and in fact they seem to do just that, as we demonstrate in the chapters ahead. The reader may judge the results of our search.