Chapter 3

Passage of Radiation Through Matter

In everyday life we constantly use our understanding of the passage of matter through matter. We do not try to walk through a closed steel door, but we brush through if the passage is only barred by a curtain. We stroll through a meadow full of tall grass but carefully avoid a field of cacti. Difficulties arise if we do not realize the appropriate laws; for example, driving on the right-hand side of a road in England or Japan can lead to disaster. Similarly, a knowledge of the passage of radiation through matter is a crucial part in the design and the evaluation of experiments. The present understanding has not come without surprises and accidents. The early X-ray pioneers burned their hands and their bodies; many of the early cyclotron physicists had cataracts. It took many years before the exceedingly small interaction of the neutrino with matter was experimentally observed because it can pass through a light year of matter with only small attenuation. Then there was the old cosmotron beam at Brookhaven which was accidentally found a few km away from the accelerator, merrily traveling down Long Island.

The passage of charged particles and of photons through matter is governed primarily by *atomic* physics. True, some interactions with nuclei occur. However, the main energy loss and the main scattering effects come from the interaction with the atomic electrons. We shall therefore give few details and no theoretical derivations in the present chapter but shall summarize the important concepts and equations.

3.1 Concepts

Consider a well-collimated beam of monoenergetic particles passing through a slab of matter. The properties of the beam after passage depend on the nature of the particles and of the slab, and we first consider two extreme cases, both of great interest. In the first case, shown in Fig. 3.1(a), a particle undergoes many interactions. In each interaction, it loses a small amount of energy and suffers a small-angle scattering. In the second, shown in Fig. 3.1(b), the particle either passes unscathed through the slab or it is eliminated from the beam in one “deadly” encounter. The first case applies, for instance, to heavy charged particles, and the

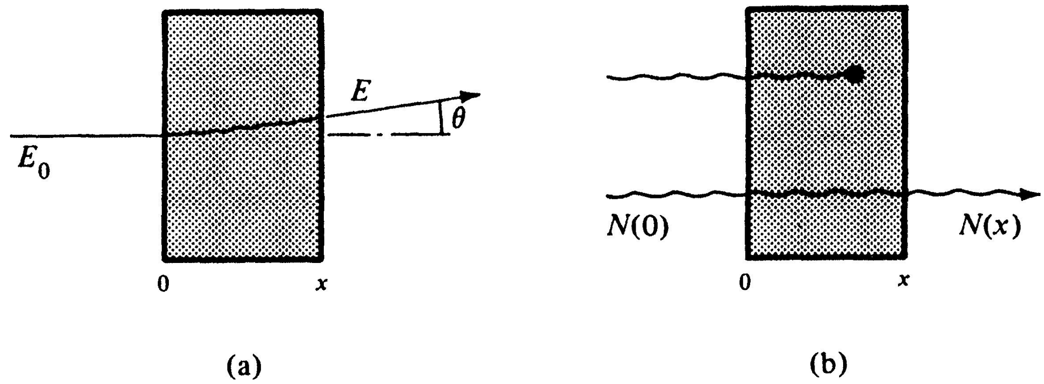

Figure 3.1: Passage of a well-collimated beam through a slab. In (a), each particle suffers many interactions; in (b), a particle is either unharmed or eliminated.

second one approximates the behavior of photons. (Electrons form an intermediate case.) We shall now discuss the two cases in more detail.

Many Small Interactions. Each interaction produces an energy loss and a deflection. Losses and deflections add up statistically. After passing through an absorber the beam will be degraded in energy, will no longer be monoenergetic, and will show an angular spread. Characteristics of the beam before and after passage are shown in Fig. 3.2. The number of particles left in the beam can be observed as a function of the absorber thickness x. Up to a certain thickness, essentially all particles will be transmitted. At some thickness, some of the particles will no longer emerge; at a thickness R_0, called the mean range, half of the particles will be stopped, and finally, at sufficiently large thickness, no particles will emerge. The behavior of the number of transmitted particles versus absorber thickness is shown in Fig. 3.3. The fluctuation in range is called range straggling.

"All-or-Nothing" Interactions. If an interaction eliminates the particle from the beam, the characteristics of the transmitted beam are different from the one just discussed. Since the transmitted particles have not undergone an interaction, the transmitted beam has the same energy and angular spread as the incident one. In each elementary slab of thickness dx the number of particles undergoing interactions is proportional to the number of incident particles, and the coefficient of proportionality is called the absorption coefficient μ:

$$dN = -N(x)\mu dx.$$

Integration gives

$$N(x) = N(0)e^{-\mu x}. \tag{3.1}$$

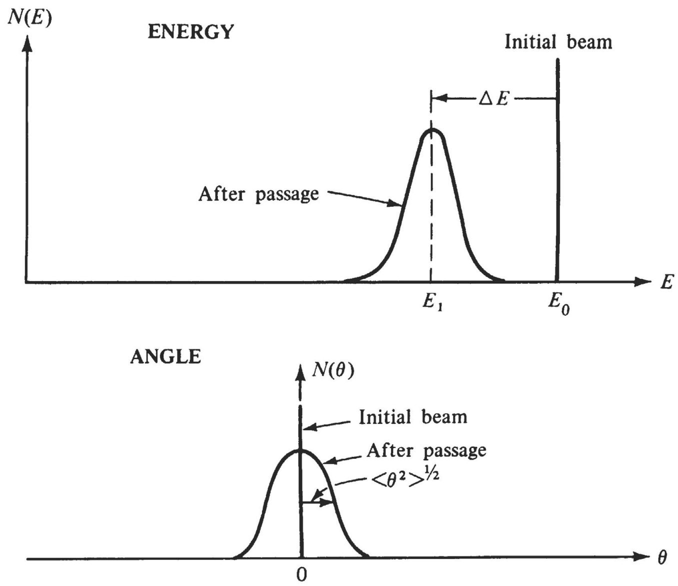

Figure 3.2: Energy and angular distribution of a beam of heavy charged particles before and after passing through an absorber.

The number of transmitted particles decreases exponentially, as indicated in Fig. 3.4. No range can be defined, but the *average distance* traveled by a particle before undergoing a collision is called the *mean free path*, and it is equal to $1/\mu$.

3.2 Heavy Charged Particles

Heavy charged particles lose energy mainly through collisions with bound electrons via Coulomb interactions. The electrons can be lifted to higher discrete energy levels (excitation), or they can be ejected from the atom (ionization). Ionization dominates if the particle has an energy large compared to atomic binding energies. The rate of energy loss due to collisions with electrons has been calculated classically by Bohr and quantum mechanically by Bethe and by Bloch.[(1)] The result, called the Bethe equation, is

$$-\frac{dE}{dx} = \frac{4\pi n z^2 Z^2 e^4}{m_e v^2}\left[\ln\frac{2m_e v^2}{I[1-(v/c)^2]} - \left(\frac{v}{c}\right)^2\right]. \tag{3.2}$$

Here $-dE$ is the energy lost in a distance dx, n the number of electrons per cm^3 in the stopping substance and Z its atomic number; m_e the electron mass; ze the charge and v the speed of the particle and I is the mean excitation potential of the atoms of the stopping substance. (Eq. (3.2) is an approximation, but it suffices for our purpose.)

[1]N. Bohr, *Phil. Mag.* **25**, 10 (1913); H. A. Bethe, *Ann. Physik* **5**, 325 (1930); F. Bloch, *Ann. Physik* **16**, 285 (1933).

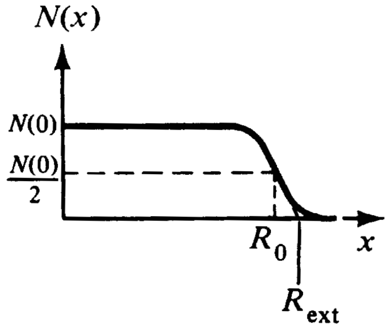

Figure 3.3: Range of heavy charged particles. $N(x)$ is the number of particles passing through an absorber of thickness x. R_0 is the mean range; R_{ext} is called the extrapolated range.

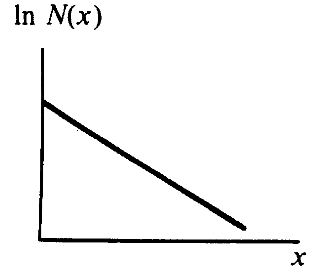

Figure 3.4: In all-or-nothing interactions, the number of transmitted particles, $N(x)$, decreases exponentially with the absorber thickness x.

In practical applications, the thickness of an absorber is not measured in length units but in terms of ρx, where ρ is the density of the absorber. ρx is usually given in g/cm^2, and it can be found experimentally by determining the mass and the area of the absorber and taking the ratio of the two. The specific energy loss tabulated or plotted is then

$$\frac{dE}{d(\rho x)} = \frac{1}{\rho}\frac{dE}{dx}.$$

Figure 3.5 gives the specific energy loss of protons, pions, and muons in several materials as a function of the momentum p. Figure 3.5 and Eq. (3.2) show the salient features of the energy loss of heavy particles in matter clearly. The specific energy loss is proportional to the number of electrons in the absorber and proportional to the *square* of the particle charge. At a certain energy, for protons about 1 GeV, an *ionization minimum* occurs. Below the minimum, $dE/d(\rho x)$ is proportional to $1/v^2$. Consequently, as a nonrelativistic particle slows down in matter, its energy loss increases. However, Eq. (3.2) breaks down when the particle speed becomes comparable to, or less than, the speed of the electrons in the atoms. The energy loss then decreases again, and the curves in Fig. 3.5 turn down below about 1 MeV. Above the ionization minimum, $dE/d(\rho x)$ increases slowly. It is often useful to remember that the energy loss at the minimum and for at least two decades above is about the same for all materials and that it is of the order

$$-\frac{dE}{d(\rho x)}(\text{at minimum}) \approx 1.6z^2 \text{ MeV/g cm}^{-2}. \tag{3.3}$$

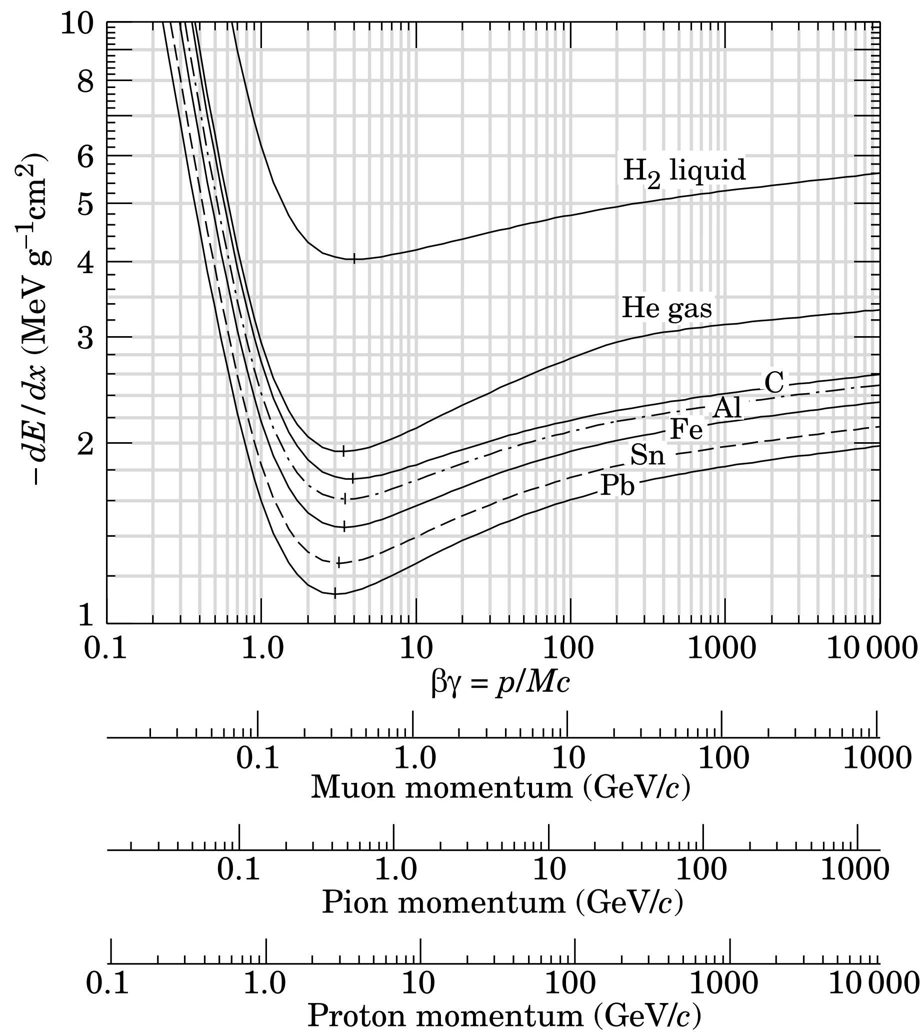

Figure 3.5: Specific energy loss, $dE/d(\rho x)$, for protons, pions, and muons in several materials.[From PDG.]

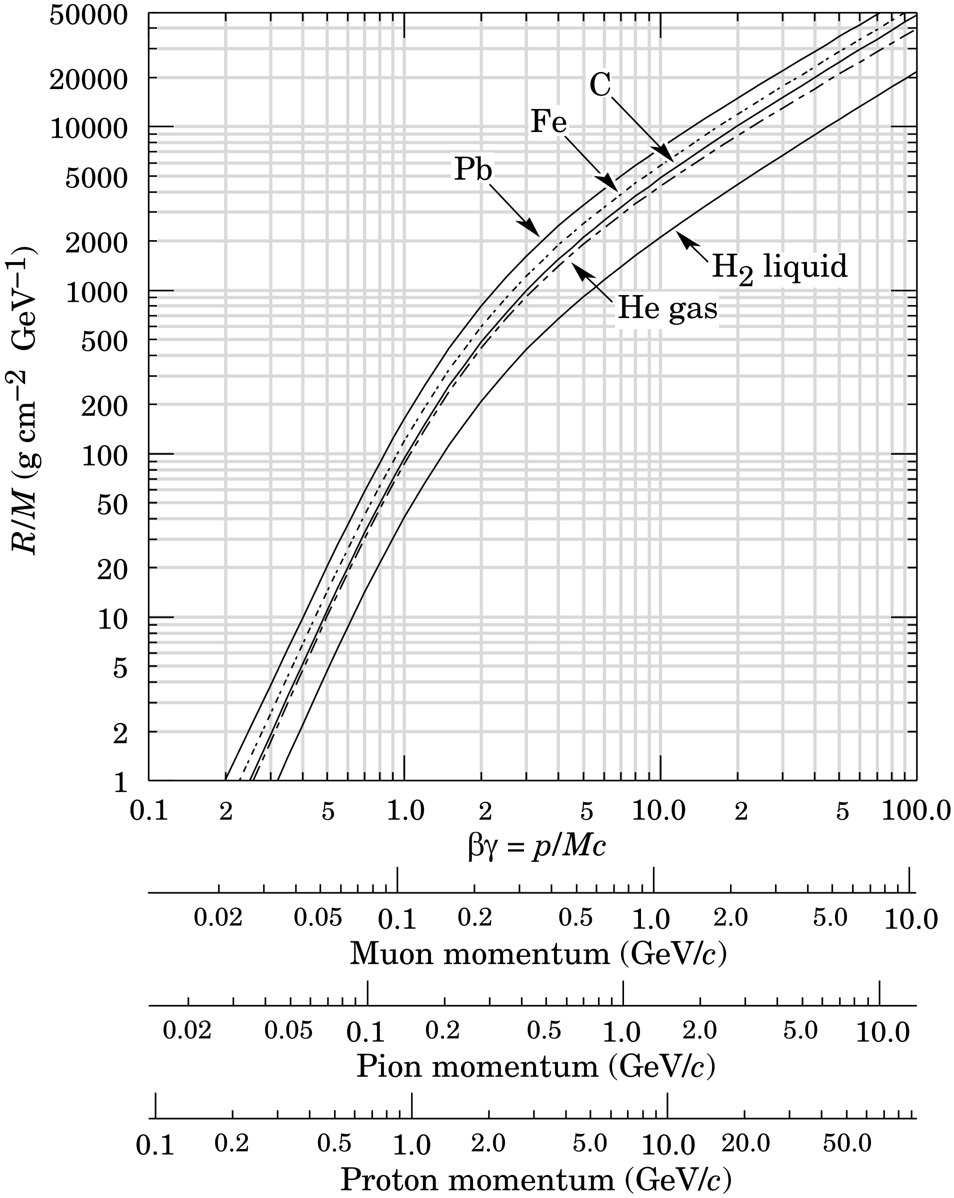

Figure 3.6: Range of particles in liquid hydrogen (bubble chamber), helium gas, carbon, iron, and lead. For example, for a pion of momentum 230 MeV/c, $\beta\gamma = 1.4$. For lead we read $R/M \approx$ 400 g cm^{-2} GeV^{-1}, and so the range is ≈ 56 g cm^{-2}. [From PDG.]

Equation (3.2) also shows that the specific energy loss does not depend on the mass of the particle (provided it is much heavier than the electron) but only on its charge and speed. The curves in Fig. 3.5 therefore are valid also for particles other than the protons if the energy scale is appropriately shifted.

The *range* of a particle in a given substance is obtained from Eq. (3.2) by integration:

$$R = \int_{T_0}^{0} \frac{dT}{(dT/dx)}. \tag{3.4}$$

Here T is the kinetic energy and the subscript 0 refers to the initial value. Some useful information concerning range and specific energy loss is summarized in Fig. 3.6.

Two more quantities shown in Fig. 3.2, the spread in energy and the spread in angle, are important in experiments, but they are not essential for a first view of the subatomic world. We shall therefore not discuss them here; the relevant information can be found in the references given in Section 3.6.

3.3 Photons

Photons interact with matter chiefly by three processes:

1. Photoelectric effect.
2. Compton effect.
3. Pair production.

A complete treatment of the three processes is rather complicated and requires the tools of quantum electrodynamics. The essential facts, however, are simple. In the photoelectric effect, the photon is absorbed by an atom, and an electron from one of the shells is ejected. In the Compton effect, the photon scatters from an atomic electron. In pair production, the photon is converted into an electron–positron pair. This process is impossible in free space because energy and momentum cannot be conserved simultaneously when a photon decays into two massive particles. It occurs in the Coulomb field of a nucleus which is needed to balance energy and momentum.

The energy dependences of processes 1–3 are very different. At low energies, below tens of keV, the photoelectric effect dominates (which accounts for the sharp edges), the Compton effect is small, and pair production is energetically impossible. At an energy of $2m_ec^2$, pair production becomes possible, and it soon dominates completely. Two of the three processes, photoelectric effect and pair production, eliminate the photons undergoing interaction. In Compton scattering, the scattered photon is degraded in energy. The all-or-nothing situation described in Section 3.1 and depicted in Fig. 3.1(b) is therefore a good approximation, and the transmitted beam should show an exponential behavior, as described by Eq. (3.1). The absorption coefficient μ is a sum of three terms,

$$\mu = \mu_{\text{photo}} + \mu_{\text{Compton}} + \mu_{\text{pair}} \tag{3.5}$$

and each term can be computed accurately.

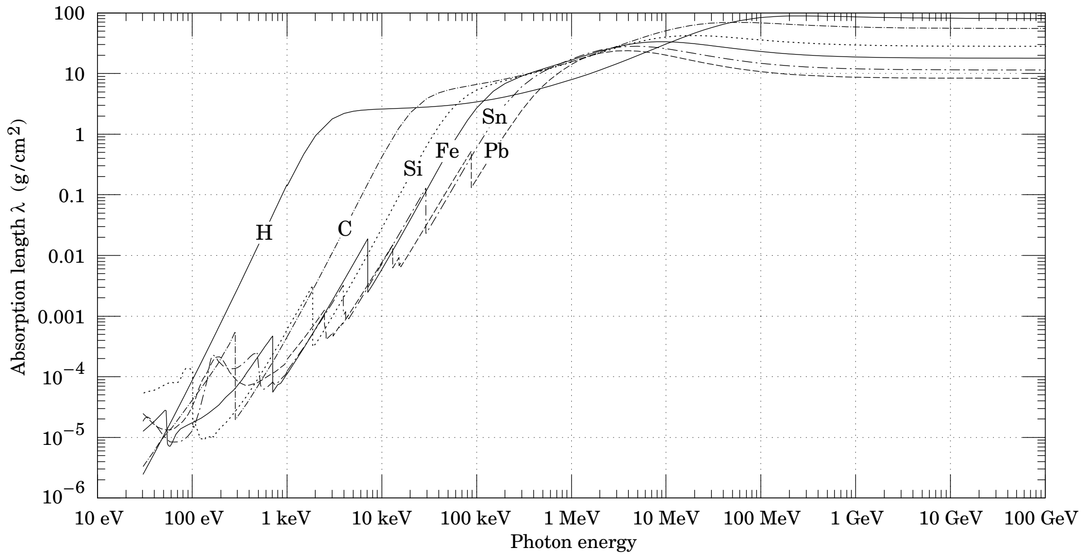

Figure 3.7: Mean free path ($\lambda = \rho/\mu$) versus photon energy. [From PDG.]

3.4 Electrons

The energy-loss mechanism of electrons differs from that of heavier charged particles for several reasons. The most important difference is energy loss by radiation; this mechanism is unimportant for heavy particles but dominant for high-energy electrons. Radiation makes it necessary to consider two energy regions separately. At energies well below the *critical energy* E_c, given approximately by

$$E_c \approx \frac{600 \text{ MeV}}{Z}, \tag{3.6}$$

excitation and ionization of the bound absorber electrons dominate. [In Eq. (3.6), Z is the charge number of the absorber's atoms.] Above the critical energy, radiation loss takes over. We shall treat the two regions separately.

Ionization Region ($E < E_c$) In this region, the energy loss of an electron and a proton of equal speed are nearly the same and Eq. (3.2) can be taken over with some small modifications. There is, however, one major difference, as sketched in Fig. 3.8. The path of the heavy particle is straight and the $N(x)$ against x curve is as given in Fig. 3.3. The electron, owing to its small mass, suffers many scatterings with considerable angles. The behavior of the number of transmitted electrons versus absorber thickness is sketched in Fig. 3.8. An extrapolated range R_p is defined as shown in Fig. 3.8. Between about 0.6 and 12 MeV the extrapolated range in aluminum is well represented by the linear relation

$$R_p(\text{in g/cm}^2) = 0.526 E_{\text{kin}}(\text{in MeV}) - 0.094. \tag{3.7}$$

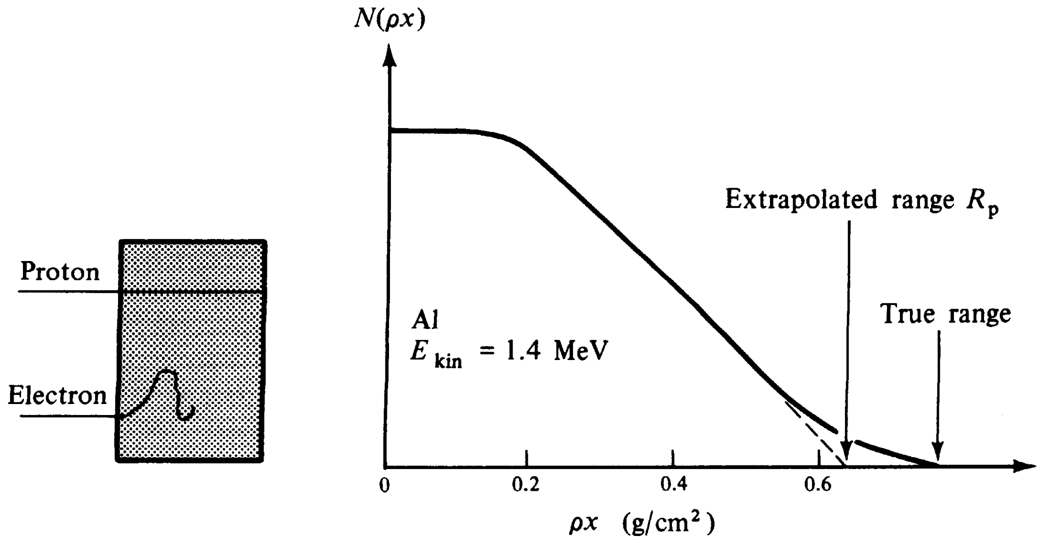

Figure 3.8: Passage of a proton and an electron with equal total pathlength through an absorber. The $N(x)$ against x behavior for electrons is given at right.

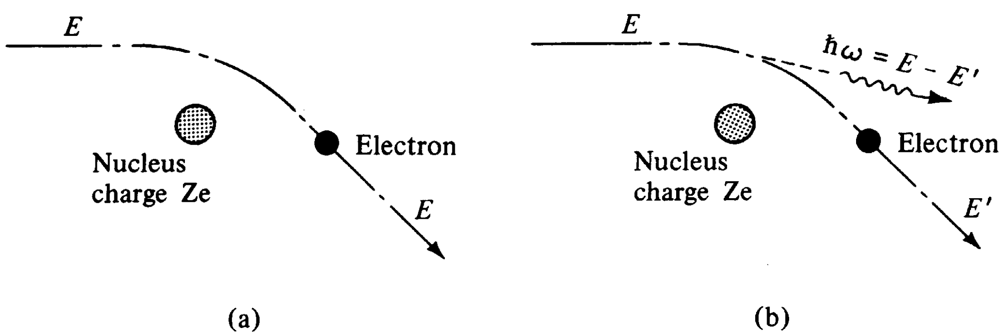

Figure 3.9: Coulomb scattering. (a) Elastic scattering. (b) The accelerated electron radiates and loses energy in the form of a photon (Bremsstrahlung).

Radiation Region ($E > E_c$) A charged particle passing by a nucleus of charge Ze experiences the Coulomb force and it is deflected (Fig. 3.9(a)). The process is called *Coulomb scattering.* The deflection accelerates (decelerates) the passing particle. As pointed out in Section 2.6, acceleration produces radiation. In the case of electrons in a synchrotron, it is called *synchrotron radiation;* in the case of charged particles scattered in the Coulomb field of nuclei, it is called *Bremsstrahlung* (braking radiation). Equations (2.21) and (2.22) show that, for equal acceleration, the energy carried away by photons will be proportional to $(E/mc^2)^4$. Bremsstrahlung is thus an important energy-loss mechanism for electrons, but it is very small for heavier particles, such as muons, pions, and protons.

Table 3.1: VALUES OF THE CRITICAL ENERGY E_c AND THE RADIATION LENGTH X_0 FOR VARIOUS SUBSTANCES.

Material	Z	Density (g/cm^3)	Critical Energy (MeV)	Radiation Length (g/cm^2)	Radiation Length (cm)
H_2 (liquid)	1	0.071	340	62.8	887
He (liquid)	2	0.125	220	93.1	745
C	6	1.5	103	43.3	28
Al	13	2.70	47	24.3	9.00
Fe	26	7.87	24	13.9	1.77
Pb	82	11.35	6.9	6.4	0.56
Air		0.0012	83	37.2	30870
Water		1	93	36.4	36.4

Actually, Eq. (2.21) has been calculated by using classical electrodynamics. Bremsstrahlung, however, must be treated quantum mechanically. Bethe and Heitler have done so, and the essential results are as follows.[(2)] The number of photons with energies between $\hbar\omega$ and $\hbar(\omega + d\omega)$ produced by an electron of energy E in the field of a nucleus with charge Ze is proportional to Z^2/ω:

$$N(\omega)d\omega \propto Z^2 \frac{d\omega}{\omega}. \tag{3.8}$$

Owing to the emission of these photons, the electron loses energy, and the distance over which its energy is reduced by a factor e is called the *radiation* or *attenuation length* and conventionally denoted by X_0. In terms of X_0, the radiative energy loss for large electron energies is

$$-\left(\frac{dE}{dx}\right)_{\text{rad}} \approx \frac{E}{X_0} \quad \text{or} \quad E = E_0 e^{-x/X_0}. \tag{3.9}$$

The radiation length is given either in g/cm^2 or in cm; a few values of X_0 and of the critical energy E_c are given in Table 3.1.

According to Eq. (3.9), a highly energetic electron loses its energy exponentially and after about seven radiation lengths has only 10^{-3} of its initial energy left. However, concentrating on the primary electron is misleading. Many of the Bremsstrahlung photons have energies greatly in excess of 1 MeV and can produce electron-positron pairs (Section 3.3). In fact, the mean free path, that is, the average distance, X_p, traveled by a photon before it produces a pair, is also related to the radiation length:

$$X_p = \frac{9}{7} X_0. \tag{3.10}$$

[2] H. A. Bethe and W. Heitler, *Proc. R. Soc. (London)* **A146**, 83 (1934).

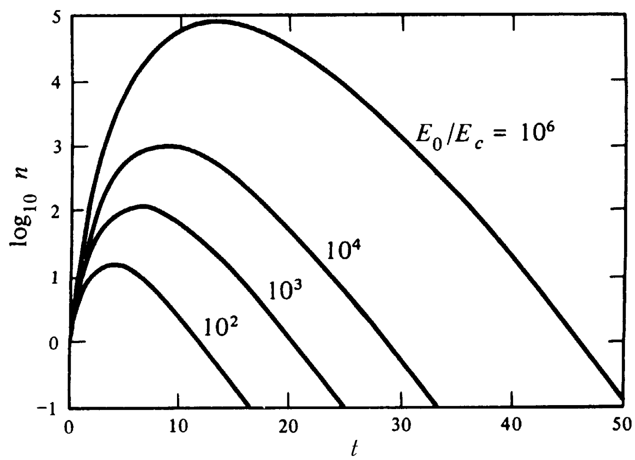

Figure 3.10: Number n of electrons in a shower as a function of the thickness traversed, t, in radiation lengths. [These curves were taken from the work of B. Rossi and K. Greisen, *Rev. Modern Phys.* **13**, 240 (1941).]

In successive steps, a high-energy electron creates a *shower*. (Of course a shower can also be initiated by a photon.) The detailed theory of such a shower is very complicated and in practice computer calculations are performed. Figure 3.10 shows the number n of electrons in a shower as a function of the thickness of the absorber. The energy E_0 of the incident electron is measured in units of the critical energy; the thickness is expressed in units of the radiation length X_0. Figure 3.10 expresses the development and death of a shower: The increase in the number of electrons is very rapid at the beginning. As the cascade progresses, the average energy per electron (or per photon) becomes smaller. At some point it becomes so small that the photons can no longer produce pairs, and the shower dies.

3.5 Nuclear Interactions

If the passage of particles through matter were governed entirely by the phenomena described in Sections 3.1–3.4, neutral particles would pass through matter without being affected, and muons and protons of the same energy would nearly have the same range. The facts, however, are different; the electrically neutral neutrons have a strong short-distance interaction with matter, and high-energy protons have a much shorter range than muons. The reason for this behavior, and for the discrepancy between naive expectation and reality, is the neglect of nuclear interactions. The treatment in Sections 3.1–3.4 is based entirely on the electromagnetic interaction, and nonelectromagnetic forces between the nucleus and the passing particle are neglected. These interactions, the hadronic and the weak ones, form the central topic of subatomic physics and they will be explored and described in the following parts.

3.6 References

The basic ideas underlying the computation of the energy loss of charged particles in matter are described lucidly in N. Bohr, "Penetration of Atomic Particles Through Matter," *Kgl. Danske Videnskab. Selskab Mat-fys Medd.* **XVIII**, No. 8 (1948), and in E. Fermi, *Nuclear Physics*, notes compiled by J. Orear, A. H. Rosenfeld, and R. A. Schluter, University of Chicago Press, Chicago, (1950); J.F. Ziegler, J.P. Biersack, and W. Littmark, *Stopping Powers and Ranges* Pergamon Press, New York, 1985; M.A. Kumak and E.F. Komarov, *Radiation from Charged Particles in Solids*, transl. G. Kurizki, Amer. Inst. Phys., New York, 1989; see also PDG for an up-to-date review and further references.

Problems

3.1. An accelerator produces a beam of protons with kinetic energy of 100 MeV. For a particular experiment, a proton energy of 50 MeV is required. Compute the thickness of

(a) a carbon and

(b) a lead absorber,

both in cm and in g/cm^2, necessary to reduce the beam energy from 100 to 50 MeV. Which absorber would be preferable? Why?

3.2. A counter has to be placed in a muon beam of 100-MeV kinetic energy. No muons should reach the counter. How much copper is needed to stop all muons?

3.3. We have stated that the transmission of charged particles through matter is dominated by atomic, and not nuclear, interactions. When is this statement no longer true; i.e., when do nuclear interactions become important?

3.4. A beam stop is required at the end of accelerators to prevent the particles from running wild. How many meters of solid dirt would be required at FNAL to completely stop the 200 GeV protons, assuming only electromagnetic interactions? Why is the actual beam stop length less?

3.5. Cosmic-ray muons are still observed in mines that are more than 1 km underground. What is the minimum initial energy of these muons? Why are no cosmic-ray protons or pions observed in these underground laboratories?

3.6. Discuss and understand the simplest derivation of Eq. (3.2).

3.7. Show that the mean free path of a particle undergoing exponential absorption as described by Eq. (3.1) is given by $1/\mu$.

3.8. A beam of 1-mA protons of kinetic energy of 800 MeV passes through a 1-cm^3 copper cube. Compute the maximum energy deposited per sec in the copper. Assume the cube to be thermally insulated, and compute the temperature rise per sec.

3.9. Compare the energy loss of nonrelativistic π^+, K^+, d, $^3He^{2+}$, $^4He^{2+} \equiv \alpha$ to that of protons of the same energy in the same material.

3.10. In an experiment, alpha particles of 200 MeV energy enter a scattering chamber through a copper foil that is 0.1 mm thick.

   (a) Use the form of Eq. (3.2) to find approximately the energy of the proton beam that has the same energy loss as the α beam.

   (b) Compute the energy loss.

3.11. Use Eq. (3.2) and Fig. 3.5 to sketch the ionization along the path of a heavy charged particle (Bragg curve).

3.12. Use Eq. (3.2) to calculate numerically the energy loss of a 20 MeV proton in aluminum ($I = 150$ eV).

3.13. A radioactive source emits gamma rays of 1.1 MeV energy. The intensity of these gamma rays must be reduced by a factor 10^4 by a lead container. How thick (in cm) must the container walls be?

3.14. ^{57}Fe has a gamma ray of 14 keV energy. A source is contained in a metal cylinder. It is desired that 99% of the gamma rays escape the cylinder. How thin must the walls be made if the cylinder is

   (a) Aluminum?

   (b) Lead?

3.15. A source emits gamma rays of 14 and 6 keV. The 6 keV gamma rays are 10 times more intense than the 14 keV rays. Select an absorber that cuts the intensity of the 6 keV rays by a factor of 10^3 but affects the 14 keV rays as little as possible. What is your choice? By what factor is the 14 keV intensity reduced?

3.16. The three processes discussed in Section 3.3 are not the only interactions of photons. List and briefly discuss other types of photon interactions.

3.17. A radioactive source contains two gamma rays of equal intensity with energies of 85 and 90 keV, respectively. Compute the intensity of the two gamma lines after passing through a 1 mm lead absorber. Explain your result.

3.18. Electrons of 1 MeV kinetic energy should be stopped in an aluminum absorber. How thick, in cm, must the absorber be?

3.19. What is the energy of an electron that has approximately the same total (true) pathlength as a 10 MeV proton?

3.20. An electron of 10^3GeV energy strikes the surface of the ocean. Describe the fate of the electron. What is the maximum number of electrons in the resulting shower? At which depth, in m, does the maximum occur?

3.21. A 10-GeV electron passes through a 1-cm aluminum plate. How much energy is lost?

3.22. Show that pair production is not possible without the presence of a nucleus to take up momentum.

3.23. Show that the maximum energy that can be transferred to an electron in a single collision by a non-relativistic particle of kinetic energy T and mass $M(M \gg m_e)$ is $(4m_e/M)T$.