> Τό πᾶν άλλ′ έστι τι τό ὸλον παρά τά μόρια
> The whole is more than the sum of its parts
>
> Aristotle Metaphysica 1045a

Chapter 1

The phenomenology of complex systems

1.1 Complexity, a new paradigm

Complexity is part of our ordinary vocabulary. It has been used in everyday life and in quite different contexts for a long time and suddenly, as recently as 15 years ago it became a major field of interdisciplinary research that has since then modified considerably the scientific landscape. What is in the general idea of complexity that was missing in our collective knowledge -one might even say, in our collective consciousness- which, once recognized, conferred to it its present prominent status? What makes us designate certain systems as “complex” distinguishing them from others that we would not hesitate to call “simple”, and to what extent could such a distinction be the starting point of a new approach to a large body of phenomena at the crossroads of physical, engineering, environmental, life and human sciences?

For the public and for the vast majority of scientists themselves science is usually viewed as an algorithm for predicting, with a theoretically unlimited precision, the future course of natural objects on the basis of their present state. Isaac Newton, founder of modern physics, showed more than three centuries ago how with the help of a few theoretical concepts like the law of universal gravitation, whose statement can be condensed in a few lines, one can generate data sets as long as desired allowing one to interpret the essence of the motion of celestial bodies and predict accurately, among others, an eclipse of the sun or of the moon thousands of years in advance. The impact of this historical achievement was such that, since then, scientific thinking has been dominated by the *Newtonian paradigm* whereby the world is reducible to a few fundamental elements animated by a regular, reproducible and hence predictable behavior: a world that could in this sense be qualified as fundamentally simple.

During the three-century reign of the Newtonian paradigm science reached a unique status thanks mainly to its successes in the exploration of the very small and the very large: the atomic, nuclear and subnuclear constitution of matter on the one side; and cosmology on the other. On the other hand man's intuition and everyday experience are essentially concerned with the intermediate range of phenomena involving objects constituted by a large number of interacting subunits and unfolding on his own, macroscopic, space and time scales. Here one cannot avoid the feeling that in addition to regular and reproducible phenomena there exist other that are, manifestly, much less so. It is perfectly possible as we just recalled to predict an eclipse of the sun or of the moon thousands of years in advance but we are incapable of predicting the weather over the region we are concerned more than a few days in advance or the electrical activity in the cortex of a subject a few minutes after he started performing a mental task, to say nothing about next day's Dow Jones index or the state of the planet earth 50 years from now. Yet the movement of the atmosphere and the oceans that governs the weather and the climate, the biochemical reactions and the transport phenomena that govern the functioning of the human body and underlie, after all, human behavior itself, obey to the same dispassionate laws of nature as planetary motion.

It is a measure of the fascination that the Newtonian paradigm exerted on scientific thought that despite such indisputable facts, which elicit to the observer the idea of "complexity", the conviction prevailed until recently that the irregularity and unpredictability of the vast majority of phenomena on our scale are not authentic: they are to be regarded as temporary drawbacks reflecting incomplete information on the system at hand, in connection with the presence of a large number of variables and parameters that the observer is in the practical impossibility to manage and that mask some fundamental underlying regularities.

If evidence on complexity were limited to the intricate, large scale systems of the kind mentioned above one would have no way to refute such an assertion and fundamental science would thus have nothing to say on complexity. But over the years evidence has accumulated that quite ordinary systems that one would tend to qualify as "simple", obeying to laws known to their least detail, in the laboratory, under strictly controlled conditions, generate unexpected behaviors similar to the phenomenology of complexity as we encounter it in nature and in everyday experience: Complexity is not a mere metaphor or a nice way to put certain intriguing things, it is a phenomenon that is deeply rooted into the laws of nature, where systems involving large numbers of interacting subunits are ubiquitous.

This realization opens the way to a systematic search of the physical and mathematical laws governing complex systems. The enterprise was crowned with success thanks to a multilevel approach that led to the development of highly original methodologies and to the unexpected cross-fertilizations and blendings of ideas and tools from nonlinear science, statistical mechanics and thermodynamics, probability theory and numerical simulation. Thanks to the progress accomplished complexity is emerging as the new, post-Newtonian paradigm for a fresh look at problems of current concern. On the one side one is now in the position to gain new understanding, both qualitative and quantitative, of the complex systems encountered in nature and in everyday experience based on advanced modeling, analysis and monitoring strategies. Conversely, by raising issues and by introducing concepts beyond the traditional realm of physical science, natural complexity acts as a source of inspiration for further progress at the fundamental level. It is this sort of interplay that confers to research in complexity its unique, highly interdisciplinary character.

The objective of this chapter is to compile some representative facts illustrating the phenomenology associated with complex systems. The subsequent chapters will be devoted to the concepts and methods underlying the paradigm shift brought by complexity and to showing their applicability on selected case studies.

1.2 Signatures of complexity

The basic thesis of this book is that a system perceived as complex induces a characteristic phenomenology the principal signature of which is *multiplicity*. Contrary to elementary physical phenomena like the free fall of an object under the effect of gravity where a well-defined, single action follows an initial cause at any time, several outcomes appear to be possible. As a result the system is endowed with the capacity to choose between them, and hence to explore and to adapt or, more generally, to evolve. This process can be manifested in the form of two different expressions.

- The *emergence*, within a system composed of many units, of global traits encompassing the system as a whole that can in no way be reduced to the properties of the constituent parts and can on these grounds be qualified as “unexpected”. By its non-reductionist character emergence has to do with the creation and maintenance of hierarchical structures in which the disorder and randomness that inevitably exist at the local level are controlled, resulting in states of order and long range coherence. We refer to this process as *self-organization*. A classical example of this behavior is provided by the communication and control networks in living matter, from the subcellular to the organismic level.

- The intertwining, within the same phenomenon, of large scale regularities and of elements of "surprise" in the form of seemingly erratic evolutionary events. Through this coexistence of order and disorder the observer is bound to conclude that the process gets at times out of control, and this in turn raises the question of the very possibility of its long-term prediction. Classical examples are provided by the all-familiar difficulty to issue satisfactory weather forecasts beyond a horizon of a few days as well as by the even more dramatic extreme geological or environmental phenomena such as earthquakes or floods.

If the effects generated by some underlying causes were related to these causes by a simple proportionality -more technically, by *linear* relationships- there would be no place for multiplicity. Nonlinearity is thus a necessary condition for complexity, and in this respect nonlinear science provides a natural setting for a systematic description of the above properties and for sorting out generic evolutionary scenarios. As we see later nonlinearity is ubiquitous in nature on all levels of observation. In macroscopic scale phenomena it is intimately related to the presence of feedbacks, whereby the occurrence of a process affects (positively or negatively) the way it (or some other coexisting process) will further develop in time. Feedbacks are responsible for the onset of cooperativity, as illustrated in the examples of Sec. 1.4.

In the context of our study a most important question to address concerns the transitions between states, since the question of complexity would simply not arise in a system that remains trapped in a single state for ever. To understand how such transitions can happen one introduces the concept of *control parameter*, describing the different ways a system is coupled to its environment and affected by it. A simple example is provided by a thermostated cell containing chemically active species where, depending on the environmental temperature, the chemical reactions will occur at different rates. Another interesting class of control parameters are those associated to a *constraint* keeping the system away of a state of equilibrium of some sort. The most clearcut situation is that of the state of thermodynamic equilibrium which, in the absence of phase transitions, is known to be unique and lack any form of dynamical activity on a large scale. One may then choose this state as a reference, switch on constraints driving the system out of equilibrium for instance in the form of temperature or concentration differences across the interface between the system and the external world, and see to what extent the new states generated as a response to the constraint could exhibit qualitatively new properties that are part of the phenomenology of complexity. These questions, which are at the heart of complexity theory, are discussed in the next section.

1.3 Onset of complexity

The principal conclusion of the studies of the response of a system to changes of a control parameter is that the onset of complexity is not a smooth process. Quite to the contrary, it is manifested by a cascade of transition phenomena of an explosive nature to which is associated the universal model of *bifurcation* and the related concepts of *instability* and *chaos*. These catastrophic events are not foreseen in the fundamental laws of physics in which the dependence on the parameters is perfectly smooth. To use a colloquial term, one might say that they come as a "surprise".

Figure 1.1 provides a qualitative representation of the foregoing. It depicts a typical evolution scenario in which, for each given value of a control parameter λ, one records a certain characteristic property of the system as provided, for instance, by the value of one of the variables X (temperature, chemical concentration, population density, etc.) at a given point. For values of λ less than a certain limit λ_c only one state can be realized. This state possesses in addition to uniqueness the property of *stability*, in the sense that the system is capable of damping or at least of keeping under control the influence of the external perturbations inflicted by the environment or of the internal fluctuations generated continuously by the locally prevailing disorder, two actions to which a natural system is inevitably subjected. Clearly, complexity has no place and no meaning under these conditions.

The situation changes radically beyond the critical value λ_c. One sees that if continued, the unique state of the above picture would become unstable: under the influence of external perturbations or of internal fluctuations the system responds now as an amplifier, leaves the initial "reference" state and is driven to one or as a rule to several new behaviors that merge to the previous state for $\lambda = \lambda_c$ but are differentiated from it for λ larger than λ_c. This is the phenomenon of bifurcation: a phenomenon that becomes possible thanks to the nonlinearity of the underlying evolution laws allowing for the existence of multiple solutions (see Chapter 2 for quantitative details). To understand its necessarily catastrophic character as anticipated earlier in this section it is important to account for the following two important elements.

(a) An experimental measurement -the process through which we com-

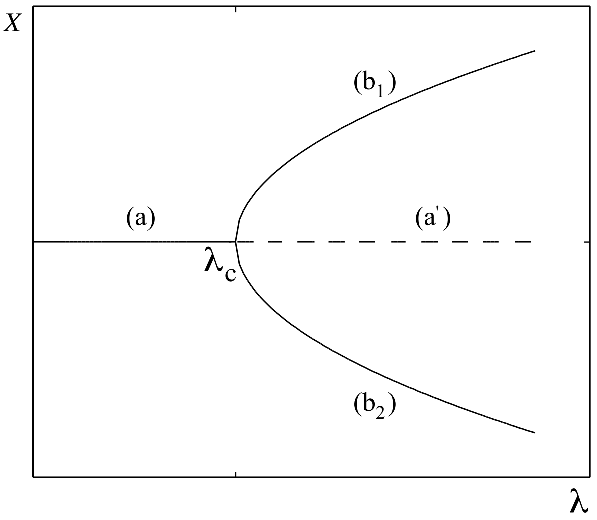

Fig. 1.1. A bifurcation diagram, describing the way a variable X characterizing the state of a system is affected by the variations of a control parameter λ. Bifurcation takes place at a critical value λ_c beyond which the original unique state (a) loses its stability, giving rise to two new branches of solutions (b_1) and (b_2).

municate with a system- is necessarily subjected to finite precision. The observation of a system for a given value of control parameter entails that instead of the isolated point of the λ axis in Fig. 1.1 one deals in reality with an "uncertainty ball" extending around this axis. The system of interest lies somewhere inside this ball but we are unable to specify its exact position, since for the observer all of its points represent one and the same state.

(b) Around and beyond the criticality λ_c we witness a selection between the states available that will determine the particular state to which the system will be directed (the two full lines surrounding the intermediate dotted one -the unstable branch in Fig. 1.1- provide an example). Under the conditions of Fig. 1.1 there is no element allowing the observer to determine beforehand this state. Chance and fluctuations will be the ones to decide. The system makes a series of attempts and eventually a particular fluctuation takes over. By stabilizing this choice it becomes a historical object, since its subsequent evolution will be conditioned by this critical choice. For the observer, this pronounced *sensitivity to the parameters* will signal its inability to predict the system's evolution beyond λ_c since systems within the uncertainty ball, to him identical in any respect, are differentiated and end up in states whose distance is much larger than the limits of resolution of the experimental measurement.

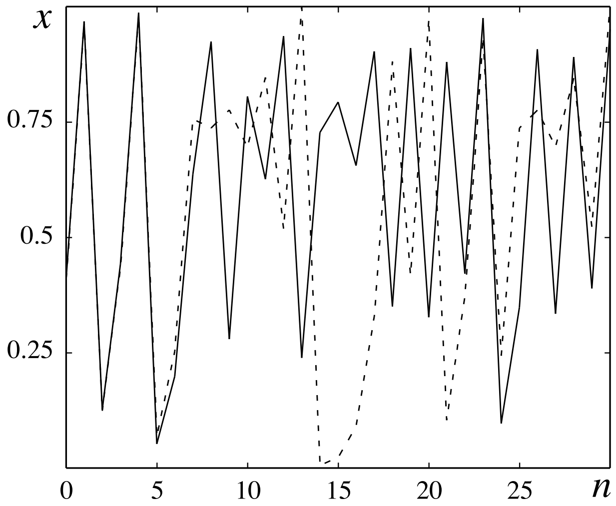

Fig. 1.2. Illustration of the phenomenon of sensitivity to the initial conditions in a model system giving rise to deterministic chaos. Full and dashed lines denote the trajectories (the set of successive values of the state variable X) emanating from two initial conditions separated by a small difference $\epsilon = 10^{-3}$.

We now have the basis of a mechanism of generation of complexity. In reality this mechanism is the first step of a cascade of successive bifurcations through which the multiplicity of behaviors may increase dramatically, culminating in many cases in a state in which the system properties change in time (and frequently in space as well) in a seemingly erratic fashion, not any longer because of external disturbances or random fluctuations as before but, rather, as a result of deterministic laws of purely intrinsic origin. The full line of Fig. 1.2 depicts a *time series* -a succession of values of a relevant variable in time- corresponding to this state of *deterministic chaos*. Its comparison with the dotted line reveals what is undoubtedly the most spectacular property of deterministic chaos, the *sensitivity to the initial conditions*: two systems whose initial states are separated by a small distance, smaller than the precision of even the most advanced method of experimental measurement, systems that will therefore be regarded by the observer as indistinguishable (see also point (a) above) will subsequently diverge in such a way that the distance between their instantaneous states (averaged over many possible initial states, see Chapters 2 and 3) will increase exponentially. As soon as this distance will exceed the experimental resolution the systems will cease to be indistinguishable for the observer. As a result, it will be impossible to predict their future evolution beyond this temporal horizon.

We here have a second imperative reason forcing us to raise the question of predictability of the phenomena underlying the behavior of complex systems.

All elements at our disposal from the research in nonlinear science and chaos theory lead to the conclusion that one cannot anticipate the full list of the number or the type of the evolutionary scenarios that may lead a system to complex behavior. In addition to their limited predictability complex systems are therefore confronting us with the fact that we seem to be stuck with a mode of description of a limited universality. How to reconcile this with the requirement that the very mission of science is to provide a universal description of phenomena and to predict their course? The beauty of complex systems lies to a great extent in that despite the above limitations this mission can be fulfilled, but that its realization necessitates a radical reconsideration of the concepts of universality and prediction. We defer a fuller discussion of this important issue to Chapters 2 and 3.

1.4 Four case studies

1.4.1 Rayleigh-Bénard convection

Consider a shallow layer of a fluid limited by two horizontal plates brought to identical temperatures. As prescribed by the second law of thermodynamics, left to itself the fluid will tend rapidly to a state where all its parts along the horizontal are macroscopically identical and where there is neither bulk motion nor internal differentiation of temperatures: $T = T_1 = T_2, T_2$ and T_1 being respectively the temperatures of the lower and upper plate. This is the state we referred to in Sec. 1.2 as the state of thermodynamic equilibrium.

Imagine now that the fluid is heated from below. By communicating to it in this way energy in the form of heat one removes it from the state of equilibrium, since the system is now submitted to a constraint $\Delta T = T_2 - T_1 > 0$, playing in this context the role of the control parameter introduced in Sec. 1.2. As long at ΔT remains small the flux of energy traversing the system will merely switch on a process of heat conduction, in which temperature varies essentially linearly between the hot (lower) zone and the cold (upper) one. This state is maintained thanks to a certain amount of energy that remains trapped within the system -one speaks of *dissipation*- but one can in no way speak here of complexity and emergence, since the state is unique and the differentiation observed is dictated entirely by the way the constraint has been applied: the behavior is as "simple" as the one in the state of equilibrium.

If one removes now the system progressively from equilibrium, by increas-

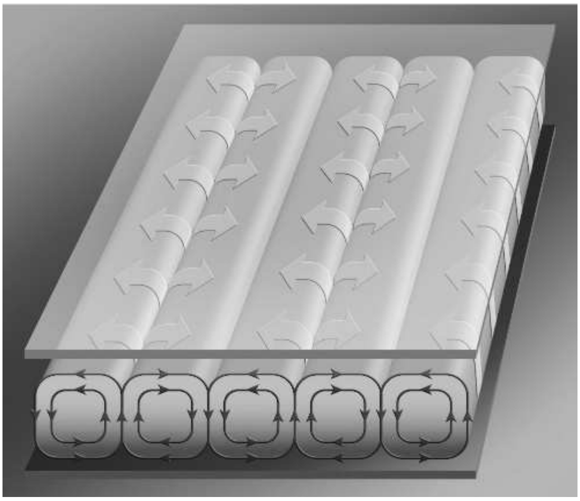

Fig. 1.3. Rayleigh-Bénard convection cells appearing in a liquid maintained between a horizontal lower hot plate and an upper cold one, below a critical value of the temperature difference ΔT (see Color Plates).

ing ΔT, one suddenly observes, for a critical value ΔT_c, the onset of bulk motion in the layer. This motion is far from sharing the randomness of the motion of the individual molecules: the fluid becomes structured and displays a succession of cells along a direction transversal to that of the constraint, as seen in Fig. 1.3. This is the regime of thermal, or Rayleigh-Bénard convection. Now one is entitled to speak of complexity and emergence, since the spatial differentiation along a direction free from any constraint is the result of processes of internal origin specific to the system, maintained by the flow of energy communicated by the external world and hence by the dissipation. We have thus witnessed a particular manifestation of emergence, in the form of the birth of a *dissipative structure.* In a way, one is brought from a static, geometric view of space, to one where space is modeled by the dynamical processes switched on within the system. One can show that the state of rest is stable below the threshold ΔT_c but loses its stability above it while still remaining a solution -in the mathematical sense of the term- of the evolution laws of the fluid. As for the state of thermal convection, it simply does not exist below ΔT_c and inherits above it the stability of the state of rest. For $\Delta T = \Delta T_c$ there is degeneracy in the sense that the two states merge. We here have a concrete illustration of the generic phenomenon of bifurcation introduced in Sec. 1.3, see Fig. 1.1. Similar phenomena are observed in a wide range of laboratory scale systems, from fluid mechanics to chemical kinetics, optics, electronics or materials science. In each case one encoun-

FOUNDATIONS OF COMPLEX SYSTEMS - Nonlinear Dynamics, Statistical Physics, Information and Prediction

ters essentially the same phenomenology. The fact that this is taking place under perfectly well controlled conditions allows one to sort out common features and set up a quantitative theory, as we see in detail in the subsequent chapters.

A remarkable property of the state of thermal convection is to possess a characteristic space scale -the horizontal extent of a cell (Fig. 1.3) related, in turn, to the depth of the layer. The appearance of such a scale reflects the fact that the states generated by the bifurcation display *broken symmetries*. The laws of fluid dynamics describing a fluid heated from below and contained between two plates that extend indefinitely in the horizontal direction remain invariant -or more plainly look identical- for all observers displaced to one another along this direction (translational invariance). This invariance property is shared by the state realized by the fluid below the threshold ΔT_c but breaks down above it, since a state composed of a succession of Bénard cells displays an intrinsic differentiation between its different parts that makes it less symmetrical than the laws that generated it. A differentiation of this sort may become in many cases one of the prerequisites for further complexification, in the sense that processes that would be impossible in an undifferentiated medium may be switched on. In actual fact this is exactly what is happening in the Rayleigh-Bénard and related problems. In addition to the first bifurcation described above, as the constraint increases beyond ΔT_c the system undergoes a whole series of successive transitions. Several scenarios have been discovered. If the horizontal extent of the cell is much larger than the depth the successive transition thresholds are squeezed in a small vicinity of ΔT_c. The convection cells are first maintained globally but are subsequently becoming fuzzy and eventually a regime of *turbulence* sets in, characterized by an erratic-looking variability of the fluid properties in space (and indeed in time as well). In this regime of extreme spatio-temporal chaos the motion is ordered only on a local level. The regime dominated by a characteristic space scale has now been succeeded by a *scale-free* state in which there is a whole spectrum of coexisting spatial modes, each associated to a different space scale. Similar phenomena arise in the time domain, where the first bifurcation may lead in certain types of systems to a strictly periodic clock-like state which may subsequently lose its coherence and evolve to a regime of deterministic chaos in which the initial periodicity is now part of a continuous spectrum of coexisting time scales.

As we see throughout this book states possessing a characteristic scale and scale-free states are described, respectively, by exponential laws and by power laws. There is no reason to restrict the phenomenology of complexity to the class of scale free states as certain authors suggest since, for one thing, coherence in living matter is often reflected by the total or partial synchronization of the activities of the individual cells to a dominant temporal or spatial mode.

In concluding this subsection it is appropriate to stress that configurations of matter as unexpected a priori as the Bénard cells, involving a number of molecules (each in disordered motion !) of the order of the Avogadro number $N \approx 10^{23}$ are born spontaneously, inevitably, at a modest energetic and informational cost, provided that certain conditions related to the nature of the system and the way it is embedded to its environment are fulfilled. Stated differently the overall organization is not ensured by a centralized planification and control but, rather, by the "actors" (here the individual fluid parcels) present. We refer to this process as the *bottom-up* mechanism.

1.4.2 Atmospheric and climatic variability

Our natural environment plays a central role in this book, not only on the grounds of its importance in man's everyday activities but also because it qualifies in any respect as what one intuitively means by complex system and forces upon the observer the need to cope with the problem of prediction. Contrary to the laboratory scale systems considered in the previous subsection we have no way to realize at will the successive transitions underlying its evolution to complexity. The best one can expect is that a monitoring in the perspective of the complex systems approach followed by appropriate analysis and modeling techniques, will allow one to constitute the salient features of the environment viewed as a dynamical system and to arrive at a quantitative characterization of the principal quantities of interest.

To an observer caught in the middle of a hurricane, a flood or a long drought the atmosphere appears as an irrational medium. Yet the atmospheric and climatic variables are far from being distributed randomly. Our environment is structured in both space and time, as witnessed by the stratification of the atmospheric layers, the existence of global circulation patterns such as the planetary waves, and the periodicities arising from the daily or the annual cycle. But in spite of this global order one observes a pronounced superimposed variability, reflected by marked deviations from perfect or even approximate regularity.

An example of such a variability is provided by the daily evolution of air temperature at a particular location (Fig. 1.4). One observes small scale irregular fluctuations that are never reproduced in an identical fashion, superimposed on the large scale regular seasonal cycle of solar radiation. A second illustration of variability pertains to the much larger scale of global climate. All elements at our disposal show indeed that the earth's climate has undergone spectacular changes in the past, like the succession of glacial-

FOUNDATIONS OF COMPLEX SYSTEMS - Nonlinear Dynamics, Statistical Physics, Information and Prediction

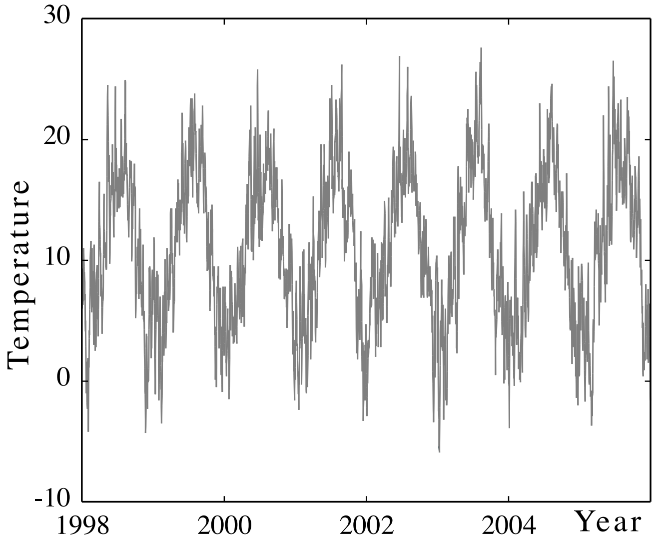

Fig. 1.4. Mean daily temperature at Uccle (Brussels) between January 1st, 1998 and December 31, 2006.

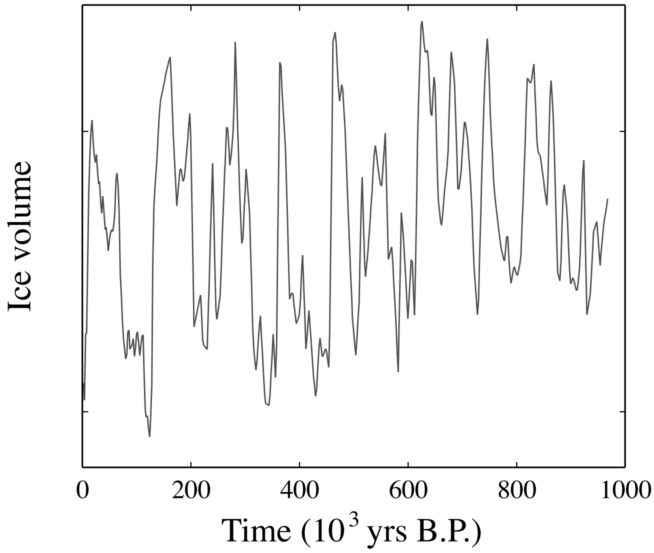

Fig. 1.5. Evolution of the global ice volume on earth during the last million years as inferred from oxygen isotope data.

interglacial periods. Figure 1.5 represents the variation of the volume of continental ice over the last million years as inferred from the evolution of the composition of marine sediments in oxygen 16 and 18 isotopes. Again, one is struck by the intermittent character of the evolution, as witnessed by a marked aperiodic component masking to a great extent an average time scale of 100 000 years that is sometimes qualified as the Quaternary glaciation "cycle". An unexpected corollary is that the earth's climate can switch between quite different modes over a short time in the geological scale, of the order of a few thousand years.

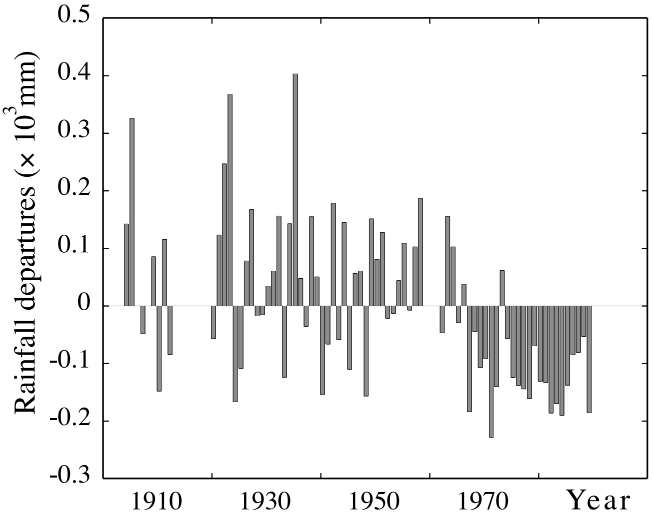

Fig. 1.6. Yearly rainfall departures from the long time average value at Kaédi (Mauritania) between 1904 and 1987.

Figure 1.6 depicts another example of climatic variability and regime switching, on a scale that is intermediate between those in Figs 1.4 and 1.5. It has to do with the time variation of the precipitation in western Sahel, and signals the onset of a regime of drought in this region, a phenomenon known to occur in several other areas of the globe. Again, one is struck by the irregular character of the process. The new element as compared to Figs 1.4 and 1.5 is that in the language of statistics the signal is no longer stationary: rather than succeeding each other without exhibiting a systematic trend, the states are here undergoing an abrupt transition between a regime of a quasi-normal and a weak rainfall that one can locate using traditional statistical analysis around the mid-1960's. It is likely that observations over a much longer time scale will reestablish the stationarity of the process, in the sense that the state of drought will sooner or later be succeeded by a quasi-normal state which will subsequently switch again to a state of drought, and so forth.

A fundamental consequence of the aperiodicity of the atmospheric and climate dynamics is the well-known difficulty to make reliable predictions. Contrary to simple periodic or multiperiodic phenomena for which a long term prediction is possible, predictions in meteorology are limited in time. The most plausible (and currently admitted) explanation is based on the realization that a small uncertainty in the initial conditions used in a prediction scheme (usually referred as "error") seems to be amplified in the course of the evolution. Such uncertainties are inherent in the process of experimental measurement, as pointed out already in Sec. 1.3. A great deal of effort is devoted in atmospheric sciences in the development of *data assimilation* techniques aiming to reduce them as much as possible (cf. also Sec. 5.4), but it is part of the laws of nature that they will never be fully eliminated. This brings us to the picture drawn in connection with Fig. 1.2, suggesting that the atmosphere displays sensitivity to the initial conditions because it is in a state of deterministic chaos. This conjecture seems to be compatible both with the analysis of the data available and with the modeling of atmospheric dynamics. This aspect is discussed more amply in Chapters 5 and 6, but one may already notice at this stage that much like experiment, modeling is also limited in practice by a finite resolution (of the order of several kilometers) and the concomitant omission of "subgrid" processes like e.g. local turbulence. Furthermore, many of the parameters are not known to a great precision. In addition to initial errors prediction must thus cope with *model errors*, reflecting the fact that a model is only an approximate representation of nature. This raises the problem of sensitivity to the parameters and brings us to the picture drawn in connection with Fig. 1.1. If the dynamics were simple like in the part of Fig. 1.1 left to λ_c neither of these errors would matter. But this is manifestly not the case. Initial and model errors can thus be regarded as probes revealing the fundamental instability and complexity underlying the atmosphere.

In all the preceding examples it was understood that the characteristic parameters of the atmosphere remained fixed. Over the last years there has been growing interest in the response of the weather and climate to changing parameter values - for instance, as a result of anthropogenic effects. In the representation of Fig. 1.1, the question would then be, whether the underlying dynamical system would undergo transitions to new regimes and if so, what would be the nature of the most plausible transition scenarios. This raises a whole new series of problems, some of which will be taken up in the sequel.

As pointed out earlier in this subsection, in certain environmental phenomena the variability is so considerable that no underlying regularity seems to be present. This property, especially pronounced in hydrology and in particular in the regime of river discharges, entails that the average and other quantifiers featured in traditional statistics are irrelevant. An ingenious way to handle such records, suggested some time ago by Harold Hurst, is to monitor the way the distance R between the largest and smallest value in a certain time window τ -usually referred to as *the range*- varies with τ. Actually, to deal with a dimensionless quantity one usually reduces R by the standard deviation C around the mean measured over the same interval. A most surprising result is that in a wide spectrum of environmental records R/C varies with τ as a power law of the form τ^H, where the *Hurst exponent* H turns out to be close to 0.70. To put this in perspective, for records generated by statistically independent processes with finite standard deviation, H is bound to be 1/2 and for records where the variability is organized around a characteristic time scale there would simply not be a power law at all. Environmental dynamics provides therefore yet another example of the coexistence of phenomena possessing a characteristic scale and of scale free ones.

An interesting way to differentiate between these processes is to see how the law is changing upon a transformation of the variable (here the window τ). For an exponential law, switching from τ to $\lambda\tau$ (which can be interpreted as a change of scale in measuring τ) maintains the exponential form but changes the exponent multiplying τ, which provides the characteristic scale of the process, by a factor of λ. But in a power law the same transformation keeps the exponent H invariant, producing merely a multiplicative factor. We express this by qualifying this law as *scale invariant*. The distinction breaks down for nonlinear transformations, for which a power law can become exponential and vice versa.

As we see later deterministic chaos can be associated with variabilities of either of the two kinds, depending on the mechanisms presiding in its generation.

1.4.3 Collective problem solving: food recruitment in ants

In the preceding examples the elements constituting the system of interest were the traditional ones considered in physical sciences: molecules, volume elements in a fluid or in a chemical reagent, and so forth. In this subsection we are interested in situations where the actors involved are living organisms. We will see that despite this radical change, the principal manifestations of complexity will be surprisingly close to those identified earlier. Our discussion will focus on social insects, in particular, the process of food searching in ants.

Ants, like bees, termites and other social insects represent an enormous ecological success in biological evolution. They are known to be able to accomplish successfully number of collective activities such as nest construction, recruitment, defense etc. Until recently the view prevailed that in such highly non-trivial tasks individual insects behave as small, reliable automatons executing a well established genetic program. Today this picture is fading and replaced by one in which adaptability of individual behavior, collective interactions and environmental stimuli play an important role. These elements are at the origin of a two-scale process. One at the level of the individual, characterized by a pronounced probabilistic behavior, and another at the level of the society as a whole, where for many species despite the inefficiency and unpredictability of the individuals, coherent patterns develop at the scale of the entire colony.

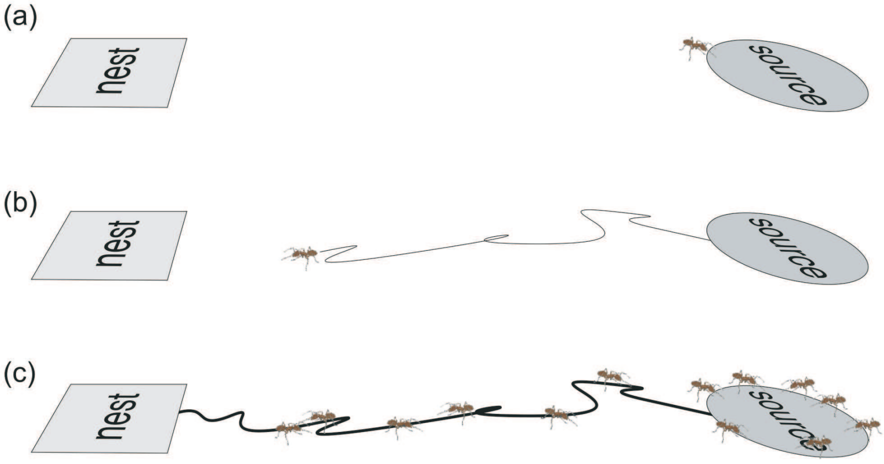

Fig. 1.7. Schematic representation of recruitment: (a) discovery of the food source by an individual; (b) return to the nest with pheromone laying; (c) the pheromone trail stimulates additional individuals to visit the source, which contribute to its reinforcement by further pheromone laying.

Let us see how these two elements conspire in the process of food searching by ants. Consider first the case where a single food source (for instance a saccharose solution) is placed close to the nest, as in Fig. 1.7 (here and in the sequel laboratory experiments emulating naturally occurring situations while allowing at the same time for detailed quantitative analyses are instru-

mental). A "scout" discovers the source in the course of a random walk. After feeding on the source it returns to the nest and deposits along the way a chemical signal known as *trail pheromone*, whose quantity is correlated to the sugar concentration in the source. Subsequently a process of *recruitment* begins in which two types of phenomena come into play:
- a first mechanism in which the scout-recruiter and/or the trail stimulate individuals that were till then inactive to go out of the nest;
- and a second one where the trail guides the individuals so recruited to the food source, entailing that as recruited individuals will sooner or later become recruiters in their turn the process will be gradually amplified and a substantial traffic will be established along the trail.

Consider now the more realistic situation where the colony disposes of several food sources. A minimal configuration allowing one to study how it then copes with the problem of choice to which it is confronted is depicted in Fig. 1.8a: two equivalent paths leading from the nest to two simultaneously present identical food sources. In a sufficiently numerous colony after a short period of equal exploitation a bifurcation, in the precise sense of Fig. 1.1, is then observed marking a preferential exploitation of one of the sources relative to the other, to its exhaustion (Fig. 1.8b). Thereafter the second source is fully colonized and its exploitation is intensified. When the colony is offered two sources with different sugar concentrations and the richest source is discovered before or at the same time as the poorer one, it is most heavily exploited. But when it is discovered after the poorer one, it is only weakly exploited. This establishes the primordial importance of the long-range cooperativity induced by the presence of the trail.

It is tempting to conjecture that far from being a curiosity the above phenomenon, which shares with the Rayleigh-Bénard instability the property of spontaneous emergence of an a priori highly unexpected behavioral pattern, is prototypical of a large class of systems, including socio-economic phenomena in human populations (see also Sec. 1.4.4 below). The key point lies in the realization that nature offers a bottom-up mechanism of organization that has no recourse to a central or hierarchical command process as in traditional modes of organization. This mechanism leads to collective decisions and to problem solving on the basis of (a) the local information available to each "agent"; and (b) its implementation on global level without the intervention of an information-clearing center. It opens the way to a host of applications in the organization of distributed systems of interacting agents as seen, for example, in communication networks, computer networks and networks of mobile robots or static sensory devices. Such analogy-driven considerations can stimulate new ideas in a completely different context by serving as archetypes. They are important elements in the process of model

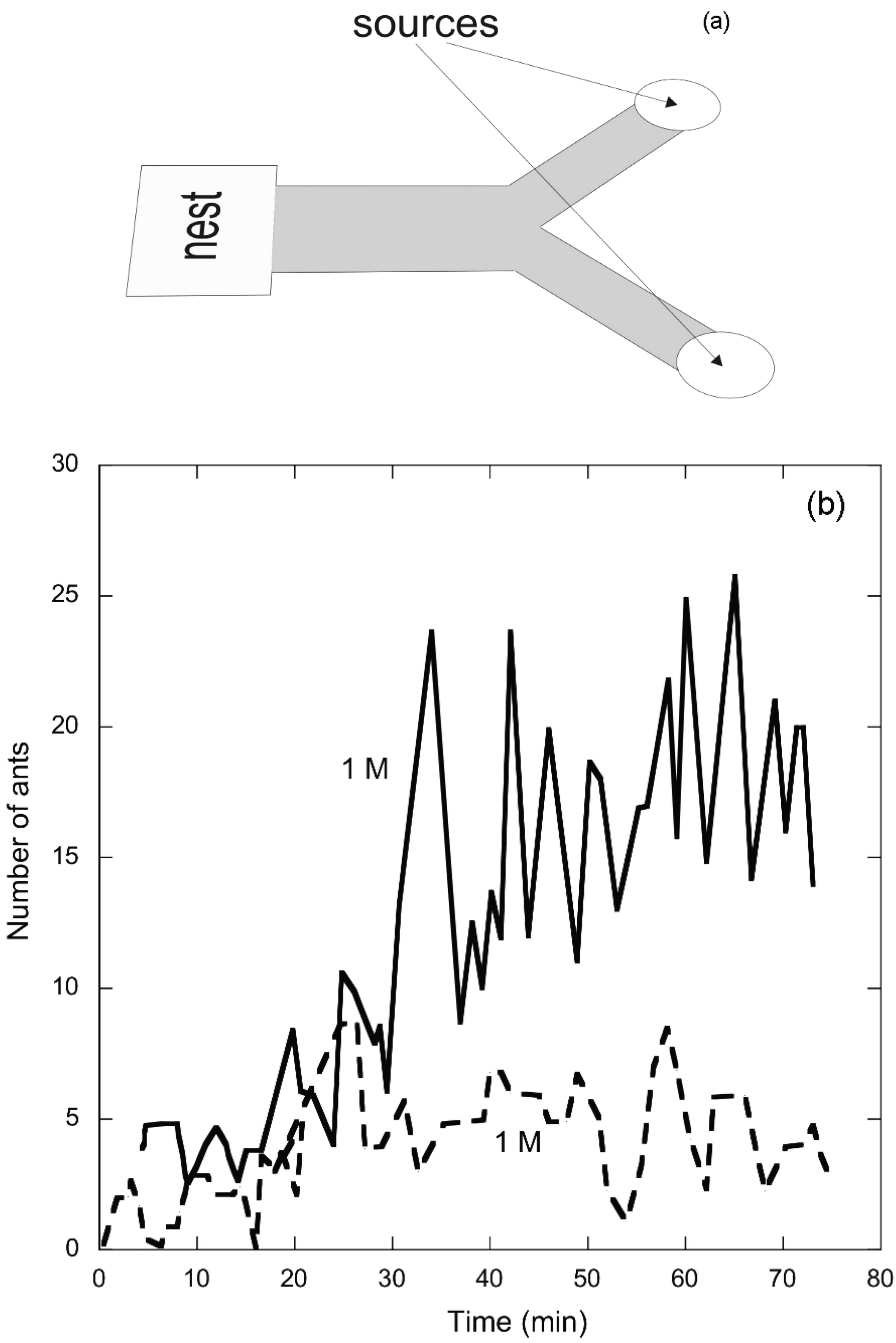

Fig. 1.8. (a) A typical experimental set up for the study of the process of choice between two options. (b) Time evolution of the number of individuals (here ants of the species *Lasius niger*) exploiting two equivalent (here 1 molar saccharose rich) food sources offered simultaneously, in an experimental set up of the type depicted in Fig. 1.8(a).

building -an essential part of the research in complex systems- in situations in which the evolution laws of the variables involved may not be known to any comparable degree of detail as in physical systems.

1.4.4 Human systems

We now turn to a class of complexity related problems in which the actors involved are human beings. Here the new element that comes into play is the presence of such concepts as strategy, imitation, anticipation, risk assessment, information, history, quite remote at first sight from the traditional vocabulary of physical science. The expectation would be that thanks to the rationality underlying these elements, the variability and unpredictability should be considerably reduced. The data at our disposal show that this is far from being the case. Human systems provide, in fact, one of the most authentic prototypes of complexity. They also constitute a source of inspiration for raising number of new issues, stimulating in turn fundamental research in the area of complex systems.

A first class of instances pertains to cooperativity (imitation) driven socio-cultural phenomena. They usually lead to bifurcations very similar to those considered in the previous subsection in which the variability inherent in the dynamics of the individuals is eventually controlled to yield an emergent pattern arising through a sharp transition in the form of a bifurcation. The propagation of rumors or of opinions is the most classical example in this area, but in recent years some further unexpected possibilities have been suggested, such as the genesis of a phonological system in a human society. Ordinarily, the inherent capacity of humans to emit and recognize sounds and to attribute them to objects is advanced as the most plausible mechanism of this process. On the other hand, consider a population of N individuals capable to emit M sounds to designate a given object. When two individuals pronouncing sounds i and j meet, each one of them can convince, with certain probabilities, the other that his sound is more appropriate to designate the object. This switches on a cooperativity in the process of competition between the options available very similar to that between the two trails in Fig. 1.8a, leading to the choice of one of them by the overwhelming part of the population (being understood that N is large enough). This scenario opens interesting perspectives, which need to be implemented by linguistic analyses and real-time experiments.

Competition between different options is also expected to underlie the origin of a variety of spatial patterns and organizational modes observed in human systems. An example is provided by the formation and the evolution of urban structures, as certain areas specialize in specific economic activities and as residential differentiation produces neighborhoods differing in their living conditions and access to jobs and services. In many cases this occurs as a spontaneous process of endogenous origin. In addition to this evolutionary scenario central planning may be present as well and provide a bias in the individual decision making. It is, however, most unlikely that under present conditions it will supersede the bottom-up mechanism operating in complex systems: the chance of a modern Deinokrates or a modern Constantine the Great designing from scratch an Alexandria or a Constantinople-like structure are nowadays practically nil.

It is, perhaps, in the domain of economic and financial activities that the specificity of the human system finds its most characteristic expression. In addition to steps involving self-organization and emergence through bifurcation one witnesses here the entrance in force of the second fingerprint of complexity, namely, the intertwining of order and disorder. This raises in turn the problem of prediction in a most acute manner. The economics of the stock market provides a striking example. On October 19, 1987 the Dow Jones index of New York stock exchange dropped by 22.6%. This drop, the highest registered ever in a single day, was preceded by three other substantial ones on October 14, 15, 16. Impressive as they are, such violent phenomena are far from being unique: financial history is full of stock market crises such as the famous October 1929 one in which on two successive days the values were depreciated cumulatively by 23.1%.

The first reaction that comes to mind when witnessing these events is that of irrationality yet, much like in our discussion of subsection 1.4.2, the evidence supports on the contrary the idea of perfectly rational attitudes being at work. Ideally, in a market a price should be established by estimating the capacity of a company to make benefits which depends in turn on readily available objective data such as its technological potential, its developmental strategy, its current economic health and the quality of its staff. In reality, observing the market one realizes that for a given investor these objective criteria are in many instances superseded by observing the evolution of the index in the past and, especially, by watching closely the attitude of the other investors at the very moment of action. This may lead to strong cooperative effects in which a price results in from an attitude adopted at a certain time, and is subsequently affecting (e.g. reinforcing) this very attitude (which was perhaps initially randomly generated). As a matter of fact this largely endogenous mechanism seems to be operating not only during major crises but also under "normal" conditions, as illustrated by Fig. 1.9 in which the "real" (full line) versus the "objective" (dashed line) value of a certain product in the New York stock exchange is depicted for a period of about 50 years. It may result in paradoxical effects such as the increase of

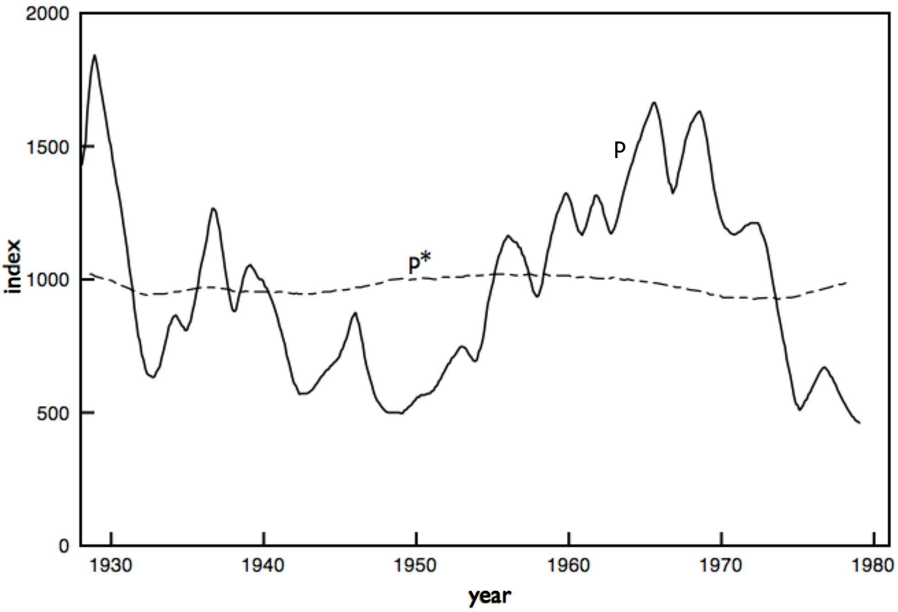

Fig. 1.9. Dow Jones industrial average p and a posteriori estimated rational price p^* of the New York stock market during the period 1928 to 1979. Raw data have been detrended by dividing by the systematic growth factor.

a certain value merely because the investors anticipate at a certain moment that this is indeed going to happen, though it has not happened yet! In this logic the product that is supposed to guarantee this high value might even be inferior to others, less well quoted ones. That such a priori unexpected events actually occur with appreciable probability is reminiscent of the comments made in subsections 1.4.1 and 1.4.3 in connection with the emergence of Rayleigh-Bénard cells and pheromone trails. It suggests that key manifestations of economic activities are the result of constraints acting on the system and activating intrinsic nonlinearities, as a result of which the concept of economic equilibrium often becomes irrelevant. Of equal importance is also the variability of the individual agents, reflected by the presence of different goals and strategies amongst them (cf. also Sec. 3.7).

It is important to realize that the speculative character of the process underlying Fig. 1.9 coexists with regular trends reflected by the generally admitted existence of economic cycles. While the latter are manifested on a rather long time scale, the behavior on a wide range covering short to intermediate scales seems rather to share the features of a scale free process. Again the situation looks similar in this respect to that encountered in the previous subsections. An analysis of the range of variability normalized by its standard deviation confirms this, with Hurst exponents H close to 0.5 for products most easily subject to speculation, and higher for products that are less negotiable. As mentioned in connection with subsection 1.4.2 this implies that the corresponding processes are, respectively, uncorrelated and subjected to long range correlations.

An alternative view of financial fluctuations is provided by the construction of their histograms from the available data. Let P_t be the present price of a given stock. The stock price return r_t is defined as the change of the logarithm of the stock price in a given time interval Δt, $r_t = \ln P_t - \ln P_{t-\Delta t}$. The probability that a return is (in absolute value) larger than x is found empirically to be a power law of the form

$$P(|r_t| > x) \approx x^{-\gamma_t} \tag{1.1}$$

with $\gamma_t \approx 3$. This law which belongs to the family of probability distributions known as Pareto distributions holds for about 80 stocks with Δt ranging from one minute to one month, for different time periods and for different sizes of stocks. It may thus be qualified as "universal" in this precise sense. The scale invariant (in Δt and in size) behavior that it predicts in the above range suggests that large deviations can occur with appreciable probability, much more appreciable from what would be predicted by an exponential or a Gaussian distribution. As a matter of fact such dramatic events as the 1929 and 1987 market crashes conform to this law. Surprisingly, Pareto's law seems also to describe the distribution of incomes of individuals in a country, with an exponent that is now close to 1.5.

In an at first sight quite different context, power laws concomitant to self-similarity and scale free behavior are also present whenever one attempts to rank objects according to a certain criterion and counts how the frequency of their occurrence depends on the rank. For instance, if the cities of a given country are ranked by the integers 1, 2, 3,... according to the decreasing order of population size, then according to an empirical discovery by George Zipf the fraction of people living in the nth city varies roughly as

$$P(n) \approx n^{-1} \tag{1.2}$$

Zipf has found a similar law for the frequency of appearance of words in the English prose, where $P(n)$ represents now the relative frequency of the nth most frequent word ("the", "of", "and" and "to" being the four successively more used words in a ranking that extends to 10 000 or so).

Eq. (1.2) is parameter free, and on these grounds one might be tempted to infer that it applies universally to all populations and to all languages. Benoît Mandelbrot has shown that this is not the case and proposed a two-parameter extension of Zipf's law accounting for the differences between subjects and languages, in the form

$$P(n) \approx (n + n_0)^{-B} \tag{1.3}$$

where n_0 plays the role of a cutoff.

1.5 Summing up

The fundamental laws of nature governing the structure of the building blocks of matter and their interactions are deterministic: a system whose state is initially fully specified will follow a unique course. Yet throughout this chapter we have been stressing multiplicity as the principal manifestation of complexity; and have found it natural -and necessary- to switch continuously on many occasions between the deterministic description of phenomena and a probabilistic view.

Far from reflecting the danger of being caught in a contradiction already at the very start of this book this opposition actually signals what is going to become the leitmotiv of the chapters to come, namely, that when the fundamental laws of nature are implemented on complex systems the deterministic and the probabilistic dimensions become two facets of the same reality: because of the limited predictability of complex systems in the sense of the traditional description of phenomena one is forced to adopt an alternative view, and the probabilistic description offers precisely the possibility to sort out regularities of a new kind; but on the other side, far from being applied in a heuristic manner in which observations are forced to fit certain a priori laws imported from traditional statistics, the probabilistic description one is dealing with here is intrinsic in the sense that it is generated by the underlying dynamics. Depending on the scale of the phenomenon, a complex system may have to develop mechanisms for controlling randomness in order to sustain a global behavioral pattern thereby behaving deterministically or, on the contrary, to thrive on randomness in order to acquire transiently the variability and flexibility needed for its evolution between two such configurations.

Similarly to the determinism versus randomness, the structure versus dynamics dualism is also fading as our understanding of complex systems is improving. Complex systems shape in many respects the geometry of the space in which they are embedded, through the dynamical processes that they generate. This intertwining can occur on the laboratory time scale as in the Rayleigh-Bénard cells and the pheromone trails (1.4.1, 1.4.3); or on the much longer scale of geological or biological evolution, as in e.g. the composition of the earth's atmosphere or the structure of biomolecules.

Complexity is the conjunction of several properties and, because of this, no single formal definition doing justice to its multiple facets and manifestations can be proposed at this stage. In the subsequent chapters a multilevel approach capable of accounting for these diverse, yet tightly intertwined elements will be developed. The question of complexity definition(s) will be taken up again in the end of Chapter 4.