

Chapter 1

IN MEMORY OF PROFESSOR S. S. CHERN

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The year was 1930 and I was just turning eight. It was autumn when Mr Shiing-Shen Chern (陈省身) entered a Ph.D program in Mathematical Science at Tsinghua University. Incidentally, my father, Mr Yang Ko-Chuen (杨克纯), was a professor in the mathematics department and my family were living on the campus. I remember seeing Mr Chern on a few occasions. He often came by our home while I was in primary four. Many years later, Mr Chern and I carved out our separate careers in science. But as fate would have it, both of us finally came full circle and “reunited” once again in the academic world. Today, I would like to dedicate this piece to the memory of Mr Chern. Looking back, it suddenly struck me that all those years when we were running the marathon of our dreams, even though the routes we took were very different, we were in fact moving towards the same goal.

After graduation, Mr Chern went to Europe to further his studies. He returned home in 1937 and became a mathematics Professor at Southwest Associated University in Kunming. I was an undergraduate physics student in 1938–1942. Mr Chern was a young professor and was popular with the students. I attended his classes on differential geometry. He was a very serious teacher and his lessons were very well-structured and presented. I especially enjoyed his lessons on the geometry of curves and surfaces. But what I could not forget were the little things. Like the time I tried to prove the existence of conformal relations between any 2 dimensional geometry and the flat plane. Though I was able to convert metric tensor to $A^2 d\mu^2 + B^2 d\omega^2$, I was unable to prove that A and B could be made equal. After numerous attempts, I was getting really frustrated. It was Mr Chern who

gave me the missing piece of the puzzle — complex variables. The problem was solved instantly. I will never forget that feeling of sudden illumination.

In 1943, Mr Chern was offered a research position at the Institute for Advanced Study in Princeton. During his two-year stint there, he successfully led research on differential geometry to a whole new level. And it swiftly became one of the major components of basic mathematics in the twentieth century.

Mr Chern became a professor at the University of Chicago in 1949. His next stop was Berkeley in the early 1960s. During the sixties and seventies, the two of us met up quite often in Chicago, Princeton and Berkeley. I was also a regular stay-in guest at his house in the city of El Cerrito which was just located north of Berkeley. Needless to say, I knew his family very well. He and his wife, Shih-ning Cheng (郑士宁), had a son Paul (陈伯龙) and a daughter May (陈璞). Mr Chern and I talked about practically everything under the sun — friends, relatives, family matters or politics, you name it. Even though I knew Mr Chern was widely regarded as one of the greatest mathematician of the twentieth century, the two of us hardly ever discussed our work.

Mathematics and physics used to share a close relationship aeons ago. But all of this has changed since the mid-nineteenth century. As mathematicians and physicists gradually move towards cutting-edge R&D initiatives, the gap between the two has widened and they have eventually become independent of each other. In “The Revolution in Mathematics” which was published in the *American Mathematical Monthly* 68, the former chairman of the mathematics department of the University of Chicago, Professor Marshall Stone (1903–1989) wrote:

“... since 1900 in our conception of mathematics or in our view concerning it, the one which truly involves a revolution in ideas is the discovery that mathematics is entirely independent of the physical world.”

The article was first published in 1961. During the same period, Mr Chern was already regarded as the father of modern differential geometry. He was the first mathematician who succeeded in bridging local geometry and topology. But as Professor Stone suggested, it was a time when everybody believed that modern mathematics had nothing to do with “the physical world”.

But as it has turned out, what Professor Stone said could not be further from the truth. Exterior differential forms or fiber bundles is very much intertwined with gauge field theory of theoretical physics. It was during

the early seventies that I began to see the intricate link between the two disciplines. I had related my experience of this understanding in an article in 1980:

“In 1975, impressed with the fact that gauge fields are connections on fiber bundles, I drove to the house of Shiing-Shen Chern in El Cerrito near Berkeley. (I had taken courses with him in the early 1940’s when he was a young professor and I an undergraduate student at the National Southwest Associated University in Kunming, China. That was before fiber bundles had become important in differential geometry and before Chern had made history with his contributions to the generalized Gauss–Bonnet theorem and the Chern classes.) We had much to talk about: friends, relatives, China. When our conversation turned to fiber bundles, I told him that I had finally learned from Jim Simons the beauty of fiber-bundle theory and the profound Chern–Weil theorem. I said I found it amazing that gauge fields are exactly connections on fiber bundles, which the mathematicians developed *without reference to the physical world*. I added, “this is both thrilling and puzzling, since you mathematicians dreamed up these concepts out of nowhere.” He immediately protested, “No, no. These concepts were not dreamed up. They were natural and real.”

Why would the creator of the universe use “natural and real” concepts of abstract mathematics to create the physical world? I suppose this question will never be answered. But I penned a poem <In praise of the Chern class> in the seventies:

天衣岂无缝 匠心剪接成
 浑然归一体 广邃妙绝伦
 造化爱几何 四力纤维能
 千古寸心事 欧高黎嘉陈

As a matter of fact, the line “creator loves geometry” had already been stated by Einstein. After the 1970s this idea was gradually accepted widely by theoretical physicists.

Mr Chern once said that he would not be held responsible for what was said in my poem. It is true I did not consult him when I wrote the poem. But I do believe not only that there is a link between physics and geometry, that differential geometry holds the key to theoretical physics, but also that beyond the geometrical concepts that Mr Chern and I had helped to introduce into physics, there are more abstract, more intrinsic,

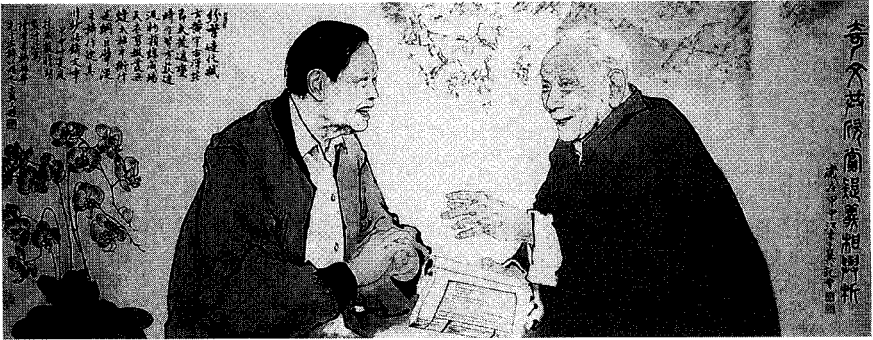


Fig. 1. Chinese artist Fan Zeng's impression of S. S. Chern and C. N. Yang.

but “natural and real” mathematical concepts, that are favored by the creator of the universe.

The renowned Chinese artist Fan Zeng is a good friend of both Mr Chern and me. He had presented a huge painting to Nankai University in the summer of 2004, with a poem (Figure 1):

纷繁造化赋玄黄
 宇宙浑茫即大荒
 递变时空皆有数
 迁流物类总成场
 天衣剪掇丛无缝
 太极平衡律是纲
 巨擘从来诗作魄
 真情妙语铸文章

Mr Fan's poetry and painting reminded me of a passage in an article written by Mr Chern in 1987 about our long relationship:

“Fifty odd years, from Lianda to Nankai, the creator was kind to us.”

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Translated by Tan Hwee Chiang