Chapter 1

Real Option Based Equity Valuation Models: An Empirical Analysis

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This paper provides empirical evidence in support of real option based equity valuation models that relate share price to accounting earnings and book value. Our empirical results are generally consistent with the predictions of several models, all of which are based on real options theory. However, we find that the basic model, which includes components related to put and call options, fits the data more efficiently and parsimoniously than do models modified for the level of firm efficiency (i.e., accounting profitability measured as the return on common equity book value). We also find that the fit of the basic model and the derived coefficients vary with firm efficiency as measured by accounting profitability. We also test for the impact of capital structure on equity valuation and find some evidence for the relevance of debt for loss firms (i.e., low efficiency firms) and growth firms. We find anomalous results for loss firms, consistent with previous research, and provide an explanation for them. Our research contributes to the valuation literature by studying the empirical validity of a general real option based model and thus extends previous empirical studies that were based more or less on an options approach. Our contribution is significant in that there have been many theoretical papers on real options, but few empirical studies of the predictions of these models.

Keywords: Real options; valuation; equity valuation; clean surplus.

1. Introduction

The valuation of equity securities is of fundamental importance in accounting and finance, and has been the subject of theoretical and empirical study over many years. There have been a considerable number of papers that have examined the relationship between the market value of equity and various accounting numbers reported in the financial statements. For example, Landsman (1986), Barth (1991), and Shevlin (1991) examine the role of balance sheet measures in equity valuation. Other studies such as Ball and Brown (1968), Barth, Beaver, and Landsman (1992), Collins and Kothari (1989), and Collins, Maydew, and Weiss (1997) examine an alternative income statement approach to equity valuation based on earnings. In a more complete framework, Ohlson (1995) and Feltham and Ohlson (1995) combine the two approaches and show that, under a certain reasonable set of assumptions, a firm's value can be modeled as a function of both the book value of equity and the level of earnings.

Although considerable progress has been made, there remain some fundamental questions that have still not been completely resolved. These include (1) the real option fraction of equity value to expand or contract the scale of operations; (2) financial implications of measures such as dividend payout, capital structure, and capital expenditure (Rees, 1997); and (3) to a lesser extent, the negative price-earnings anomaly observed for loss firms in the current paper and in Jan and Ou (1995), Burgstahler and Dichev (1997), and Kothari and Zimmerman (1995). Collins, Pincus, and Xie (1999) provide a reasonable explanation and suggest adding the book value of equity to the simple earnings model.

In his seminal paper, Myers (1977) conceptualized the idea of viewing a firm's growth opportunities as real options. He provides the theoretical framework to value a firm as income generating assets-in-place plus the value of growth opportunities arising from future discretionary investments. Although there has been extensive research on theoretical real option models and applications since Myers' (1977) article, there have been only a few empirical studies in the real options literature. More specifically, Paddock, Siegel, and Smith (1988), Bailey (1991), Quigg (1993), and Moel and Tufano (2002) compare the net present value (NPV) with real options models. McConnell and Muscarella (1985) investigate market reaction to positive NPV projects, and Belkaoui (2000) uses a general regression model with corporate reputation, multinationality, size, profitability, leverage, and systematic risk as variables to estimate growth opportunities. In addition, Burgstahler and Dichev (1997) include an adaptation option (i.e., the value of the option to convert a firm's resources to more productive alternatives) in an equity valuation model and Berger, Ofek, and Swary (1996) consider an abandonment option.

The basic purpose of this paper is to extend our knowledge of the relationship of accounting numbers, specifically book value and earnings, to the market value of equity using real option based valuation models. Following Zhang (2000), this empirical study tests the predictions of a number of valuation models derived by supplementing standard valuation models with real options theory. We run regressions of the various valuation models for our full sample and several sub-samples stratified based on profitability levels. We show that the predictions of the various models hold generally for our sample but that Zhang's (2000) basic valuation model seems superior to his modified models. Because of apparent empirical anomalies in some situations, we have examined the assumptions and predictions of the real option based models more closely. In addition, we consider the financial implications of capital structure by modifying the operational version of Zhang's (2000) basic model to test the value relevance of debt for our sample stratified on profitability. Finally, we show that, although a sub-sample of firms' (i.e., loss firms) coefficients have a negative sign, our empirical findings are not anomalous but rather quite consistent with the more detailed expectations from the model.

The current paper makes several contributions to the valuation literature. First, it provides empirical evidence to support theoretical results based on real options theory. Prior empirical findings are based on the earnings capitalization model and the more complete, but intuitive, valuation models that include earnings and book value as explanatory variables. This contribution is significant since there have been few empirical studies that have tested predictions rooted in real options theory. Second, we incorporate capital structure considerations that are ignored in Zhang's (2000) basic model and discuss the value relevance of debt in equity valuation for cross-sectional stratified sub-samples. Third, despite apparent anomalies, our empirical results are consistent with those of previous research and provide further evidence on the variability of coefficients in valuation models and suggest that Collins, Pincus, and Xie's (1999) warning on the interpretation of the coefficient of earnings be extended. This coefficient appears to depend not just on whether earnings are positive or negative but also on the profitability of the firm. Finally, the current research contributes to the valuation literature by illustrating the convergence of two different theoretical valuation approaches that explain the value relevance of earning and book value.

The rest of this paper is organized as follows: Section 2 provides the theoretical background of the basic valuation model, derives predictions, and discusses prior research. Section 3 discusses real option based equity valuation models for analyzing the cross-sectional behavior of the properties of the valuation function. It also develops predictions for the signs of the coefficients of the operational regression models. Section 4 provides details of the samples used in the study. Section 5 describes statistical analyses and discusses the major findings and results. Section 6 discusses and provides empirical evidence on the relevance of capital structure in equity valuation. Section 7 resolves the anomalous relationship between earnings and equity valuation. Finally, Section 8 provides conclusions and discusses the limitations of this study.

2. Background and Prior Research

Recent research has shown that the basic earnings capitalization model to estimate a firm's value is not satisfactory because it yields anomalous empirical results for companies with negative earnings (loss firms) (Hayn, 1995; Jan and Ou, 1995). Burgstahler and Dichev (1997) developed and empirically tested an option-style valuation model, and showed that both book value and earnings contribute to explaining equity value. They also show that the relationship is convex in both earnings and book value, and that the relative explanatory power of earnings and book value vary with accounting profitability. Collins, Pincus, and Xie (1999) supplement the basic earnings capitalization model with book value in order to address the loss firm anomaly. With their revised model, they show that the anomalous results disappear and that the earnings coefficient of the basic capitalization model is biased upward (downward) for profit (loss) firms when the beginning of the period book value of net assets is not included in the empirical tests (Collins, Pincus, and Xie, 1999).

Burgstahler and Dichev (1997) introduce the notion that market value comprises two elements of value. These are adaptation value, which exemplifies the potential use of existing resources for alternative purposes, and recursion value, which assumes the continued use of existing resources for current purposes. They model market value as a function of a fixed adaptation value plus a call option on the recursion value. Collins, Pincus, and Xie (1999) specifically address the anomalous negative coefficient of earnings in the basic earnings capitalization model and motivate the addition of book value by appealing to Ohlson's (1995) valuation model and the clean surplus relation. Their model suggests that earnings be supplemented by book value because it serves as a proxy for expected future normal earnings and abandonment value, i.e. a put option.

More recently, Zhang (2000) developed a formal theoretical model for equity valuation in a real options framework. Zhang (2000) makes quite reasonable assumptions and shows that the Ohlson (1995) and Feltham and Ohlson (1995, 1996) valuation approach can be modified to incorporate the options to either abandon or grow a business, i.e. to include both put and call options.

His model shows that the basic earnings capitalization model may be complemented by an abandonment (put) option or a growth (call) option, depending on the efficiency of the business. In addition, Zhang (2000) shows how the basic model can be modified for different levels of efficiency and derives several specific additional models for relating equity value to accounting numbers.

In Zhang's (2000) basic model, the equity value depends on anticipated future actions, specifically abandonment or discretionary additional investments. The decision as to which action to take depends on a firm's efficiency and growth potential. In conservative accounting settings, equity value is shown to be a function of two accounting variables (earnings and book value) and measurement bias. If accounting measures are assumed to be free from bias, the model produces the following valuation function:

$$V_t = B_t P_d(q) + kX_t + GC_e(q), \tag{A}$$

where V_t is the market value of equity at time t; B_t, the book value of equity at time t; X_t, the accounting earnings for the current period ending at time t; G, the amount invested in new opportunities because of growth potential; k, the capitalization factor $= 1/(R-1)$; R, 1 plus the risk-free rate of interest; q, the operational definition of firm efficiency level;

$$P_d(q) = \frac{1}{R(R-1)} \int_{\underline{\nu}}^{q_d^* - q_t} [q_d^* - q_t - \nu_{t+1}] f(\nu_{t+1}) - \mathrm{d}\nu_{t+1}$$

is the value of the put option set, that is, to discontinue operations; and

$$C_e(q) = \frac{1}{R(R-1)} \int_{q_e^* - q_t}^{\overline{\nu}} [\nu_{t+1} + q_t - q_e^*] f(\nu_{t+1}) - \mathrm{d}\nu_{t+1}$$

is the value of the call option set, that is, to expand operations.

In the mathematical expressions for a firm's call and put options, q_t and q_{t+1} are the internal rates of return of cash investment at time t and $t+1$, which represents a firm's operating efficiency; q_d^* is the lower bound of operating efficiency that will trigger discontinuation of the firm's operation (i.e., $q_{t+1} < q_d^*$); q_e^* is the upper bound of operating efficiency that will trigger an expansion of the firm's operation (i.e., $q_{t+1} > q_e^*$); ν_{t+1} is a zero mean noise term pertaining to operational efficiency that cannot be predicted; $f(\nu_{t+1})$ is the probability density function of operational efficiency defined over the region $\nu_{t+1} \in [\underline{\nu}, \overline{\nu}]$

with a zero mean noise term given by $\int_{\underline{\nu}}^{\overline{\nu}} \nu f(\nu) - d\nu = 0$. The variable q_t is analogous to the underlying asset in option terminology and has a time series behavior $q_{t+1} = q_t + \nu_{t+1}$, i.e. it follows a random walk.

In order to investigate the cross-sectional differences in the behavior of the valuation function, Zhang (2000) considers three types of firms that differ in efficiency and/or growth potential:

(i) *Low efficiency firms* have a high probability of discontinuing and a low probability of growth. For these firms, the put option $P_d(.)$ is valuable, and so $BP_d(q)$ accounts for a significant portion of the total value, whereas the call option $C_e(.)$ is negligible.
(ii) *Steady state firms* have a sufficiently high efficiency that the probability of discontinuing is low, but there is no growth potential. They are expected to stay on the current course of operations, i.e., current earnings will continue in perpetuity, and both $P_d(.)$ and $C_e(.)$ are negligible.
(iii) *High efficiency firms* have a high growth potential. For these firms, the call option $C_e(.)$ is valuable, and so the value due to current earnings is supplemented by $GC_e(.)$, which makes up a significant portion of the total value, whereas $P_d(.)$ is negligible.

3. Real Option Based Equity Valuation Models

3.1. *Model 1*

We transform Zhang's (2000) basic valuation Model A for any firm i, which assumes that accounting measures are free of bias, to the following regression model (Model 1):

$$V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}, \tag{1}$$

where V_{it}, B_{it}, and X_{it} are the same as defined before, $\alpha_1 = GC_e(q)$, $\beta_1 = P_d(q)$, $\gamma_1 = 1/r_f$, and ε_{it} is the error term.

Since $G \geq 0$, $r_f = R - 1 > 0$ and the put and call options cannot take negative values, we have the following predictions for the sign of the parameters:

- The coefficient related to the call option will be zero or positive for all firms ($\alpha_1 \geq 0$).
- The coefficient related to the put option will be positive for all firms ($\beta_1 > 0$).

- The coefficient related to the current earnings will be positive and equal for all firms ($\gamma_1 > 0 = \text{constant}$).

The form of Model 1 suggests that the contribution of various terms of the valuation function will vary with the efficiency of operations, as proxied by profitability, q, of the firm. Analysis of the dependence of the coefficients of Model 1 on profitability based on the properties summarized in Appendix A shows that the following relations hold:

- $\partial\alpha_1/\partial q = GC_e'(q)$,
- $\partial\beta_1/\partial q = P_d'(q)$,
- $\partial\gamma_1/\partial q = 0$.

From the aforementioned terms, we make the following predictions of the relative magnitude of the coefficients in Model 1 at different levels of efficiency[1] (q):

- The coefficient of the call option term (α_1) will be largest for growth firms and smallest for low efficiency firms.
- The coefficient of the put option term (β_1) will be largest for low efficiency firms and smallest for growth firms.
- The coefficient of the current earnings term (γ_1) will be the same for all firms.

3.2. *Models 2–4*

Zhang (2000) suggests that it would be appropriate to examine separately sub-samples that are homogeneous with respect to firm efficiency. Zhang (2000) uses X_t/B_{t-1}, i.e., the firm's current period profitability (return on equity) as measured by accounting numbers, as a proxy for q_t and makes a number of other assumptions to derive from Model 1 plausible regression models for firms with different levels of efficiency. To derive his models, Zhang (2000) assumes that the book value is the same at the beginning and end of the year ($B_t = B_{t-1}$). Although this is a reasonable assumption in most cases, it may cause some empirical problems if the earnings represent a large percentage of the book value, which could happen if the book value is small.

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[1] The predictions made here are consistent with predictions 1, 3, 5–7 in Zhang (2000).

For low efficiency firms, the following regression model (Model 2) is derived:

$$V_{it} = \alpha_2 + \beta_2 B_{it} + \gamma_2 X_{it} + \delta_2 \left(\frac{X_{it}^2}{B_{it}} \right) + \varepsilon_{it}, \tag{2}$$

where[2]

$$\alpha_2 = \left[\frac{1}{1 + r_f} + C_c'(r_f) - C_c''(r_f) r_f \right] (\Delta u);$$

$$\beta_2 = \left[\frac{1 - c_d}{1 + r_f} + C_c(r_f) - C_c'(r_f) r_f + \frac{C_c''(r_f) r_f^2}{2} \right] \left(1 + \frac{u}{B}\right);$$

$$\gamma_2 = \left[\frac{1}{1 + r_f} + C_c'(r_f) - C_c''(r_f) r_f \right];$$

$$\delta_2 = \frac{C_c''(r_f)}{2};$$

$0 < c_d < 1$ is the cost of discontinuation; u, the accounting bias between the accounting and economic values; $\Delta u = u_t - u_{t-1}$, the bias between accounting and economic earnings; and, ε_{it} is the error term.

Since it is assumed that accounting measures are free of bias, $\Delta u = 0$ and $u = 0$. Note that

$$C_c(q) = \frac{1}{R(R-1)} \int_{q_d^* - q_t}^{\overline{\nu}} [\nu_{t+1} + q_t - q_d^*] f(\nu_{t+1}) - \mathrm{d}\nu_{t+1}$$

is the call option to continue operations for low efficiency firms obtained using the put-call parity condition.

Based on the properties of the valuation function developed by Zhang (2000), which are summarized in Appendix A, plus the fact that options cannot take a negative value, the following signs are predicted for the regression parameters of Model 2[3]:

- $\alpha_2 = 0$.
- The sign of β_2 cannot be determined ($\beta_2 > 0$ or $\beta_2 < 0$ depending on the magnitudes of $C_c(.)$, $C_c'(.)$, and $C_c''(.)$).

[2]Note that the expressions for the coefficients here involve $C_c(r_f)$ rather than $C_e(q)$ as in Model 1.

[3]Note that these predictions are based on the assumption that accounting numbers are unbiased. If there is a bias, the major change is that α_2 and α_3 may be < 0 or > 0. See Zhang (2000, pp. 281–282) where $u > 0$ but Δu may be < 0 or > 0.

- The sign of γ_2 cannot be determined ($\gamma_2 > 0$ or $\gamma_2 < 0$ depending on the magnitudes of $C_c'(.)$ and $C_c''(.)$).
- $\delta_2 > 0$.

For steady state firms, the following regression model (Model 3) is derived:

$$V_{it} = \alpha_3 + \gamma_3 X_{it} + \varepsilon_{it}, \tag{3}$$

where $\alpha_3 = \Delta u/r_f$, $\gamma_3 = 1/r_f$, and ε_{it} is the error term.

The properties referred to the preceding terms yield the following predictions for the signs of the regression parameters of Model 3:

- $\alpha_3 = 0$,
- $\gamma_3 > 0$.

For high efficiency firms, the following regression model (Model 4) is derived:

$$V_{it} = \alpha_4 + \gamma_4 X_{it} + \theta_4 \left(\frac{X_{it}}{B_{it}}\right) + \lambda_4 \left(\frac{X_{it}}{B_{it}}\right)^2 + \varepsilon_{it}, \tag{4}$$

where

$$\alpha_4 = G\left[C_e(r_f) - C_e'(r_f)r_f + \frac{C_e''(r_f)r_f^2}{2}\right] + \frac{\Delta u}{r_f},$$

$$\gamma_4 = \frac{1}{r_f}, \quad \theta_4 = G\left[C_e'(r_f) - C_e''(r_f)\right], \quad \lambda_4 = \frac{G}{2}C_e''(r_f),$$

and $\varepsilon_{it}$is the error term.

The properties referred to the aforementioned terms plus the fact that $G \geq 0$ yield the following predictions for the signs of the regression parameters for Model 4:

- The sign of α_4 cannot be determined ($\alpha_4 > 0$ or $\alpha_4 < 0$ depending on the magnitude of $C_e(.)$, $C_e'(.)$, and $C_e''(.)$).
- $\gamma_4 > 0$.
- The sign of θ_4 cannot be determined ($\theta_4 > 0$ or $\theta_4 < 0$ depending on the magnitude of $C_e'(.)$ and $C_e''(.)$).
- and $\lambda_4 \geq 0$.

In Table 1, we summarize the sign predictions for the four models.

Table 1. Predictions for all models used in the study.

Model type	α_i	β_i B	γ_i X	δ_i X^2/B	θ_i X/B	λ_i $(X/B)^2$
Model 1						
Low efficiency firms	$\cong 0$	> 0	> 0			
Steady state firms	> 0	≥ 0	> 0			
Growth firms	> 0	> 0	> 0			
Model 2						
Low efficiency firms	$\cong 0$	> 0 or < 0	> 0 or < 0	> 0		
Model 3						
Steady state firms	$\cong 0$		> 0			
Model 4						
Growth firms	> 0 or < 0		> 0		> 0 or < 0	> 0

Notes:
Model 1: $V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}$.
Model 2: $V_{it} = \alpha_2 + \beta_2 B_{it} + \gamma_2 X_{it} + \delta_2(X_{it}^2/B_{it}) + \varepsilon_{it}$.
Model 3: $V_{it} = \alpha_3 + \gamma_3 X_{it} + \varepsilon_{it}$.
Model 4: $V_{it} = \alpha_4 + \gamma_4 X_{it} + \theta_4(X_{it}/B_{it}) + \lambda_4(X_{it}/B_{it})^2 + \varepsilon_{it}$.

4. Sample and Variables

The sample is drawn from the COMPUSTAT database of active US firms over the period 1988–2002 inclusive (i.e., 15 years of annual data for 10,357 companies representing 155,355 firm-year observations included in the active COMPUSTAT US file). The following data items are collected for each firm from the COMPUSTAT database:

(1) The stock price at the fiscal year end adjusted for stock splits and stock dividends occurring during the fiscal year (COMPUSTAT item number A199; mnemonic PRCCF). This variable is coded "V" in the current study.
(2) The total common equity interest in the company, including common stock outstanding adjusted for treasury stocks, capital surplus, and retained earnings (COMPUSTAT item number A60; mnemonic CEQ).
(3) The number of common shares outstanding at the year end, excluding treasury stocks and scrip (COMPUSTAT item number A25; mnemonic CSHO).
(4) The income before extraordinary items and discontinued operations available for common equity net of preferred stock dividend requirements and

before adding savings due to common stock equivalents (COMPUSTAT item number A237; mnemonic IBCOM).

(5) Total Debt (TD) = [total long-term debt, plus current liabilities (COMPUSTAT mnemonic DT; no item number exists for this variable)] + preferred stocks (COMPUSTAT item number A130; mnemonic PSTK) + minority interest (COMPUSTAT item number A38; mnemonic MIB).

Data items 2–5 are used to calculate the following variables (all on a per share basis):

- B_{it} = CEQ/CSHO is the book value per share for firm i at time t.
- X_{it} = IBCOM/CSHO is the earnings per share before extraordinary items and before discontinued operations for firm i at time t.
- X_{it}/B_{it-1} is the accounting return on the beginning book value, which is used as a proxy for profitability q.
- TDBV_{it} = (CEQ + TD)/CSHO is the total of the book value of common equity plus debt per share for firm i at time t.
- TD_{it} = TD/CSHO is the total debt per share for firm i at time t.

After excluding firm-years that have missing data and negative book values plus outliers [boundaries for inclusion are ± 3 standard deviations from the median for the variables earnings (X_{it}) and profitability (X_{it}/B_{it-1})], the final sample consists of 64,796 firm-year observations, of which 20,100 (31.0%) have negative earnings.[4] To test for capital structure considerations (i.e., relevance of debt), the sample size is further reduced to 63,026 firm-years due to missing values for debt and debt-related variables.

5. Analyses and Results

5.1. *Descriptive statistics*

The mean, median, and standard errors for the variables used in Models 1 through 4 are given in Table 2. The median and mean values are noticeably different for all variables, and the standard errors are relatively small compared to the mean values for all variables.

[4]This proportion is somewhat higher than the value of 22.8% in the final sample of Collins, Pincus, and Xie (1999). This is presumably because our sample includes firm-years from years around the turn of the millennium when there was generally poor economic performance.

Table 2. Descriptive statistics — full sample.

Variable/statistic	Mean	Median	Standard error
V	14.36	9.50	0.06
B	7.60	5.07	0.03
X	0.23	0.35	0.01
X/B	−0.12	0.08	0.00
X^2/B	0.90	0.10	0.03
$(X/B)^2$	0.68	0.02	0.02

Table 3. Correlation coefficients — full sample.

Item	V	B	X	X/B	X^2/B	$(X/B)^2$
V	1					
B	0.63*	1				
X	0.10*	0.10*	1			
X/B	0.14*	0.15*	0.37*	1		
X^2/B	0.08	0.04	−0.66*	−0.37*	1	
$(X/B)^2$	−0.06	−0.10*	−0.20*	−0.76*	0.38*	1

*The correlation coefficient is significant at the 0.01 level.

The correlation coefficients among the six variables for the full sample used in this study are given in Table 3. For the full sample, there is a significant correlation at the 1% level between share price and book value per share (63%) as expected, but the correlations between share price and earnings per share (10%) and book value per share and earnings per share (10%) respectively are relatively low. The correlation coefficients for positive and negative earnings firms in Tables 4 and 5, respectively, provide more insight into the association of stock price and book value with earnings per share. For profitable firms, the results in Table 4 show that share price is positively and significantly correlated with book value per share (64%) and with earnings per share (62%); i.e., each of the two independent variables displays the same level of correlation with stock price. The results for low efficiency (loss firms), in Table 5, show a different correlation pattern; stock price is positive and significantly correlated with book value per share (53%), but negative and significantly correlated with earnings per share (−37%). Also, the results of Table 5 indicate a negative significant correlation between book value per share and earnings per share (−45%). Tables 3–5 also report significant correlations among other independent variables X_{it}/B_{it}, X_{it}^2/B_{it}, and $(X_{it}/B_{it})^2$, which are expected to be fairly highly correlated. The correlation coefficients between variables with the same

Table 4. Correlation coefficients — positive earnings firms.

Item	V	B	X	X/B	X^2/B	$(X/B)^2$
V	1					
B	0.64*	1				
X	0.62*	0.67*	1			
X/B	0.03	−0.12*	0.15*	1		
X^2/B	0.18*	0.12*	0.53*	0.47*	1	
$(X/B)^2$	−0.01	−0.04	0.03	0.85*	0.34*	1

*The correlation coefficient is significant at the 0.01 level.

Table 5. Correlation coefficients — negative earnings firms.

Item	V	B	X	X/B	X^2/B	$(X/B)^2$
V	1					
B	0.53*	1				
X	−0.37*	−0.45*	1			
X/B	0.06	0.19*	0.23*	1		
X^2/B	0.17*	0.11*	−0.75*	−0.39*	1	
$(X/B)^2$	−0.04	−0.12*	−0.15*	0.91*	0.37*	1

*The correlation coefficient is significant at the 0.01 level.

power of earnings (e.g., X_{it} with X_{it}/B_{it} and X_{it}^2/B_{it} with $(X_{it}/B_{it})^2$ are positive, while those between variables having even and odd powers of earnings (e.g., X_{it} with X_{it}^2/B_{it} and X_{it}/B_{it} with $(X_{it}/B_{it})^2$) are negative, which suggests the need for further investigation.

Of utmost importance in these results is the correlation between share price and earnings per share, which is positive for positive earnings firms, but negative for negative earnings firms, while both are far removed from those for the full sample in Table 3. These correlation results, thus, indicate that examining only the correlation coefficients for the full sample masks the differences that show clearly in the positive and negative earnings sub-samples. This suggests some fundamental difference between the positive and negative earnings firms that may impact the results for regression Models 1 through 4.

5.2. *Diagnostic statistics*

In this section, we assess our sample data to explore for the presence of serial/autocorrelation and heteroscedasticity problems. Our analysis of full and sub-samples reported in the results section reveals that our data are free from autocorrelation as the Durbin–Watson d statistic is always close to 2; the lowest value of d is 1.86; the highest value of d is 2.01[5] (Gujarati, 1992). In addition, we tested the data used in this study using Park's test (see Gujarati, 1992) and found no evidence of heteroscedasticity. The results of our tests indicate an R^2 of 0.00 and a t-value of 0.00 for the variables (X_{it}) and (B_{it}), an indication of homoscedasticity.

5.3. *Results from Model 1*

Firms in the full sample were ranked according to accounting profitability (X_{it}/B_{it-1}), the proxy for firm efficiency (q), and separated into three approximately equal-sized sub-samples. The low efficiency sub-sample consisted of 20,100 firm-years with negative earnings (loss firms), whereas the steady state and high growth sub-samples each consisted of 22,348 firm-years reporting positive earnings. The full sample plus the three sub-samples were fitted separately to the regression equation for Model 1, with the results as reported in Table 6.

Table 6. Estimated regression coefficients (t-statistics are listed below the coefficients) for Model 1.

Profitability (q): full sample and sub-samples	α	β B_i	γ X_i	Model F-value	Prob. of F-value	Model adjusted R^2
Full Sample:	5.57	1.15	0.20	20,957	0.00	39%
64,796 firm-years	84.10	202.27	12.15			
Low efficiency-loss firms: 20,100 firm-years	3.79	0.90	−0.58	4,332	0.00	30%
	34.62	68.10	−26.02			
Steady state:	3.62	0.75	4.45	13,392	0.00	55%
22,348 firm-years	37.65	68.25	33.89			
Growth: 22,348	7.34	1.08	2.52	9,207	0.00	45%
firm-years	61.37	54.69	26.56			

Notes: Model 1: $V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}$. All coefficients are significant at the 0.00 level.

[5] In general, based on Gujarati (1992), the presence or absence of positive or negative autocorrelation depends on the calculated d statistics. Positive or negative autocorrelation is said to be present if the value of d is close to zero or 4, respectively. As the value of the d statistic inches close to 2, the more likely it is that autocorrelation is not present.

The fit of Model 1 to the data for the full sample and the three sub-samples is quite good as shown by the reasonable R^2 values and the large F values, all significant at the 0.00 level. The fit is clearly poorest, although statistically significant, for the low efficiency sub-sample, that is, the negative earnings (loss) firms.

The coefficients for all four regressions in Table 6 are significant at the 0.00 level. In addition, the t-values of the intercept (α) for the three sub-samples are all statistically significant. For the full sample, all coefficients are significant and consistent with the predicted signs. For the three sub-samples, all coefficients are also significant and consistent with the predicted signs except for the coefficient of earnings (γ) for the low efficiency (loss) firms, which is negative and significant rather than positive as predicted. These regression results are consistent with the correlation coefficients that are given in Tables 3–5.

The coefficient for the intercept (α) for the growth firms (7.34) is larger than those for the low efficiency (3.79) and steady state firms (3.62), which are close to each other in value. This is consistent with the prediction that the call option is most valuable for the growth firms but not for the other firms. The coefficients of the book value (β) are close to 1 for the full sample and for the three sub-samples. The coefficient for the book value (β) is larger for the low efficiency firms than for the steady state firms, consistent with the expectation that the put option should be more important for low efficiency firms. However, it is unexpectedly large for growth firms.[6] Contrary to expectations, the coefficients of earnings (γ) are not the same for the three sub-samples. The results in Table 6 show that the earnings coefficients increase quite markedly from the low efficiency firms to the steady state firms and then decrease for the growth firms, rather than being the same for all sub-samples as predicted. The fact that the coefficient of earnings (γ) is larger for the steady state firms is consistent with the expectation that current earnings are more important for them than for growth firms.

At this stage, several points should be noted: First, it seems clear that analysis of the full (pooled) sample masks important differences among the firms.

[6]Although the magnitude is unexpected according to the predictions of the model, it may be rationalized as follows: First, there is no reason that a growth firm cannot have a put value. Second, it may be argued that a growth firm is perceived more favorably than low efficiency and steady state firms so that the put value of its assets exceed their accounting book value, whereas the put values of low efficiency and steady state firms are less than their accounting book values.

Stratification by profitability shows important differences that go beyond differences in earnings and so can be usefully incorporated in empirical analyses. Second, the effect of differences in profitability on the coefficient of the book value (β) is not completely consistent with the predictions of the basic options based valuation model. Third, the coefficient of earnings (γ) not only differs among sub-samples of firms but is significantly negative for the low efficiency firms (−0.58 with a t-value of −26.02, significance: 0.00), contrary to prediction.

5.4. *Results from Models 2–4*

The results of fitting the low efficiency, steady state and growth sub-samples described earlier separately to Zhang's (2000) modified Models 2–4, respectively, are presented in Table 7. The results in Table 7 show the adjusted R^2 values of 30% for low efficiency, 45% for steady state, and 38% for growth

Table 7. Estimated regression coefficients (t-statistics are listed below the coefficients) for Models 2–4.

Profitability (q) sub-samples	α intercept	β B_i	γ X_i	δ X^2/B	θ X_i/B_i	λ $(X_i/B_i)^2$
Low efficiency-loss firms (Model 2): 20,100 firm-years; adjusted R^2 = 30%; F-value = 2,892	3.80	0.89	−0.66	−0.03		
(0.00)	34.72	61.64	−18.16	−2.84		
Steady state (Model 3): 22,348 firm-years; adjusted R^2 = 45%; F-value = 18,309	5.99		11.65			
(0.00)	60.88		135.31			
Growth (Model 4): 22,348 firm-years; adjusted R^2 = 38%; F-value = 4,663	11.18		6.88		−9.01	1.10
(0.00)	66.72		118.24		−14.51	10.25

Notes
Model 2: $V_{it} = \alpha_2 + \beta_2 B_{it} + \gamma_2 X_{it} + \delta_2(X_{it}^2/B_{it}) + \varepsilon_{it}$.
Model 3: $V_{it} = \alpha_3 + \gamma_3 X_{it} + \varepsilon_{it}$.
Model 4: $V_{it} = \alpha_4 + \gamma_4 X_{it} + \theta_4(X_{it}/B_{it}) + \lambda_4(X_{it}/B_{it})^2 + \varepsilon_{it}$.

firms. A comparison of data in Tables 6 and 7 shows that R^2 is the same for low efficiency firms, and lower for the other two sub-samples.

The predictions for the coefficients from fitting Model 2 for the low efficiency (loss) firms are not very specific so that testing them extensively is not possible. Contrary to predictions, the intercept coefficient (α) is non-zero and the return coefficient (δ) is negative. The coefficient for the book value (β) for Model 2 is positive and significant (0.89, t-value: 61.64), consistent with the book value being a primary determinant of the value for loss firms. The coefficient of earnings (γ) is negative and significant (-0.66, t-value: -18.16), which seems surprising. Both of these observations are consistent with what was found using Model 1. The results of fitting Model 3 for the steady state firms are consistent with the prediction for the coefficient of earnings (γ), which is positive and significant (11.65, t-value: 135.31). This shows that earnings are very important in determining the share price for steady state firms as expected. But the intercept coefficient (α) is positive rather than zero as predicted.

The results of fitting Model 4 for the growth firms are also consistent with predictions in that the only two specifically predicted signs, for (γ) and (δ), are correct. The coefficient of earnings (γ) is positive and significant (6.88, t-value: 118.24) showing that earnings are very important in determining the market value of growth firms also. The magnitude and the significance of the intercept term (α) (11.18, t-value: 66.72) suggest that the call option is also very important in determining the market value of these firms, as expected. The profitability coefficient (θ) is negative and significant (-9.01, t-value: -14.51) and significantly contributes to valuation. Also, the square of the profitability coefficient (λ) is positive and significant (1.10, t-value: 10.25). Both of these results are consistent with predictions.

5.5. *Comparison of results from Model 1 to results from Models 2–4*

The values of the various coefficients and R^2 obtained from fitting the three sub-samples to Model 1 and separately to Models 2–4 are summarized in comparative format in Table 8. A comparison of the results from Models 1 and 2 for the low efficiency firms shows that Model 2 has the same explanatory power as Model 1 even though it has an additional explanatory variable. Further, the coefficients (α), (β) and (γ) in these two models are quite close. The negative value for the return coefficient (δ) is opposite to the predicted value, and its

Table 8. Summary comparison of estimated regression coefficients and R^2 for low efficiency, steady state, and high efficiency sub-samples using Models 1–4.

Profitability (q) sub-samples	Model	α	β	γ	δ	θ	λ	$R^2(\%)$
Low efficiency (loss firms)	1	3.79	0.90	−0.58	–	–	–	30
	2	3.80	0.89	−0.66	−0.03	–	–	30
Steady state	1	3.62	0.75	4.45	–	–	–	55
	3	5.99	–	11.65	–	–	–	45
Growth	1	7.34	1.08	2.52	–	–	–	45
	4	11.18	–	6.88	–	−9.01	1.10	38

magnitude is small, suggesting that it does not make a great contribution to valuation.

A comparison of the results of Models 1 and 3 for the steady state firms shows that Model 3 has a noticeably lower explanatory power than does Model 1. Although earnings makes a very important contribution to valuation in Model 3, the large changes in the intercept and earnings coefficients (α) and (γ) from those in Model 1 suggest that the former Model 3 is not properly specified. The behavior here parallels that observed by Collins, Pincus, and Xie (1999) in that book value appears to be a correlated omitted variable in Model 3, leading to an upward bias in the coefficient of earnings (γ).

A comparison of the results of Models 1 and 4 for the growth firms shows that Model 4 also has a noticeably lower explanatory power than does Model 1, even though it has two terms in place of the book value in Model 1. In addition, the coefficients (α) and (γ) differ between the two models, suggesting that Model 4 is also not well specified.

One reason for the poor performance of Models 2 and 4 may be the assumption that the book value is the same at the beginning and the end of the year ($B_t = B_{t-1}$) as was mentioned earlier. But both Models 3 and 4 do not appear to be well specified, i.e., the omitted variable problem leads to biased coefficients. In particular, the book value term, which is related to a put option, plays an important role for both steady state and growth firms and should not be omitted. This suggests that the absence of the variable "book value" is the major problem, not whether the book value is measured at the beginning or the end of the year in Models 2 and 4. The overall conclusion is that the basic valuation Model 1 captures the information relevant for valuation in a more efficient and parsimonious manner than do Models 2–4 and should be the basis for any further analysis.

5.6. *Further analysis of full sample*

A further analysis of the sample data was undertaken in order to investigate the apparently anomalous behavior of the low efficiency firms identified before. The sample was ranked from the lowest to the highest accounting profitability (X_t/B_{t-1}) and split into deciles. (Note that the first three deciles, i.e., those with the lowest profitability, include all the loss firms that gave the anomalous results identified earlier). This procedure was motivated by the predictions made earlier that the coefficients α and β should vary with profitability. The formation of deciles that are more homogeneous in profitability should fit the data more efficiently and parsimoniously.

The results of fitting Model 1 for deciles are presented in Table 9. The fit to Model 1 for all deciles is quite good as measured by the R^2 and F values, although they vary noticeably among deciles. Also, all coefficients are significant at the 0.00 level, except for the coefficient of earnings (X_{it}) for decile 2, which is not significantly different from zero.

As stated in Section 3, this study predicts that the intercept coefficient (α) (i.e., call option) should be positive and increase with profitability. The results in Table 9 show that α is positive in all deciles and generally increases as expected with profitability. The results in Table 9 also support this paper's prediction that the coefficient for the book value (β) is positive for all deciles and shows a general decrease with profitability although the actual results reveal that the trend is not completely clear or smooth. The results in Table 9 do not support our prediction that the coefficient of earnings (γ) is positive and is the same for all deciles. As the data in Table 9 indicate, the coefficient of earnings (γ) is positive as predicted only for the seven highest profitability deciles. Conversely, the coefficient (γ) is negative and significant for deciles 1 and 2, while not being significantly different from zero for decile 3. The results, however, show a generally increasing trend with profitability. These results are consistent with the negative sign found for the coefficient of earnings (γ) for the analysis of the low efficiency (i.e., loss) firms using Model 2 and with the correlation coefficients in Table 5.

The stratification implemented in Table 9 may lead to two problems: First, it may not create homogeneous strata due to the fact that the stratification is based

Table 9. Estimated regression coefficients (t-statistics are listed below the coefficients) for Model 1 — full sample split into deciles on profitability (q).

Profitability (q) deciles	Firm-years	α Intercept	βB_i	γX_i	Model adjusted R^2(%)	Model F-value
Full sample	64,796	5.57	1.15	0.20	39	20,957
		84.10	202.27	12.15		
1 (Lowest)	6,480	4.61	1.67	−0.11	33	1,627
		21.32	37.80	−2.89		
2	6,480	3.68	0.97	−0.21	27	1,182
		20.18	26.77	−2.29(0.02)		
3	6,480	3.31	0.86	−0.01	36	1,852
		20.87	53.90	−0.07(0.94)		
4	6,480	3.38	0.86	1.65	46	2,743
		19.68	48.03	3.07		
5	6,480	3.25	0.80	3.46	56	4,178
		19.10	22.43	6.37		
6	6,480	4.15	0.85	3.14	53	3,624
		21.35	14.53	5.26		
7	6,480	4.18	1.08	2.09	60	4,909
		22.64	15.94	3.84		
8	6,480	4.78	0.48	7.21	58	4,514
		24.72	6.58	14.81		
9	6,480	6.12	0.21	8.90	55	3,912
		28.76	2.57(0.01)	20.98		
10	6,476	10.08	1.56	0.48	31	1,455
		41.65	26.79	2.92		

Notes: Model 1: $V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}$. Coefficients are significant at the 0.00 level except as indicated in brackets.

on a sample split into 10 equally sized groups and does not, accordingly, result in a homogeneous profitability in, or a smooth change in profitability between, strata. Second, each of the 10 strata may not represent a homogeneous pool of firm-years. It is possible that the empirical results in Table 9 are influenced by a high level of intra-decile variability in profitability (q). As a result, an alternative approach to stratification of the full sample is also employed. Firms with profitability less than -1.00 and greater than $+1.00$ were put into separate sub-samples for further analysis. The firms with a profitability in the range -1.00 to $+1.00$ were divided into 10 sub-samples, each with a profitability range of 0.20. The results of fitting each of these sub-samples to Model 1 are reported in Table 10.[7]

The data in Table 10 indicate that the number of observations in each new stratum varies considerably among the strata, with the highest number in the stratum 0.00–0.20 (32,434 firm-years) and the lowest in the stratum 0.80–1.00 (274 firm-years). In addition, the empirical results in Table 10 reveal that Model 1 fits the data quite well as measured by R^2 and F values, although they also vary noticeably among strata. The coefficients (α) and (β) are positive and statistically significant for all sub-samples. They also show the expected variation with profitability, although the trends are once more not completely smooth. The earnings coefficient (γ) is seemingly erratic in behavior. For the sub-samples with a profitability above +0.40, it is insignificant. For the sub-samples with a negative profitability, the behavior is mixed. It is not significant in the range −0.40 to 0.00, positive and significant in the range −0.80 to −0.40, and negative and significant in the range below −0.80. The results for the negative profitability strata are consistent with the results in Table 9. The fact that the model produces poor results for large negative and positive profitabilities is not unreasonable as it is unlikely that Model 1, or any relatively simple model, would fit well over a wide range of profitability. Rather, it is reasonable to expect Model 1 to fit the data over a "reasonable" or "narrower" range of profitability. The empirical results in Table 10 indicate that Model 1 produces better results in the range between −0.20 and +0.20.

Based on the aforementioned remarks, we re-examine the Model 1 fit for a narrower profitability range (−0.20 to +0.20), which is actually quite a wide range of profitability (q) as it is unlikely that a firm would consistently have a profitability outside that range in the normal course of events. The observations in the profitability range −0.20 to +0.20 were separated into 10 sub-samples, each covering a profitability range of 0.04. The observations in each of these sub-samples were fitted to Model 1, yielding the results reported in Table 11. The number of observations varies a fair amount among the sub-samples, from a low of 1,214 firm-years (−0.20 to −0.16 profitability range) to a high of 7,734 firm-years (0.12–0.16 profitability range). The fit of the data is quite reasonable for all deciles as measured by the R^2 and F-values, and is much

[7]We refitted Model 1 for nine industry groups based on the first digit SIC code. The results of the analysis indicate some industry effect since few independent variables are positive and significant for some industry groups.

Table 10. Estimated regression coefficients (t-statistics are listed below the coefficients) for of Model 1: Full sample split on profitability (q) range −1.00 to +1.00.

Profitability (q) deciles	Firm-years	α Intercept	βB_i	γX_i	Model adjusted R^2(%)	Model F-value
Full sample	64,796	5.57	1.15	0.20	39	20,957
		84.10	202.27	12.15		
< −1.00	1,769	6.82	1.77	−0.50	32	415
		13.57	18.31	−6.70		
−1.00 to −0.80	791	3.80	1.78	−0.29	31	176
		5.97	13.69	−3.43		
−0.80 to −0.60	1,603	3.96	1.82	0.19	37	465
		11.51	20.55	2.99		
−0.60 to −0.40	2,558	3.38	2.44	1.11	44	997
		11.60	30.88	11.70		
−0.40 to −0.20	4,309	3.63	1.00	−0.18	26	749
		16.33	18.58	−1.59(0.11)		
−0.20 – 0.00	9,070	3.46	0.88	−0.03	34	2,360
		24.44	57.92	−0.30 (0.77)		
0.00 – 0.20	32,434	4.00	0.66	5.73	56	20,838
		48.85	69.92	63.44		
0.20 – 0.40	9,044	7.33	0.72	5.59	47	3,964
		37.66	11.87	19.70		
0.40 – 0.60	1,571	10.82	1.46	1.00	25	266
		20.25	6.14	1.50(0.13)		
0.60 – 0.80	571	12.08	0.55	1.60	24	93
		14.79	2.42	2.73(0.01)		
0.80 – 1.00	274	8.39	2.30	0.15	46	119
		7.58	7.16	0.25(0.81)		
>1.00	802	9.24	1.47	0.03	25	131
		18.24	10.50	0.14(0.89)		

Notes: Model 1: $V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}$. Coefficients are significant at the 0.00 level except as indicated in brackets.

better for the positive profitability sub-samples than for the negative profitability sub-samples. The coefficients (α) and (β) are all positive and significant, and in general terms, show, the expected variation with changing profitability. The earnings coefficient (γ) varies with profitability. It is positive and significant for a positive profitability, not significantly different from zero near zero profitability, negative for a somewhat negative profitability and positive for a more negative profitability.

Table 11. Estimated regression coefficients (t-statistics are listed below the coefficients) for of Model 1: Sample split on profitability (q) range −0.20 to +0.20.

Profitability (q) ranges	Firm-years	α Intercept	βB_i	γX_i	Model adjusted R^2(%)	Model F-value
−0.20 to −0.16	1,214	3.46	1.22	1.03	33	300
		8.58	10.21	2.01(0.04)		
−0.16 to −0.12	1,458	4.04	1.45	3.79	27	271
		10.86	12.42	5.74		
−0.12 to −0.08	1,678	2.84	1.04	0.41	37	497
		8.92	28.86	3.61		
−0.08 to −0.04	2,113	2.80	0.75	−2.84	39	665
		10.03	17.70	−5.87		
−0.04–0.00	2,607	3.83	0.77	−1.13	37	767
		14.78	33.05	−2.21(0.03)		
0.00–0.04	4,982	3.45	0.83	1.70	46	2,096
		17.99	39.15	2.40(0.02)		
0.04–0.08	6,389	3.13	0.84	2.86	56	3,990
		18.04	24.66	5.08		
0.08–0.12	7,732	4.05	0.81	3.56	53	4,431
		23.15	16.56	7.03		
0.12–0.16	7,734	4.14	0.85	4.19	60	5,912
		24.33	13.64	8.56		
0.16–0.20	5,597	5.22	0.36	7.79	58	3,855
		25.09	4.56	15.36		

Notes: Model 1: $V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}$. Coefficients are significant at the 0.00 level except as indicated in brackets.

It appears that the fit of Model 1 to the sample and the various sub-samples is quite good over the profitability range of −0.20 to +0.20 and the coefficients (α) and (β) are generally as expected. The negative coefficient of earnings (γ) is the only major source of inconsistency with predictions.

6. Financial Management Considerations[8]

Zhang's (2000) model is based on a set of assumptions similar to those of the Ohlson (1995) and Feltham and Ohlson (1995, 1996) valuation models.

[8]We thank two anonymous referees for pointing this out, which improved an earlier version of this paper.

These models rely on some form of the Miller and Modigliani (1961) discount dividend model which assumes that current earnings are an adequate characterization of future earnings and dividends and assumes capital structure irrelevancy (Modigliani and Miller, 1958). According to Rees (1997), there are several theoretical and empirical research studies (e.g., Ross, 1977; Leland and Pyle, 1977; Ashton, 1991; McConnell and Muscarella, 1985) that argue for financial management considerations such as dividend payout, debt levels, and capital expenditure in equity valuation. In this section, we relax the debt irrelevancy assumption in Zhang's (2000) basic model to examine the relevance of debt for cross-sectional samples.

Our approach is similar to the approach used by Rees (1997) where we modify (Model 1) the operational version of Zhang's (2000) basic theoretical model given in Model A.[9] In order to examine the value relevance of debt, we restate the book value of equity of a firm i as total capital (C_i) less total debt (D_i) given by

$$B_{it} = C_{it} - D_{it}.$$

Substituting the value of (C_i), we get

$$B_{it} = (B_{it} + D_{it}) - D_{it}.$$

We next substitute the value for (B_i) in Model 1 to obtain the following regression model with debt (Model 5):

$$V_{it} = \alpha_1^1 + \beta_1^1(B_{it} + D_{it}) + \beta_1^2 D_{it} + \gamma_1^1 X_{it} + \varepsilon_{it}. \quad (5)$$

When $\beta_1^2 = -\beta_1^1$, Model 5 reduces to Model 1. Therefore, in order to test for the relevant role of debt, we predict that $|\beta_1^1| = |\beta_1^2|$ and $\beta_1^1 > 0$ and $\beta_1^2 < 0$ if the amount of debt is irrelevant to the market value of equity.

6.1. *Results from Model 5*

The results of fitting regression equation for Model 5 to the full sample plus the three profitability sub-samples of low, steady state, and growth are given in Table 12. The coefficients for all four regressions are all significant at the 0.00 level. As predicted, for the full sample and the three sub-samples, the coefficients of debt are negative ($\beta_1^2 < 0$) and of total capital (debt plus the book value of equity) are positive ($\beta_1^1 > 0$). For the full sample of 63,026

[9]Note that we do not develop a theoretical model.

Table 12. Estimated regression coefficients (t-statistics are listed below the coefficients) for Model 5.

Profitability (q): full sample and sub-sample	α	$\beta^1 B_i + D_i$	$\beta^2 D_i$	γX_i	Model F-value	Prob. of F-value	Model adjusted R^2(%)
Full Sample:							
63,026	4.77	1.33	−1.327	−0.695	10,519	0.0	33
firm-years	54.60	165.33	−147.85	−34.64			
Low efficiency-loss							
firms: 18,530	1.97	1.212	−1.291	−1.49	2435	0.0	28
firm-years	10.34	50.05	−47.11	−43.60			
Steady state:							
22,248	3.34	0.803	−0.791	4.34	8525	0.0	54
firm-years	33.62	69.90	−64.26	32.97			
Growth:							
22,248	6.812	0.956	−0.966	3.567	6083	0.0	45
firm-years	53.29	50.21	−49.67	37.24			

Notes: Model 5: $V_{it} = \alpha_1^1 + \beta_1^1(B_{it} + D_{it}) + \beta_1^2 D_{it} + \gamma_1^1 X_{it} + \varepsilon_{it}$. All coefficients are significant at the 0.00 level.

firm-years, the two coefficients are equal ($|\beta_1^1| = |\beta_1^2| = 1.33$) in value but opposite in sign. We observe similar interpretation for steady state firms where $|\beta_1^1| \approx |\beta_1^2| = 0.803 \approx 0.791$. Therefore, for both the full sample and steady state firms, our results indicate that debt is irrelevant to firm value. However, the results indicate that debt plays some role in equity valuation for both low efficiency (loss firms) and growth firms. For both these sub-samples, the coefficient of debt is larger than the coefficient for the total capital (debt plus the book value of equity) $|\beta_1^2| > |\beta_1^1|$ (i.e., a low efficiency: $1.291 > 1.212$ and a high efficiency: $0.966 > 0.956$), an indication of debt relevance. However, judging by the absolute magnitude of the difference of β_1^1 and β_1^2, debt is more relevant for loss firms than for growth firms.[10] While the role of debt in equity valuation is mixed, the empirical evidence indicates that debt is more likely to play a role as a quality indicator in equity valuation at the extremes: loss firms and growth firms.

We performed a further analysis by fitting Model 5 to sub-samples based on deciles. The results are presented in Table 13. The coefficients for all regressions

[10] Rees (1997) looked only into the direction of the difference. He has not tested for a significant difference between regression coefficients.

Table 13. Estimated regression coefficients (t-statistics are listed below the coefficients) for Model 5: Full sample split into deciles on profitability (q).

Profitability (q) deciles	Firm-years	α Intercept	$\beta^1 B_i + D_i$	$\beta^2 D_i$	γX_i	Model adjusted R^2(%)	Model F-value
Full sample	63,026	4.768	1.33	−1.327	−0.695	33	10,519
		54.604	165.33	147.85	−34.64		
1 (Lowest)	6,303	2.078	2.182	−2.348	−1.096	31	930
		4.994	28.11	−27.963	−17.317		
2	6,303	1.803	1.138	−1.233	−1.675	26	729
		6.314	19.698	−20.282	−12.04		
3	6,303	2.966	0.993	−0.994	0.167	37	1,231
		16.148	53.958	−47.195	1.58(0.11)		
4	6,303	3.148	0.826	−0.768	3.012	44	1,632
		16.425	43.811	−34.74	7.04		
5	6,303	3.052	0.741	−0.712	4.313	57	2,790
		17.874	27.243	−25.053	11.062		
6	6,303	3.757	1.002	−1.006	2.465	53	2,381
		18.36	20.02	−19.958	4.927		
7	6,303	3.83	0.919	−0.937	4.405	62	3,410
		19.892	14.936	−15.228	9.095		
8	6,303	4.96	0.421	−0.438	8.287	59	3,044
		24.232	6.698	−6.936	20.748		
9	6,303	5.852	0.628	−0.639	7.534	51	2,147
		23.286	8.998	−9.131	21.579		
10	6,299	8.966	0.647	−0.663	2.857	30	896
		35.685	19.729	−19.054	22.351		

Note: Model 5: $V_{it} = \alpha_1^1 + \beta_1^1(B_{it} + D_{it}) + \beta_1^2 D_{it} + \gamma_1^1 X_{it} + \varepsilon_{it}$. Coefficients are significant at the 0.00 level except as indicated.

are significant at the 0.00 level. As previously observed, the coefficient for total capital (debt plus the book value of equity) is larger than the coefficient for debt in deciles 4 and 5, which are steady state firms indicating value irrelevancy. The coefficient of debt is larger than the coefficient of total capital (debt plus equity) for deciles 1–3, which pertain to loss firms, indicating that debt plays some role as a measure of risk in equity valuation. A similar result is observed for growth firms (deciles 6 through 10), which indicate its value relevance for high profitability firms. Therefore, we can conclude that, similar to previous empirical research (Ross, 1977; Leland and Pyle, 1977; Rees, 1997), debt is relevant with regard to equity valuation in a real option setting.

6.2. *Impact of bias in accounting measures*

We noted earlier that the major difficulty with the coefficients from fitting the three sub-samples to Model 1 reported in Table 6 is with the coefficient of earnings (γ), which is not constant at different profitability levels as predicted by the basic theory, but is, in fact, negative for the low efficiency firms and shows a generally increasing trend with profitability. This same behavior is shown for the results based on different stratification approaches in Tables 9–13. In this section, we provide an explanation for the variation of the coefficients.

Zhang's (2000) basic model shown in Equation (A) was derived under the assumption that accounting measures are unbiased.

$$V_t = B_t P_d(q) + kX_t + GC_e(q). \tag{A}$$

Zhang's more complete model based on economic measures (Zhang, 2000, pp. 278–279) includes the effects of accounting bias as shown in Equation (B):

$$V_t = \frac{1}{R-1}(X_t + \Delta u_t) + P_d\left(\frac{X_t + \Delta u_t}{B_{t-1} + u_{t-1}}\right)(B_t + u_t) + \\ + GC_e\left(\frac{X_t + \Delta u_t}{B_{t-1} + u_{t-1}}\right). \tag{B}$$

The specific biases are as follows:

- Bias in book value = Economic value − Accounting book value = u_t.
- Bias in earnings = $\Delta u_t = u_t - u_{t-1}$.

Zhang argues that the following relationships hold under the assumption that accounting is conservative:

- The bias in book value u_t is always positive ($u_t > 0$).
- The bias in earnings Δu_t has the following behavior:
  - o $\Delta u_t < 0$ following periods of investment decline, i.e., for low efficiency firm-years;
  - o $\Delta u_t \cong 0$ following periods of constant recent investment, i.e., for steady state firm-years;
  - o $\Delta u_t > 0$ following periods of investment expansion, i.e., for growth firm-years.

Applying these expectations to Equation (B) allows some inferences about the relative magnitudes of the coefficients. For the first term in Equation (B), there should be no effect for the steady state firms because $\Delta u_t \cong 0$, whereas there should be a decrease/increase for the low efficiency/growth firms

Table 14. Effects of accounting bias on regression coefficients.

Profitability (q) sub-samples	α [Call option]	β [Put option]	γ [Current earnings]
Low efficiency firms (loss firms)	Smaller	Larger	Smaller [possibly negative]
Steady state firms	No effect	No effect	No effect
Growth firms	Larger	Smaller	Larger

because $\Delta u_t < 0/\Delta u_t > 0$, respectively. This means that the coefficient γ in Model 1 should be smaller/larger for the low efficiency/growth firms relative to the steady state firms. For the second term in Equation (B), the effect of the bias in book value u_t will be to increase the term. There will be no effect on the argument of $P_d(.)$ for the steady state firms from Δu_t, but it will be decreased/increased for the low efficiency/growth firms leading to a larger/smaller value for $P_d(.)$, respectively. This means that the coefficient β in Model 1 should be larger/smaller for the low efficiency/growth firms relative to the steady state firms. The third term in Equation (B) will have the same effect on the argument of $G_e(.)$ from the bias in earnings Δu_t. But because this is the call option term, this means that the coefficient α in Equation (B) should be smaller/larger for the low efficiency/growth firms relative to the steady state firms. These predictions are summarized in Table 14.

A re-examination of Table 7 shows that the magnitude of the coefficient α is consistent with the predictions in Table 14 in that it is essentially the same for the low efficiency and steady state firms and much larger for the growth firms, reflecting the value of the call option. The coefficient β is consistent with the predictions of Table 14 for the low efficiency and steady state firms. Note that the negative value of the coefficient β for loss firms may result from the bias of accounting earnings for the low efficiency sub-sample. Similarly, the coefficient γ is consistent with the predictions of Table 14 for the low efficiency and steady state firms. But they are not as predicted for the growth firms. The behavior of the three coefficients over the three profitability sub-samples is largely but not entirely consistent with the expectations derived from accounting bias.

7. Loss Firms

An examination of the results in Tables 6, 7, 9–13 shows that the coefficient of earnings (γ) often takes a negative value for observations with a negative profitability, which results from firms with negative earnings, i.e., the loss firms. At first glance, this appears to conflict with the results of Collins, Pincus, and Xie (1999) who showed that inclusion of the book value in the simple earnings capitalization model removed the negative coefficient of earnings for loss firms. However, there is a fundamental difference between their model and Model 1 used in the current study, namely, that their model uses the book value at the beginning of the year, whereas Zhang's (2000) model uses the book value at the end of the year. We reconcile the two models in Appendix B and show that our results are consistent with their results for loss firms.

8. Conclusions and Limitations

The results from this study lead to a number of conclusions that have implications for empirical studies and raise issues that may be addressed in further theoretical analyses. Our results extend the finding of variability of the coefficients of earnings and book value in regression models reported by Burgstahler and Dichev (1997) and reinforce the observation by Collins, Pincus, and Xie (1999) about the downward and upward biases of the coefficient of earnings for negative and positive earnings firms, respectively, if the book value is omitted from the regression model.

Our results show that Zhang's (2000) formal model that supplements the basic capitalization model by put and call option components has an empirical validity for valuation studies. We further show that his basic model (Model 1) provides a more efficient and parsimonious explanation than his modified models (Models 2–4). The results support the expectations of value for a put option, current earnings, and a call option at different profitability levels. Our results also show that there is an effect of accounting bias on the empirical results. This leads to a variation of the values of the coefficients of the three terms in the model with profitability. There is a particular impact for loss firms in that the coefficient of earnings becomes negative.

It is clear from the results found here that an analysis of the full sample masks differences that show up in analyses of the sub-samples. It seems clear that stratification of samples on some basis, profitability here, is necessary in deriving regression coefficients and in the interpretation of empirical results.

In addition, our empirical results indicate that debt is relevant to equity valuation for the low efficiency (loss) firms and growth firms. Debt is not relevant to profitable steady state firms. As a result, one can conclude that the level of debt of loss and high growth firms plays a significant role in equity valuation of these firms.

The results in this study are limited by the fundamental assumption in deriving the basic model that accounting measures are unbiased. There is also a question of whether accounting profitability is the best measure of firm efficiency to stratify the sample. Obviously, a given value of accounting profitability could be obtained from low earnings and a low book value, high earnings and a high book value, or from some intermediate combination of earnings and book value. It may also be that some measure other than accounting profitability may be useful in stratification, although the use of accounting profitability is attractive in that the data are available on a regular and reliable basis. Finally, it should be noted that regression Model 1, derived from the basic valuation model, is linear in earnings and book value, which suggests that it should only be used over a limited range of profitability as observed.

There is obviously much scope for further research. Zhang's (2000) model has been developed using firm profitability as a proxy for firm efficiency and leads naturally to stratification of samples for empirical analysis. The use of accounting profitability for stratification has shown that this is an important consideration. However, other bases for sample stratification might be more useful and merit theoretical consideration. From our study, it is also clear that further theoretical study of the impact of the bias of accounting measures on firm valuation models would be useful. Finally, the development of models that go beyond linear terms in earnings and book value may be useful in resolving some of the apparently anomalous behavior that we observed. Furthermore, these models may also provide a better explanation of firm value over a wider range of the independent variable of earnings and book value and of firm efficiency as proxied here by accounting profitability.

Appendix A. Some Basic Properties of Options

Zhang (2000) showed that the following properties, where $(.)'$ and $(.)''$ indicate the first order and the second order partial derivatives of $C_e(.)$ and $P_d(.)$ with respect to efficiency q must hold:

$$P_d'(q) = -\frac{1}{R(R-1)}\text{Prob}(\nu_{t+1} \le q_d^* - q_t) < 0,$$

$$P_d''(q) = \frac{1}{R(R-1)} f(q_d^* - q_t) > 0,$$

$$C_e'(q) = \frac{1}{R(R-1)}[1 - \text{Prob}(\nu_{t+1} \leq q_e^* - q_t)] > 0,$$

$$C_e''(q) = \frac{1}{R(R-1)} f(q_e^* - q_t) > 0,$$

$$\frac{1}{R-1} + P_d'(q) > 0.$$

Appendix B. Reconciling the Apparent Negative Earnings Anomaly

A number of studies have shown that the basic earnings capitalization model is not satisfactory when earnings are negative because the coefficient of earnings observed empirically is negative in such cases. Collins, Pincus, and Xie (1999) demonstrated that the anomalous negative coefficient of earnings for loss firms disappears when the book value of net assets is included in the empirical tests, that is, it appears that the book value is a correlated omitted variable in the basic earnings capitalization model. Yet we observe a significantly negative coefficient of earnings for loss firms in our empirical results.

The regression model that was used by Collins, Pincus, and Xie (1999, p. 44) in their empirical analysis was derived from the Ohlson (1995) and Feltham and Ohlson (1995) models and is given in Equation (B.1)[11]:

$$P_t = \alpha + \beta' X_t + \gamma' \text{BV}_{t-1}, \tag{B.1}$$

where P_t is the cum-dividend stock price; X_t the current period earnings per share; and BV_{t-1} is the book value per share at the end of year $t-1$.

The general form of their equation is presented in Equation (B.2) (Collins, Pincus, and Xie, 1999, Equation (2), p. 39)

$$P_t + d_t = \delta_0 + \delta_1 X_t + \delta_2 y_{t-1} + \varepsilon_t, \tag{B.2}$$

where P_t is now the ex-dividend price; d_t, the dividends per share; $P_t + d_t$, the cum dividend price; y_{t-1}, the beginning year book value per share; and ε_t is the noise term.

The models in Collins, Pincus, and Xie, (1999) are expressed in terms of earnings for the year and *beginning* of the year book value, whereas

[11] Note that Collins, Pincus, and Xie (1999) have interchanged the notation for the coefficients β and γ in their model relative to the usage in our models. Therefore, we designate their coefficients with a to distinguish them from ours. Also Collins, Pincus, and Xie (1999) use the book value (BV) at time $t-1$ rather than time t as in our models.

Table 15. Estimated coefficients of Model 1 for loss firms derived from empirical results of Collins, Pincus, and Xie, (1999).

Collins *et al.*	Coefficient of earnings X_t $\delta_1[\beta']$	Coefficient of book value B_{t-1} $\delta_2[\gamma']$	Difference $\delta_1 - \delta_2$
Current Study		Coefficient of book value B_t β	Coefficient of earnings X_t γ
75	−0.02	0.29	−0.31
76	−0.09	0.15	−0.24
77	0.11	0.36	−0.25
78	0.35	0.34	0.01
79	1.1	0.47	0.63
80	−0.01	0.38	−0.27
81	0.18	0.38	−0.20
82	−0.81	0.24	−1.05
83	0.24	0.54	−0.30
84	0.22	0.54	−0.32
85	0.38	0.70	−0.32
86	0.38	0.76	−0.38
87	0.16	0.69	−0.53
88	−0.12	0.54	−0.66
89	0.20	0.49	−0.29
90	0.25	0.41	−0.16
91	0.23	0.56	−0.33
92	0.06	0.68	−0.62
Mean	0.16	0.47	−0.31

Notes
Collins *et al.*'s model: $P_t = \alpha + \beta' X_t + \gamma' BV_{t-1} + \varepsilon_t$.
Model 1: $V_{it} = \alpha_1 + \beta_1 B_{it} + \gamma_1 X_{it} + \varepsilon_{it}$, $\beta = \delta_2 = \gamma'$, $\gamma = \delta_1 - \delta_2 = \beta' - \gamma'$.

Zhang's (2000) models (Equations (1)–(4) in the paper) are expressed in terms of earnings for the year and *end* of the year book value. To reconcile these two variations of the valuation model, we use the clean surplus relationship given in Equation (B.3)

$$y_t = y_{t-1} + X_t - d_t. \tag{B.3}$$

Substituting the expression for y_{t-1} from Equation (B.3) in Equation (B.2), we obtain the following relationship of cum-dividend price $P_t + d_t$ to earnings and book value at time t:

$$P_t + d_t = \delta_0 + \delta_2 d_t + (\delta_1 - \delta_2)X_t + \delta_2 y_t + \varepsilon_t. \tag{B.4}$$

Equation (B.4) is the equivalent to regression Model 1, with

$$\begin{aligned} \alpha &= \delta_1 + \delta_2 d_t = \alpha, \\ \beta &= \delta_2 = \gamma', \\ \gamma &= \delta_1 - \delta_2 = \beta' - \gamma'. \end{aligned}$$

In Table 15, we reproduce the coefficient estimates for the earnings and the book value of loss firms obtained by Collins, Pincus, and Xie, (1999, Table 4, p. 44). We then show how the results of Collins, Pincus, and Xie, (1999) appear when transformed to Model 1. After transformation, the coefficient of book value has a positive sign for the mean value and for all 18 years studied, and the coefficient of earnings has a negative sign for the mean value and for 16 of the 18 years studied. The results in Table 15 show that the coefficient of book value β found empirically for loss firms by Collins, Pincus, and Xie, (1999) is consistent with our results. More importantly, their coefficient of earnings γ for loss firms is also quite consistent with those found here for the low efficiency (loss) firms that appear in the first three deciles.

References

Ashton D. J., "Corporate Financial Policy: American Analytics and UK Taxation." *Journal of Business Finance and Accounting* 18, 465–482 (1991).

Bailey, W., "Valuing Agricultural Firms: An Examination of the Contingent Claims Approach to Pricing Real Assets." *Journal of Economic Dynamics and Control* 15, 771–791 (1991).

Ball, R. and P. Brown, "An Empirical Evaluation of Accounting Numbers." *Journal of Accounting Research* 6, 159–178 (1968).

Barth, M., "Relative Measurement Errors Among Alternative Pension Assets and Liability Measures." *Accounting Review* 66, 433–463 (1991).

Barth, M., W. Beaver and W. Landsman, " Market Valuation Implications of Net Period Pension Cost." *Journal of Accounting and Economics* 15, 22–62 (1992).

Belkaoui, A. R., *Accounting and the Investment Opportunity Set.*, Westport, Connecticut: Quorum Books (2000).

Berger, P., E. Ofek and I. Swary, "Investor Valuation of the Abandonment Option." *Journal of Financial Economics* 42, 257–287 (1996).

Burgstahler, D. and I. Dichev, "Earning, Adaptation and Equity Value." *The Accounting Review* 72, 187–215 (1997).

Collins, D. and S. P. Kothari, "An Analysis of Intertemporal and Cross-Sectional Determinants of ERCs." *Journal of Accounting and Economics* 11, 143–183 (1989).

Collins, D. W., E. Maydew and I. Weiss, "Changes in the Value-Relevance of Earnings and Book Value Over the Past Forty Years." *Journal of Accounting and Economics* 24, 39–67 (1997).

Collins, D. W., M. Pincus and H. Xie, "Equity Valuation and Negative Earnings: The Role of Book Value of Equity." *The Accounting Review* 74, 29–61 (1999).

Feltham, G. and J. A. Ohlson., "Valuation and Clean Surplus Accounting for Operating and Financial Decisions." *Contemporary Accounting Research* 11, 689–731 (1995).

Feltham, G. and J. A. Ohlson., "Uncertainty Resolution and the Theory of Depreciation Measurement." *Journal of Accounting Research* 34, 209–234 (1996).

Gujarati, D., *Essentials of Econometrics*, New York: McGraw-Hill, Inc (1992).

Hayn, C., "The Information Content of Losses." *Journal of Accounting and Economics* 20, 125–153 (1995).

Jan, C. L. and J. Ou, "The Role of Negative Earnings in the Valuation of Equity Stocks." *Working paper*, New York University and Santa Clara University (1995).

Kothari, S. P. and J. L. Zimmerman, "Price and Return Models." *Journal of Accounting and Economics* 20, 155–192 (1995).

Landsman W., "An Empirical Investigation of Pension Fund Property Rights." *The Accounting Review* 61, 44–68 (1986).

Leland, H. and D. Pyle, "Information Asymmetries, Financial Structure, and Financial Intermediation." *Journal of Finance* 32, 371–387 (1977).

McConnell, J. and C. Muscarella, "Corporate Capital Expenditure Decisions and Market Value of the Firm." *Journal of Financial Economics* 399–422 (1985).

Miller, M. and F. Modigliani, "Dividend Policy, Growth and the Valuation of Shares." *Journal of Business* 34, 411–433 (1961).

Modigliani, F. and M. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment." *American Economic Review* 48, 261–297 (1958).

Moel, A. and P. Tufano, "When are Real Options Exercised? An Empirical Examination of Mine Closings." *Review of Financial Studies* 15, 35–64 (2002).

Myers, S. C., "Determinants of Corporate Borrowings." *Journal of Financial Economics* 5, 147–175 (1977).

Ohlson. J. A., "Earnings, Book Values and Dividends in Security Valuation." *Contemporary Accounting Research* 11, 661–687 (1995).

Paddock, J., D. Siegel and J. Smith, "Option Valuation of Claims on Physical Assets: The Case of Offshore Petroleum Leases." *Quarterly Journal of Economics* 103, 479–508 (1988).

Quigg, L., "Empirical Testing of Real Option Pricing Models." *Journal of Finance* 48, 621–640 (1993).

Rees, W. P., "The Impact of Dividends, Debt and Investment on Valuation Models." *Journal of Business, Finance and Accounting* 24, 1111–1140 (1997).

Ross, S., "The Determination of Financial Structure: The Incentive Signaling Approach." *Bell Journal of Economics* 8, 23–40 (1977).

Shevlin T., "The Valuation of R&D Firms with R&D Limited Partnerships." *The Accounting Review* 66, 1–21 (1991).

Zhang, G., "Accounting Information, Capital Investment Decisions, and Equity Valuation: Theory and Empirical Implications." *Journal of Accounting Research* 38, 271–295 (2000).