



CONTENTS

Preface	vii
<hr/>	
Part. I Quantum Field Theory	1
<hr/>	
1. Quantum Mechanics	3
1.1 Hilbert Spaces	3
1.2 Canonical Formalism	4
1.3 Creation and Annihilation Operators	6
1.4 Uncertainty Principle	10
1.5 Coherent States and von Neumann Lattice	11
1.6 Squeezed Coherent State	14
1.7 Particle Number and Phase	16
1.8 Macroscopic Coherence	18
2. Quantum Field Theory	20
2.1 One-Body Hamiltonian	20
2.2 Many-Body Hamiltonian	21
2.3 Boson Field Operators	23
2.4 Quantum Field Theory	25
2.5 Fermion Field Operators	27
2.6 Electrons Interacting with Electromagnetic Field	29
3. Canonical Quantization	31
3.1 Relativistic Particles and Waves	31
3.2 Schrödinger Field	32
3.3 Real Klein-Gordon Field	37
3.4 Complex Klein-Gordon Field	42

3.5	Nöther Currents	44
4.	Spontaneous Symmetry Breaking	47
4.1	Ferromagnets	47
4.2	Real Klein-Gordon Field	51
4.3	Complex Klein-Gordon Field	55
4.4	Sigma Model	56
4.5	Schrödinger Field	58
4.6	Superfluidity	62
4.7	Goldstone Theorem	64
5.	Electromagnetic Field	67
5.1	Maxwell Equations	67
5.2	Canonical Quantization	70
5.3	Interaction with Matter Field	77
5.4	Anderson-Higgs Mechanism	79
5.5	Massive Vector Field	81
5.6	Superconductivity	83
5.7	Aharonov-Bohm Effect	89
6.	Dirac Field	94
6.1	Dirac Equation	94
6.2	Plane Wave Solutions	97
6.3	Canonical Quantization	99
6.4	Interaction with Electromagnetic Field	103
6.5	Weyl Field (Massless Dirac Field)	105
6.6	Dirac Electrons in Magnetic Field	108
7.	Topological Solitons	113
7.1	Topological Sectors	113
7.2	Classical Fields	115
7.3	Solitary Waves, Kinks and Solitons	117
7.4	Sine-Gordon Solitons	119
7.5	Vortex Solitons	123
7.6	Homotopy Classes	127
7.7	O(3) Skyrmions	132
7.8	CP ^{N-1} Skyrmions	138
8.	Anyons	144
8.1	Spin and Statistics	144
8.2	Fractional Statistics	148
8.3	Quantum Mechanics	150

8.4	Chern-Simons Gauge Theory	153
8.5	Anyon Field Operators	157
<hr/>		
Part. II	Monolayer Quantum Hall Systems	161
<hr/>		
9.	Overview of Monolayer QH Systems	163
10.	Landau Quantization	177
10.1	Planar Electrons	177
10.2	Cyclotron Motion	181
10.3	Symmetric Gauge	187
10.4	Landau Gauge	190
10.5	von Neumann Lattice	193
10.6	Electrons in N th Landau Level	195
10.7	Hall Current	199
11.	Quantum Hall Effects	204
11.1	Incompressibility	204
11.2	Integer Quantum Hall Effects	205
11.3	Fractional Quantum Hall Effects	207
11.4	Quasiparticles	210
11.5	Hall Plateaux	213
12.	Quasiparticles and Activation Energy	215
12.1	Impurity Potentials	215
12.2	Gap Energies	216
12.3	Dispersion Relation	219
12.4	Thermal Activation	222
13.	Field Theory of Composite Particles	227
13.1	Composite Particles	227
13.2	Statistical Transmutation	230
13.3	Effective Magnetic Field	233
13.4	Dressed Composite Particles	235
13.5	Composite Particles in Lowest Landau Level	239
13.6	Composite Fermions in Lowest Landau Level	242
14.	Composite Bosons and Semiclassical Analysis	246
14.1	Ground State and Laughlin Wave Function	246

14.2	Perturbative Excitations	248
14.3	Vortex Excitations	250
14.4	Field Theory of Vortex Solitons	255
14.5	Haldane-Halperin Hierarchy	258
15.	Quantum Hall Ferromagnets	261
15.1	Spin Coherence	261
15.2	Spin Degree of Freedom	263
15.3	Composite Bosons and Spin-Charge Separation	265
15.4	Spin Field, Sigma Field and CP^1 Field	268
15.5	Effective Hamiltonian	271
16.	Spin Textures	274
16.1	Spin Excitations	274
16.2	Factorizable Skyrmions	277
16.3	Skyrmion Excitation Energy	284
16.4	Experimental Evidence	287
17.	Hierarchy of Fractional QH States	291
17.1	Jain Hierarchy	291
17.2	Landau Levels of Composite Fermions	294
17.3	Beyond Principal Sequences	296
17.4	Spin Polarization	298
17.5	Gap Energies	301
18.	Edge Effects	307
18.1	Edge Currents and Bulk Currents	307
18.2	Shot Noises of Fractional Charges	308
18.3	Chiral Edge Excitations	311
18.4	Chiral Tomonaga-Luttinger Liquid	312
18.5	Electrodynamics on Edge	316
18.6	Edge Tunneling and Sine-Gordon Solitons	320
19.	Stripes and Bubbles in Higher Landau Levels	325
19.1	Higher Landau Levels	325
19.2	Haldane's Pseudopotentials	327
19.3	Effective Coulomb Interactions	328
19.4	Density Matrix Renormalization Group Method	331
19.5	Stripes, Bubbles and Wigner Crystal	332
20.	Quantum Hall Effects in Graphene	335
20.1	Unconventional QH Effects	335

20.2	Graphene and Dirac Electrons	338
20.3	Dirac Hamiltonian and Supersymmetry	343
20.4	Effective Coulomb Interactions	346
20.5	Excitonic Condensation	352
20.6	Valley Polarization	355
20.7	Multilayer Graphene Systems	360
20.8	Berry Phase and Index Theorem	365
<hr/>		
Part. III	Bilayer Quantum Hall Systems	367
<hr/>		
21.	Overview of Bilayer QH Systems	369
22.	SU(2) Pseudospin Structure	383
22.1	Bilayer Planar Electrons	383
22.2	Pseudospins	385
22.3	Tunneling Interaction	387
22.4	Imbalanced Configuration	388
22.5	Capacitance Energy	392
22.6	Compound States	393
22.7	Charge-Transferable States	394
23.	Bilayer-Locked States	400
23.1	Composite-Boson Field	400
23.2	Wave Functions	404
23.3	Ground State	405
23.4	Vortex Excitations	406
24.	Interlayer Coherence	409
24.1	Pseudospin Ferromagnet	409
24.2	Effective Hamiltonian	413
24.3	Pseudospin Waves	415
24.4	Anomalous Bilayer QH Currents	417
24.5	Pseudospin Texture	424
25.	SU(4) Quantum Hall Ferromagnets	428
25.1	SU(4) Isospin Structure	428
25.2	SU(4) Isospin Fields	431
25.3	SU(4) Isospin Waves	433
25.4	SU(4) Isospin Textures	436

25.5	Excitation Energy of SU(4) Skyrmions	440
25.6	Activation Energy Anomaly	446
26.	Bilayer Quantum Hall Systems at $\nu = 2$	454
26.1	Spin Phase, Ppin Phase and Canted Phase	454
26.2	Ground-State Energy	457
26.3	Ground-State Structure	460
26.4	Phase Diagrams	463
26.5	Experimental Data	466
26.6	SU(4) Breaking and Grassmannian Fields	471
26.7	Grassmannian $G_{4,2}$ Solitons	475
26.8	Genuine Bilayer versus Two-Monolayer Systems	479
26.9	Experimental Indication of Biskyrms	482
27.	Bilayer Quantum-Hall Junction	485
27.1	Josephson-Like Phenomena	485
27.2	Parallel Magnetic Field	487
27.3	Effective Hamiltonian	493
27.4	Commensurate-Incommensurate Phase Transition	495
27.5	Soliton Lattice	499
27.6	Anomalous Diagonal Resistivity	508
27.7	Plasmon Excitations	514
27.8	Josephson-Like Effects	516
<hr/>		
Part. IV	Microscopic Theory	521
<hr/>		
28.	Overview of Microscopic Theory	523
29.	Noncommutative Geometry	534
29.1	Noncommutative Coordinate	534
29.2	Weyl Operator and Symbol	535
29.3	Magnetic Translation	541
29.4	Density Operators	543
29.5	SU(N)-Extended W_∞ Algebra	549
29.6	Classical Fields	552
29.7	Topological Charge Density	557
29.8	Kac-Moody Algebra on Edges	559
30.	Landau Level Projection	562

30.1	Projected Coulomb Interactions	562
30.2	Monolayer QH System	565
30.3	Electron-Hole Pair Excitations	569
30.4	Electron Excitation and Hole Excitation	570
30.5	Bilayer System without Spin ($\nu = 1$)	573
30.6	Bilayer System with Spin ($\nu = 1$)	575
30.7	Bilayer System with Spins ($\nu = 2$)	578
31.	Noncommutative Solitons	584
31.1	Topological Charge and Electric Charge	584
31.2	Microscopic Skyrmion States	585
31.3	Noncommutative CP^1 Skyrmion	589
31.4	Hole and Skyrmion	592
31.5	Skyrmion Wave Functions	593
31.6	Hardcore Interaction	595
31.7	Coulomb Interaction	600
31.8	$SU(4)$ Skyrmions	605
32.	Exchange Interactions and Effective Theory	607
32.1	Exchange Hamiltonian	607
32.2	Decomposition Formula	612
32.3	Spontaneous Symmetry Breaking	615
32.4	Classical Equations of Motion	616
32.5	Four-Layer Condenser Model	618
32.6	Derivative Expansion and Effective Theory	622
32.7	Noncommutative CP^{N-1} Model	627
32.8	Equations of Motion and Hall Currents	631
32.9	Hall Currents in Pseudospin QH Ferromagnet	634
Appendices		643
A	Energy Scales	643
B	Hausdorff Formulas	645
C	Group $SU(2)$ and Pauli Matrices	647
D	Groups $SU(N)$ and $SU(2N)$	647
E	Cauchy-Riemann Equations	650
F	Green Function	651
G	Bogoliubov Transformation	652
H	Energy-Momentum Tensor	654
I	Exchange Interaction	655
J	Mermin-Wagner Theorem	659
K	Lorentz Transformation	663
L	One-Dimensional Soliton Solutions	666

M	Field-Theoretical Vortex Operators	669
N	Bosonization in One-Dimensional Space	672
O	Coulomb Energy Formulas	677
P	W_∞ Algebra	679
References		681
Index		699