PREFACE TO THE SECOND EDITION

The quantum Hall (QH) effect is one of the most fascinating and beautiful phenomena in all branches of physics. Composite bosons, composite fermions and fractional charged excitations (anyons) were among the distinguishing ideas in the original edition. Seven years have passed since the original edition. Tremendous theoretical and experimental developments are still being made in this sphere. Many novel ideas have been proposed to understand various novel experimental results, which we have included in this new edition.

First, physics in higher Landau levels is quite different from that in the lowest Landau level because the effective Coulomb interactions are different. Charge density waves such as stripe states emerge, and indeed have been observed experimentally. Second, unconventional QH effects were discovered in graphene (a single atomic layer graphite), which immediately triggered enormous theoretical and experimental studies. It is remarkable that the electron dynamics is governed by the relativistic Dirac theory and that even supersymmetric quantum mechanics plays a key role. Third, intriguing phenomena associated with the interlayer phase coherence and SU(4) QH ferromagnets in the bilayer system have been fully revealed. They include the anomalous Hall resistivity in counter flow experiments and the anomalous diagonal resistivity near the commensurate-incommensurate phase transition point. The latter would signal the formation of a soliton lattice made of sine-Gordon solitons between the two layers. They also include an SU(4) skyrmion as a quasiparticle, which changes its shape from a pseudospin SU(2) texture to a spin SU(2) texture as the density imbalance is controlled between the two layers. Fourth, the microscopic theory of the QH effect is formulated entirely within the theory of noncommutative geometry. Thus, quasiparticles are noncommutative solitons in QH ferromagnets.

This new edition provides an instructive, comprehensive and self-contained overview of the QH effect including recent developments. It is also suitable for an introduction to quantum field theory with vivid applications. For instance, the Dirac theory of electrons and holes has a remarkable realization in QH effects in graphene. A fantastic world of noncommutative geometry together with noncommutative solitons has a concrete realization in QH systems, where various imaginative ideas can be explored theoretically and tested experimentally. QH effects have proved to be so special in condensed matter physics that they are deeply connected with fundamental principles of physics and mathematics. This book is

QUANTUM HALL EFFECTS - Field Theoretical Approach and Related Topics (Second Edition)

ideal for students and researchers in condensed matter physics, particle physics, theoretical physics and mathematical physics.

In Part I, a new chapter is added for Dirac electrons, holes and supersymmetry. In Part II, two chapters are added for charge density wave states in higher Landau levels and unconventional QH effects in graphene. Part III is revised fully to meet up-to-date achievements in bilayer QH systems. Part IV is rewritten anew in view of noncommutative geometry. Furthermore, almost all parts are retouched for improvement, and a number of misprints have been corrected.

To complete the second edition I have benefited from fruitful discussions on the subject with A. Sawada, Y. Hirayama, N. Kumada, A. Fukuda, D. Terasawa, M. Morino, K. Iwata, S. Kozumi, K. Hasebe, S. Suzuki, K. Ishii, G. Tsitsishvili and others. In particular, the collaboration with A. Sawada and G. Tsitsishvili was indispensable to complete this revision. Special thanks are due to N. Shibata and M. Ezawa for providing me with the contents of Chapter 19 and 20, respectively. Further, I am grateful to N. Shibata for careful reading of the manuscript, and to M. Ezawa for allowing me to use an illustration of the Dirac cone in the cover of this book.

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January 2007

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PREFACE TO THE FIRST EDITION

Quantum Hall (QH) effects are remarkable macroscopic quantum phenomena observed in the 2-dimensional electron system. The integer QH effect was discovered in 1980 by K. von Klitzing, a century after the discovery of the classical Hall effect, for which he received the Nobel prize in 1985. The fractional QH effect was discovered in 1982 by D. Tsui, H. Störmer and A.C. Gossard. It was predicted by B. Laughlin that a quasiparticle is an anyon carrying electric charge e/m at the filling factor $\nu = 1/m$. In 1997 a direct observation of fractional charges was successfully carried out at $\nu = 1/3$ by measuring a back scattering current noise in Hall-bar experiments. B. Laughlin, D. Tsui and H. Störmer received the Nobel prize in 1998.

QH effects are so special in condensed matter physics that they are deeply connected with the fundamental principles of physics. Moreover, they present concrete realizations of various modern concepts related with topological investigations not only in physics but also in mathematics. The QH system provides us with a rare opportunity to enjoy the interplay between condensed matter physics and particle physics. It is worthwhile to make the subject as a part of the training for all graduate students in physics.

Many fancy ideas appear in QH effects. Composite particle (boson or fermion) is one of them. It is an electron bound to flux quanta. Laughlin started his seminal paper by saying that "The $\frac{1}{3}$ effect, recently discovered by Tsui, Störmer and Gossard, results from the condensation of the two-dimensional electron gas in a GaAs-$\mathrm{Ga}_x\mathrm{Al}_{1-x}$ heterostructure into a new type of collective ground state". It is our present understanding that the QH state is a condensate of composite bosons. Intriguingly a single electron is converted into a boson by acquiring flux quanta in QH states. Such a statistical transmutation is allowed in the planar geometry due to its intrinsic topological structure. Despite their bosonic low-energy properties, composite bosons obey the fermionlike exclusion principle. The hierarchy of fractional QH states is understood by the use of composite fermions.

Topological solitons play the leading role in QH effects. Indeed, charged excitations (quasiparticles) are topological solitons in the QH condensate. When quantum coherence develops with the spin wave as a Goldstone mode, quasiparticles are skyrmion O(3)-spin textures. Skyrmions were originally proposed in nuclear physics, where they are O(4)-isospin textures to be identified with nucleons. Though their relevance is still unclear in nuclear physics, their existence is firmly established in the QH system.

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Edge excitations are described by the chiral Tomonaga-Luttinger model. Electrons are topological excitations in this model, and obey a relativistic field equation. When tunneling interactions are allowed between the opposite edges, topological excitations turn into sine-Gordon solitons. Physics on the edge is "exactly solvable", due to the existence of infinitely many conservation rules. It can be a laboratory to test various results of conformal field theories.

The bilayer QH system consists of two quantum wells separated by a barrier with ~ 30 nm width. A phase transition has been observed between two distinguishable phases at a fixed filling factor by changing the electron density. When the tunneling gap is not too large, interlayer coherence develops, as is a reminiscence of the superconductor Josephson junction. Pseudoparticles are CP^3 skyrmions.

Physics confined to the lowest Landau level is curious, because the x and y coordinates of the electron position become noncommutative. It is a simplest physical system subject to noncommutative geometry. The system is characterized by the Moyal algebra or the W_∞ algebra. When restricted to the edge of the QH droplet, the Kac-Moody algebra emerges. The excitation spectrum of the QH system forms a representation of the algebra.

This book is intended to give a pedagogical and self-contained introduction to these new concepts in QH effects. It is accessible, and will be of interest, to students and researchers in condensed matter physics, particle and mathematical physics. It comprises four parts. Part I is a quick summary of quantum field theory, where I explain various concepts necessary to understand QH effects, namely, canonical quantization, quantum coherence, topological solitons, anyons and so on. I hope that this part is useful to those who are not familiar to these concepts. Readers may skip this part and come back to relevant places when necessary. Part II is devoted to monolayer QH systems, while Part III to bilayer QH systems. Some algebraic aspects of the QH system are reviewed in Part IV, where I derive some basic formulas used in Part II and Part III.

I have benefited from fruitful collaborations on the subject with A. Sawada, A. Iwazaki, H. Ohno, Y. Hirayama, K. Muraki, Y. Horikoshi, Y. Ohno, T. Saku, N. Kumada, M. Hotta, K. Sasaki, K. Hasebe, Y-S. Wu and others. I am grateful to H. Aoki, D. Yoshioka, A. MacDonald, S. Murphy and J. Eisenstein for valuable discussions. Special thanks are due to K. Shizuya, S. Katsumoto and K. Takahashi for careful reading of the manuscript. Their comments were very helpful for me to improve the manuscript.

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January 2000