Chapter 1

A Day without Yesterday

> *Man is equally incapable of seeing the nothingness from which he emerges and the infinity in which he is engulfed.*
>
> Blaise Pascal, *Pensées* (1670)

> *Not only is the universe stranger than we imagine, it is stranger than we can imagine.*
>
> Sir Arthur Eddington (1882–1944)

1.1. Instant of Creation

In an instant of creation about 14 billion years ago the universe burst forth, *creating* space where there was no space, and time when there was no time. It was so hot and so dense that not even nucleons, the building blocks of atomic nuclei, had as yet formed. Until about 1/100,000th of a second all that existed was an intense fire and primitive particles called quarks and electrons and all their heavier kin together with their antiparticles; in a sense, these were the elementary particles conceived of by two early Greek philosophers of the fifth century BC, Leucippus and his disciple, Democritus. They evidently believed that an end must come to the reduction of matter into smaller parts, which they called *atomos*. Indeed, elementary particles that we call *quarks* have been discovered at last in our own time and all attempts to reduce them further have failed.

After that first brief moment, the quarks coalesced into neutrons, protons and other versions of these nucleons, never to be free again. In that inferno the lightest elements — deuterium and helium — were forged from the neutrons and protons during the next few minutes. These two *primordial elements* make up fully one-quarter of the mass in the universe today. Almost all the remaining mass is in that simplest element — hydrogen — consisting of one proton orbited by one electron. The hydrogen nuclei that were formed in the first fraction of a second are the same that pervade the universe now.

In the very earliest moments, there must have been a slight lumpiness in an otherwise uniformly expanding universe. As the expansion progressed, gravity acted upon those lumps, attracting surrounding matter — the primordial hydrogen and helium — into great tenuous clouds of chaotically swirling gases and the radiation that was trapped inside them. As the radiation slowly leaked from their surfaces and the clouds cooled, they began to collapse and fragment to form proto-galaxies, some slowly turning in one direction, others in another, so that in sum there was no rotation.

As the proto-galaxies collapsed further, conservation of their angular momentum caused them to whirl ever faster, just as an ice skater — who goes into a whirl with arms outstretched and draws them in — whirls faster. As each cloud fragment collapsed, it was flattened like a pancake with a central bulge by the increasingly rapid rotation, like the Andromeda Galaxy shown in Figure 1.1. At last a balance was reached between the force of gravity that would cause total collapse, and the centrifugal force of rotation, that would spin matter off into space. Because of instabilities within the thin disks, the clouds fragmented further to form stars, some larger than our Sun, and many more of them smaller.

When galaxies were first observed in the first century as faint nebulous objects in the sky it was not known that they were enormous collections of stars (Figure 1.2). Al Sufi was an astronomer in the court of the Emir in Persia. He observed the great galaxy, Andromeda, as a faint nebulous patch, which he called “little cloud” in his famous *Book of Fixed Stars* in 964 A.D. And he described other *nebulae*, some of which are not galaxies at all, but rather dense clouds of gas and dust in our own galaxy, the Milky Way. The Witch Head Nebula, shown in the preface, is one of many beautiful nebula, some of which have an appearance that reminds us of an animal, or of a mythological figure.

Not until the late 1700s did William Herschel speculate that some of the hazy patches that he could see among the stars with his telescope were actually “island universes” like our own galaxy, lying far outside it and containing vast numbers of stars. For his accomplishments, William Herschel was knighted by George III of England. His sister, Caroline, too (Figure 1.3) made many important discoveries and was the first woman to be recognized as an astronomer with a salary from the King. By the end of her long life she had reaped numerous honors including the gold medal of the Royal Astronomical Society (England).

OUR PLACE IN THE UNIVERSE

Fig. 1.1 The Andromeda Galaxy (M31), was first recorded by the Persian astronomer Al Sufi (903–986 A.D.) living in the court of Emir Adud ad-Daula. He described and depicted it in his *Book of Fixed Stars* (964 A.D.) and called it the "little cloud." The galaxy is composed of about 400 billion stars. It lies relatively close to our own galaxy, the Milky Way, and is rather similar, having a central bulge and flat disk in the form of spiral arms. Relatively close in this context means 2,900,000 light-years (or equivalently a thousand billion kilometers). The two galaxies are attracting each other and will collide and pass through each other, causing distortion of each. Eventually, they will merge. Two smaller elliptical galaxies M32 and M110 (the two bright spots outside the main galaxy) are in orbit about Andromeda. The myriad foreground stars are in our own galaxy. Credit: George Healy obtained this color image of spiral galaxy M31, the Great Galaxy in Andromeda, together with its smaller elliptical satellite galaxies.

But it was not until the 1900s that astronomers using more powerful telescopes were actually able to discover that the Milky Way contains about four hundred billion stars. It is a giant among galaxies having a mass of about 600 billion to a trillion times the mass of our Sun; it is exceeded in our neighborhood only by the Andromeda Galaxy, which is generally regarded as the most distant object visible to the human eye. At a distance of 2.2 million light-years, it appears as a fuzzy patch of light in the night sky.

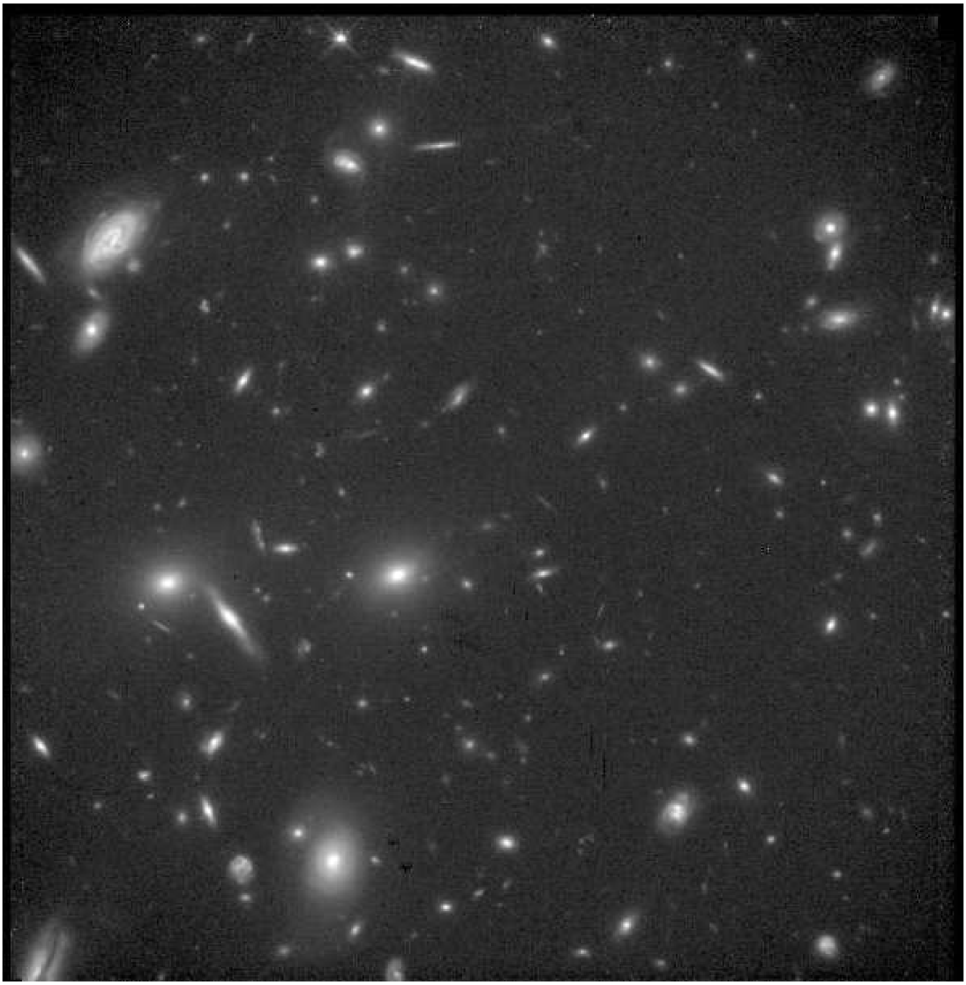

Fig. 1.2 Many galaxies, such as the one in Figure 1.1 are seen in this deep view of distant galaxies. Some are similar to our own, the spiral galaxies, seen at various angles. Others are spheroids or ellipsoids, called elliptical galaxies. Credit: NASA, ESA, and the Hubble Heritage Team (AURA/STScI).

But only the bright core of the galaxy can be seen by the eye. The full extent of the galaxy covers over 3 degrees of sky on its longest side. The Milky Way is falling towards our nearest large neighbor, the Andromeda Galaxy, M31, attracted by its great mass. In about 10 billion years the two systems will collide and merge. The end product of this merger is likely to be an elliptical or spherical galaxy with a very high concentration of stars and gas at its center, and very diffuse beyond, gradually fading into nothingness.

Fig. 1.3 Caroline Herschel (1750–1848), winner of many awards for her work in astronomy including the Gold Medal of the Royal Astronomical Society (England), and the Gold Medal of Science by the King of Prussia, first woman to be recognized and salaried as a scientist by King George III.

Ten percent of galaxies are estimated — based on modern observations — to possess planetary systems containing large Jupiter-like planets, and no doubt many others too small to detect, perhaps some like Earth. Besides our own Sun and its planets, there are 133 other known planetary systems in our galaxy, containing at least 156 planets in orbit around main sequence stars. It is very probable that there are many smaller planets in these systems, but it is easiest to detect massive planets because their effect on the host star is more apparent. Consequently, this number increases from year to year as more sensitive detection instruments are put into play.

At the center of our galaxy, as appears to be the case with others, lies an enormous black hole that is ingesting stars. It is hidden from our direct view by dust and surrounding stars spinning about it ever faster as they fall toward their fate. Our own Sun is far removed from this cannibalism. It lies in the disk of our Milky Way about two-thirds from its center. Although the Sun is moving at about 250 kilometers a second, the circumference of its orbit is so large that it has circled the center only 18 times since its birth 4.5 billion years ago.

The first generation of stars appeared several hundred million years *after the beginning.* They were supermassive — several hundred to a thousand times more massive than our Sun — and short-lived. Their intense gravity, owing to their large mass, soon crushed them, destroying atomic nuclei and sequestering a fraction of their nucleons inside a new star — a neutron star or even a black hole. Such events release enormous kinetic energy in a cataclysmic explosion called a *supernova.*

The most famous supernova was reported in 1054 to the emperor of the Sung dynasty by the imperial astronomer:

> "*I bow low. I have observed the apparition of a guest star.*
> *Its color was an iridescent yellow . . . .*"

Fearing the mighty emperor, he added a favorable interpretation: "The land will know great prosperity." This supernova was visible in daylight for 23 days in China. Anasazi Indian artists (who lived in present-day Arizona and New Mexico) created a pictograph that is thought to commemorate this amazing event; it accurately locates the position of the supernova in the night sky relative to the crescent moon which would have been its phase on the date of the explosion's appearance (Figure 1.4). Japanese historical records also noted the sudden appearance of a guest star.

This explosion, supernova 1054, created the Crab Nebula (Figure 1.5) that will be visible for thousands of years. It is a much-studied object, for it houses within it a *pulsar*, a neutron star with a mass nearly as much as our Sun; it is rotating 33 times a *second.* A very strong magnetic field is fixed in the star, which, because of the rotation, is pumping out energy like a dynamo with a luminosity of 100,000 Suns. This energy illuminates the Crab Nebula, and as well, has accelerated wisps of gas at its periphery to the very high velocity of about 1,500 kilometers per second.

The last star *observed* to explode in our own Galaxy produced the beautiful nebula called Cassiopeia in about 1667 (Figure 1.6). It is named for a mythological queen, the wife of Cepheus and the mother of Andromeda. Cassiopeia was a vain woman who thought herself more beautiful than the daughters of Nereus, god of the sea. To teach her humility, Cassiopeia was banned to the sky as a constellation of stars, hanging half of the time head downward (Figure 1.7).

The very light elements were made in the intense heat and high density that existed only in the first few minute in the life of the universe. The first traces of the all important elements for life — carbon, oxygen, and iron — were forged later in the first generation of stars. They were supermassive

Fig. 1.4 On July 4, 1054 A.D., inhabitants of Earth noticed a bright object in the sky, easily visible even in daylight — the supernova of 1054. This was the supernova which created the Crab Nebula. At that time the moon was a small crescent. Archaeologists believe the Anasazi Indians recorded the supernova event with this rock art panel in Chaco Canyon, New Mexico. Photo credit: Ron Lussier.

and appeared after several hundred million years. And the molecular dust that they cast off from their ferociously burning surfaces formed the seeds about which whole galaxies containing billions of stars eventually condensed.

The other elements were not made in significant abundance until later generations of stars — up to a few times our Sun's mass — first made their appearance. Such stars forged, and are forging, elements up to iron in their interiors but release them only upon their deaths in supernova explosions. Heavier elements like nickel and beyond are synthesized outside these stars when an intense wind of neutrinos created at the star's final collapse under gravity irradiates the hot gases that are expelled from the dying star.

In far-off galaxies, even today, supernova explosions occur, ejecting the store of elements that the exploding stars had synthesized during their lifetimes. Following countless of these starry deaths in our own galaxy, this

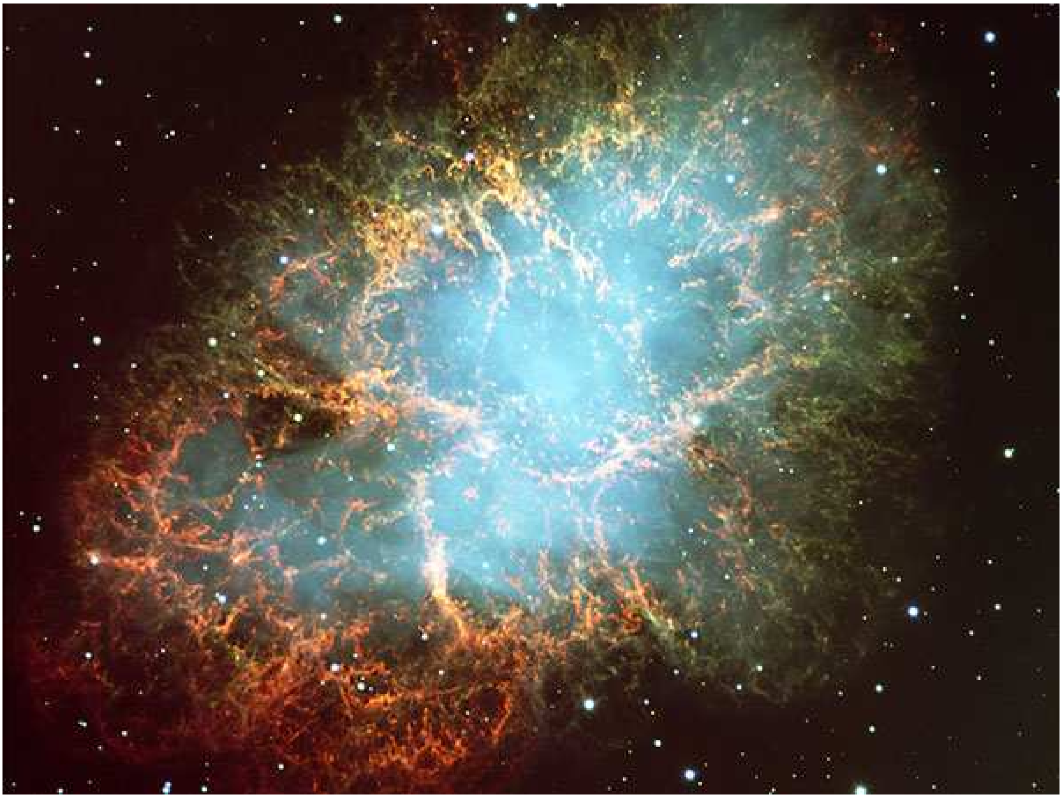

Fig. 1.5 The Crab Nebula, created by the supernova explosion of a star in 1054 and recorded by the court astronomer in China. The supernova remnant is expanding at 1,800 kilometers per second. It is about 10 light-years across. The source of the illumination is a rapidly rotating *neutron star* which has the luminosity of 100,000 Suns. Credit: European Southern Observatory/Very Large Telescope.

atomic debris has wandered for eons — mixing with that of other exploding stars — finally to coalesce to form our solar system — sun, planets, moons — and we, ourselves.

1.2. Size and Age of Earth, Sun and Milky Way

Our own experience of seeing objects in everyday life is as if light were an instantaneous messenger. For all practical purposes this is so. Light travels at the speed of 300,000 kilometers per second. But the distance between galaxies is so vast that a different measure than a mile or kilometer is needed. The unit that is useful to us here is the distance that light travels in a *year*. This unit is called the *light-year*; it is approximately 10,000 *billion*

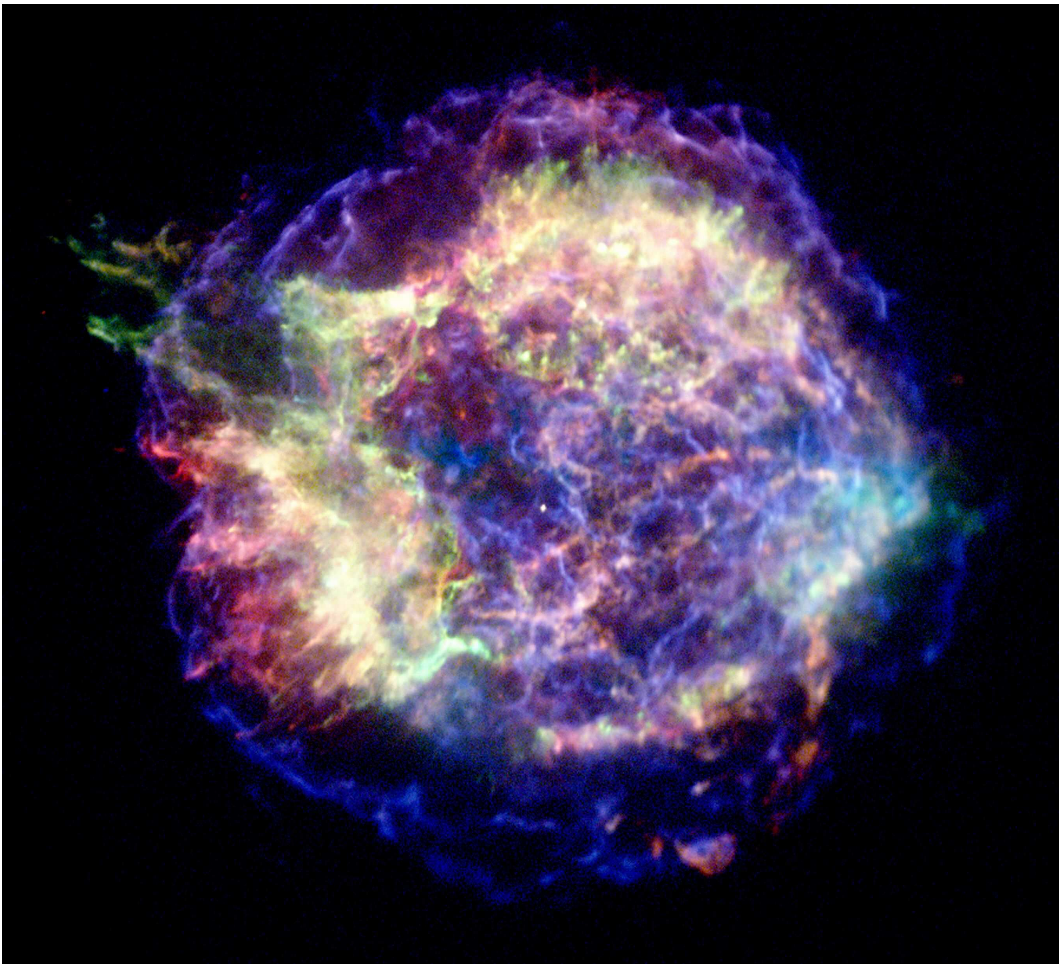

Fig. 1.6 The most recent supernova in our Galaxy occurred about 1667. The remnant, called Cassiopeia — now about 15 light-years in diameter — is seen here in x-rays in this Chandra X-Ray Observatory photo. Credit: NASA/CXC/MIT/UMassAmherst/M.D. Stage *et al.*

kilometers. A far-off galaxy is seen *now* as it was in the past by the light-travel-time from it to us. For example, the Large Magellanic Cloud — the nearest galaxy to us — is 169,000 light-years away. The light we see from it *now* has taken 169,000 years to reach us. Useful astronomical measures are given in Box 1 along with the age of the Earth, Sun, and other interesting data. (See page 30.)

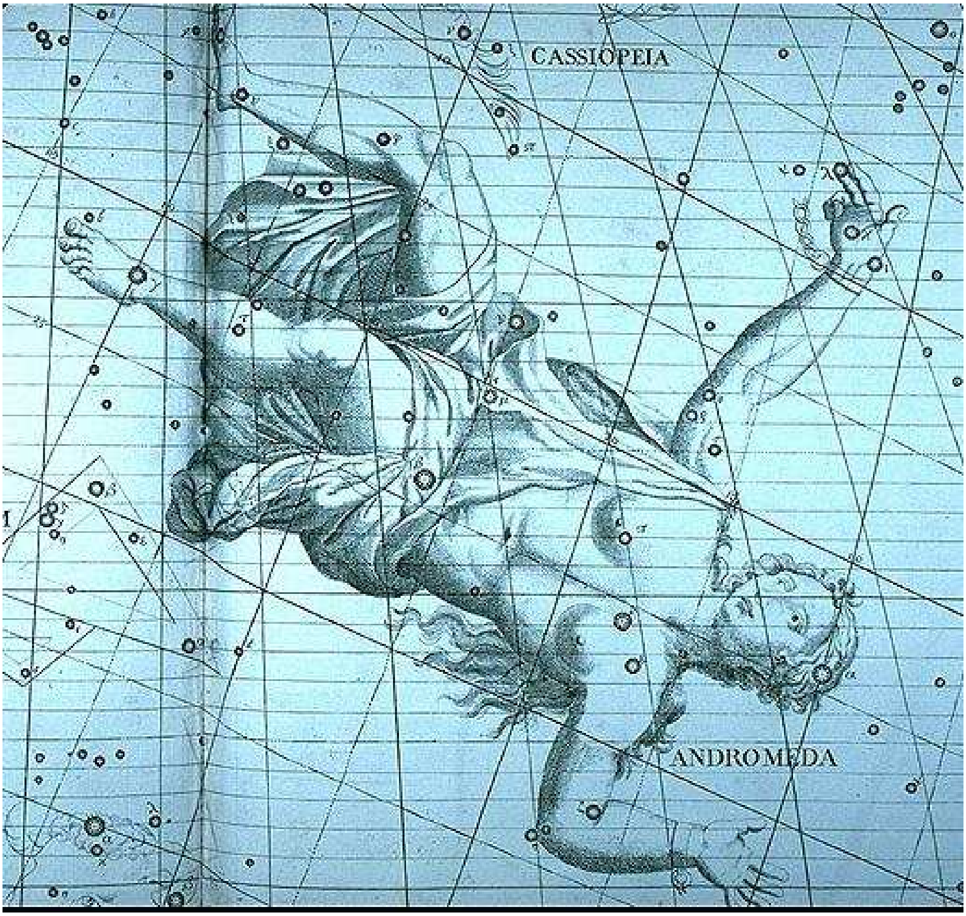

Fig. 1.7 Cassiopeia was banned to the sky as a constellation of stars, hanging half of the time head downward. This illustration of the constellation is from a book by John Flamsteed (1645–1719).

The Milky Way Galaxy in which our Sun resides, among about 12,000 billion others, is very old, about 10 billion years. Compared to our own perspective on Earth it is hard to discern with our senses, let alone comprehend with our minds the vastness of space and the sizes, ages, and masses of the various heavenly objects.

Fig. 1.8 Nasir ad-Din al-Tusi (1201–1274) was among the first of several Arabic astronomers at the observatory of Maragha in Persia who modified models based on mechanical principles by Ptolemy (87–150). This early Arabic manuscript contains the *al-Tadhkira fi ëilm al-hayía* (Memoir on Astronomy). This scholar wrote on many subjects besides astronomy, including poetry, mathematics, medicine, history, law and philosophy.

1.3. Einstein's Static Universe

> *Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life.*
>
> George Gamow, *My World Line*

At the time that Einstein discovered General Relativity (1915), permanence and an everlasting universe were fixed beliefs in Western philosophy. Hubble had not yet discovered that the universe is expanding. Einstein was greatly disturbed that his theory, as it stood, described a changing universe. He therefore added what he named the cosmological constant — denoted by the Greek symbol Λ (lambda) — so that his theory could be made to describe an unchanging universe that would neither collapse under its own gravity, nor coast forever in expansion (Box 3).

1.4. Expanding Universe

A few years after Einstein's discovery of General Relativity, Edwin Hubble, an astronomer in California, shattered the age-old belief in the constancy of the universe. He discovered through his observations that the galaxies around us are actually speeding away, and what is more, the further they are, the greater their speed. This discovery came to be known as Hubble's law. Mathematically it has a very simple expression; the velocity v of a galaxy moving away from us is proportional to its distance d and the proportionality constant is denoted by H_0, after Hubble; the relationship is $v = H_0 d$. The subscript, 0, is used to denote the *present* value of the proportionality factor, because the universe is not expanding at a constant speed. For billions of years the expansion was slowed by the force of gravity, but for the last several billion years it has been accelerating because of whatever mystery is covered by the words *cosmological constant*.

Hubble made his discovery of universal expansion by measuring what is called the redshift of light coming from nearby galaxies. The redshift, also commonly called the Doppler shift, is well known to anyone who has heard the high pitch of the whistle of an approaching train sink to a lower pitch as it passes. All wave-like motion, including light, will experience this sort of shift if it originates from a moving source. Light from a star or galaxy that is receding will appear redder: it is said to be redshifted. By measuring the redshift of other galaxies beyond our own, Hubble made his amazing discovery (Box 5).

According to our present understanding concerning the expansion, what was originally called the Doppler shift is more accurately named the *cosmological redshift*. The Doppler *redshift* refers to the elongation of light wavelengths of radiation received by an observer from an object moving away from him, and both within an *unchanging* space. The cosmological redshift of radiation from distant galaxies is caused by the elongation of light wavelengths in the *expanding* space of the universe in which the galaxies are *co-movers* in that space.

As the universe expanded, radiation, such as the light and heat we receive from the Sun, cooled in inverse proportion to the expanding size of the universe (Box 8). This we know from simple and basic laws of nature. How fast it expanded is also governed by the laws of nature, but to put numbers to it, we need to know a few — indeed a very few — important constants called the *cosmological* constants. (The three epochs of expansion can be seen in Box 7.) Their values have been discovered only during the last decade.

In the early universe, matter was in such close contact with radiation that it too cooled with the expansion in the same way as radiation until about 300,000 years. After that the primordial radiation streamed through the universe as if nothing else were there because the number of photons of light so vastly outnumbered charged particles (Box 9). We can deduce this from the discovery of Penzias and Wilson. They were trying to improve satellite communications for Bell Laboratories in New Jersey when, in 1965, they discovered something very unexpected — a faint radio signal that was coming from *all* directions of the sky.

This omni-directional radiation, known as the *Cosmic Background Radiation* (CMBR), seemed quite puzzling, for it could not be arriving from other galaxies; they are localized dots in the sky when viewed from earth. Moreover, the radiation was very cold, about 270 degrees centigrade *below* the freezing point of water. George Gamow had foreseen the likely existence of this whisper from the past. But, unaware of his foresight, Robert Dicke and Jim Peebles, two Princeton scientists, had come to the same conclusion and were actually designing receivers and antennae to detect it when word reached them of its discovery. "We've been scooped" Dicke murmured to his colleagues when a telephone call reached him with the news. This discovery won Penzias and Wilson the 1978 Nobel Prize: It was the first very convincing evidence of the hot and dense beginning, sometimes known as the *Big Bang*[a] that had been proposed, in different languages, by the Belgian theological student, Georges Lemaître in 1927, and independently by the Russian meteorologist and bomber pilot, Alexander Friedmann. Lemaître referred to the beginning as "*a day without yesterday.*"

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[a] This rather inelegant phrase for the sublime moment of the beginning of the universe was coined by Fred Hoyle, a well-known astronomer and cosmologist, on a BBC radio broadcast. That phrase is not used again in this work.

Both Friedmann and Lemaître had read Einstein's relativity papers and what was more, somehow understood without there being any evidence of it at the time that from a very dense and hot beginning, the universe burst forth to expand possibly forever. (Box 2 and Box 7.)

This is the most profound and natural implication of Einstein's General Relativity paper (that and the existence of black holes). Einstein himself had not realized this, thinking, as his contemporaries and Newton before him, that the universe was forever unchanging.

The young Lemaître, later a priest, was struggling at the time (1932) to reconcile the biblical account of creation with Einstein's theory of gravity (General Relativity). To further his task, he took up the study of astrophysics and cosmology at Cambridge University and the Massachusetts Institute of Technology. He published his early ideas on the birth of the universe in an obscure Belgian journal where it passed unnoticed by the principals in the field. Lemaître applied Einstein's theory of gravity to cosmic expansion and conceived the notion of the "primeval egg", which is the term he used to describe the universe at its beginning. His prescient notion of an eventual cosmic acceleration, which he included among the possible universes, has actually been confirmed only in the past few years and ranks among the great cosmological discoveries of all time. Friedmann too, working independently in Russia, came to the same conclusion.

It is all the more remarkable that Lemaître's publication of his audacious theory of the beginning of the world actually preceded Hubble's 1929 momentous discovery of universal expansion. After he had learned of Hubble's discovery, he quoted that work in his own subsequent publications to support his ideas. Meanwhile, Hubble, in California, was unaware of the young priest's theory, which had foreshadowed his own great discoveries.

Friedmann submitted his paper with the surprising conclusion that the universe may be expanding to the German physics journal *Zeitschrift für Physik* whose editor sought Einstein's advice as a referee. He wrote back saying; "The results concerning the non-stationary world, contained in [Friedmann's] work, appear to me suspicious. In reality it turns out that the solution given in it does not satisfy the field equations [of General Relativity]."

However, Friedmann was confident of the results that he had obtained from Einstein's theory. He wrote to Einstein beginning: "Considering that the possible existence of a non-stationary world has a certain interest, I will allow myself to present to you here the calculations I have made . . . for verification and critical assessment. . . ." Meanwhile, Einstein had already

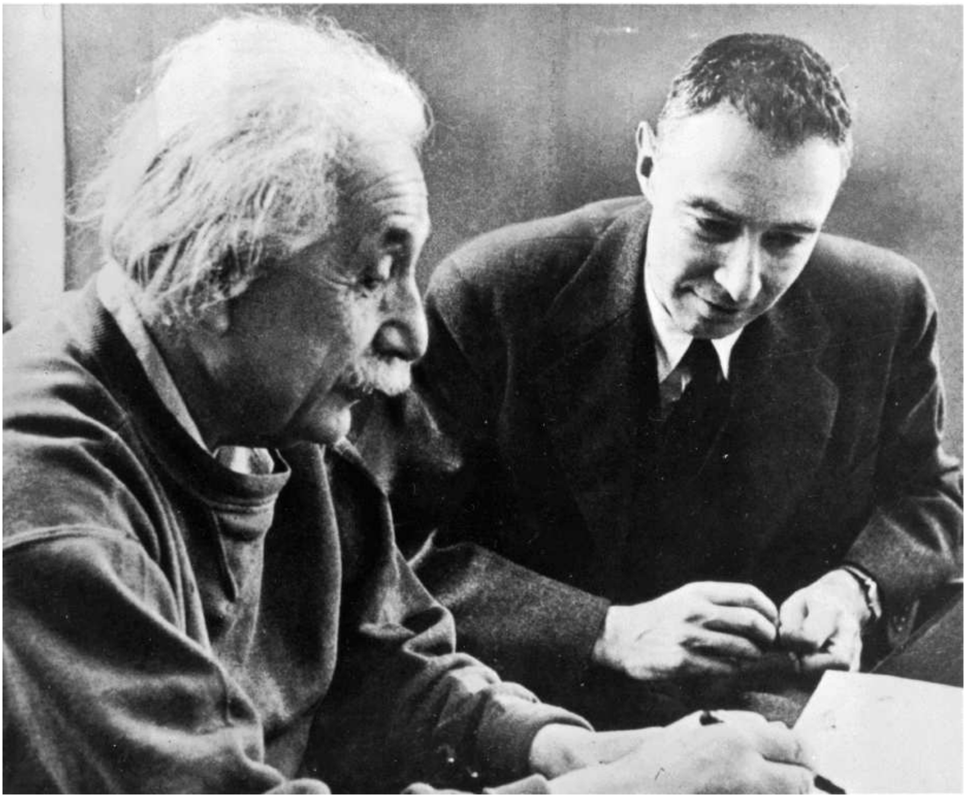

Fig. 1.9 Albert Einstein and J. Robert Oppenheimer discuss their work at the Princeton Institute for Advanced Study. By this time Einstein has long been a world figure for his paper on General Relativity, published in 1915, his name synonymous with genius. It is the greatest single contribution to our understanding of gravity, and is the means by which we can trace the history of the universe and its possible futures. Credit: Emilio Segrè Visual Archives.

left for Kyoto and did not return to Europe for several months. Then, by chance a friend of Friedmann's met Einstein at Ehrenfest's house in Leiden and described his colleague's work; Einstein saw his error and immediately wrote to the journal's editor "... my criticism [of Friedmann's paper]... was based on an error in my calculations. I consider that Mr. Friedmann's results are correct and shed new light."

Ultimately, Albert Einstein was so impressed by the work of Friedmann and Lemaître that at a meeting of Scientists in Pasadena in 1933, he rose after Lemaître's speech to say "This is the most beautiful and satisfactory explanation of creation to which I have ever listened."

1.5. Universe without Center

Edwin Hubble did not know of the theoretical work of Friedmann and Lemaître, when he discovered that the universe is expanding — that distant galaxies are rushing away — and the further they are, the faster. Moreover, he discovered that the number of galaxies increased uniformly within every angular patch of sky the deeper he looked into space, and that this was true no matter the direction. These *large-scale* properties of the universe are referred to as *homogeneity* and *isotropy*, or simply as *uniformity*. The observed *large scale* uniformity has momentous implications.

On the small scale — astronomically speaking — the nearby universe is anything but uniform. We see the Milky Way as a faint band of light across the sky. Nearby stars, those in our own galaxy, are concentrated, either in the disk or else in the central bulge; our Sun is in the disk about two-thirds from the center. In appearance, the Milky Way is similar to the Andromeda Galaxy (Figure 1.1). Even further out in space astronomers see stars concentrated in other galaxies — not randomly scattered about. And further still, galaxies are gathered into *galaxy clusters*. Gravity has had time since the beginning to rearrange the once diffuse matter from which these objects are made.

But on a much grander scale, the astronomer sees galaxy clusters at a distance of a billion and more light-years — which still corresponds to billions of years after the beginning — scattered about reasonably evenly so that what lies at a very great distance in our universe seems much the same in every direction. From these observations we learn that the universe will appear the same to an observer, *on the grand scale*, in any other galaxy as it appears to an observer in ours. This observation of *homogeneity* and *isotropy* of the universe justify what is referred to as the *Cosmological Principle*.

This principle is very important; it allows us to extend our observation of that part of the universe that we can see with our telescopes to the universe as a whole. It has *another* very interesting consequence: Any observer, located anywhere in the universe would see galaxies rushing away from him just as Hubble observed them to be rushing away from us. Therefore he might conclude that *he* was at the center. Clearly the only resolution to this paradox is that *the universe has no center*: As viewed from *any* galaxy, all distant galaxies are rushing away with a speed in proportion to their distance.[b]

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[b] By a simple example found in the Section "Questions (number 5)" one can easily perceive why Hubble's observations must be true in an expanding universe.

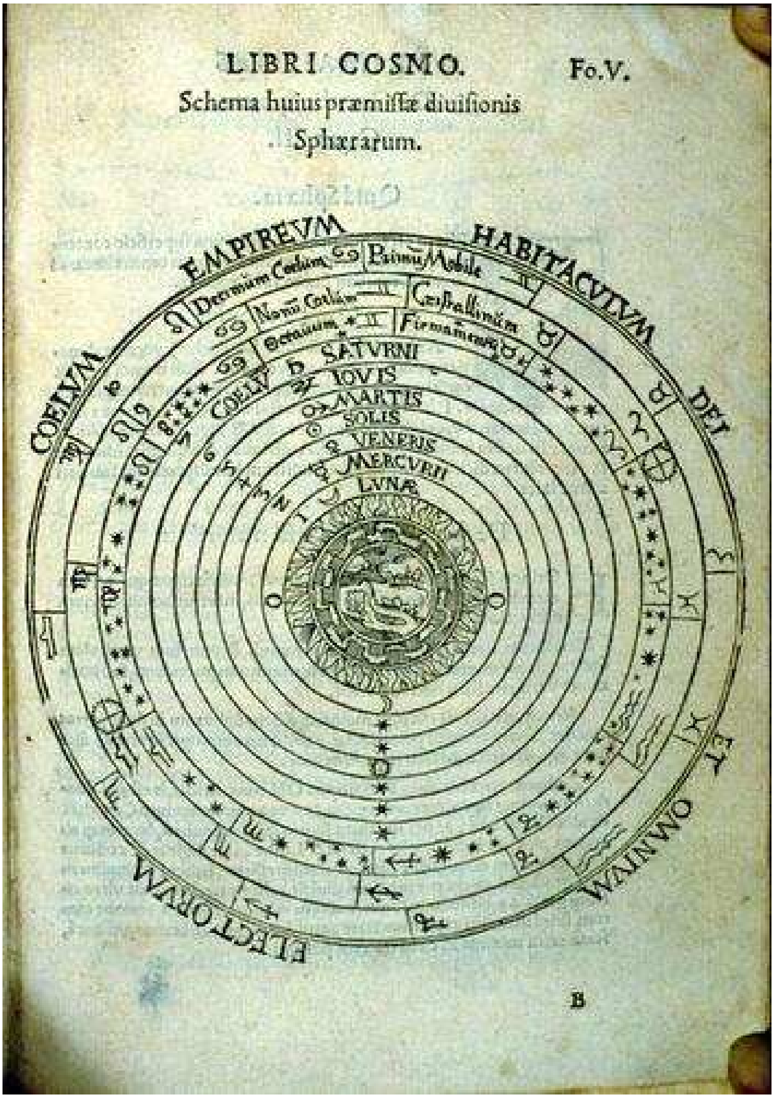

Fig. 1.10 The heavenly arrangement of Sun, Moon, and planets with the Earth as center, as was thought to constitute the world in Ptolomy's astronomy. Credit: Peter Apian, *Cosmographia* (*1540*).

That there is no center to the universe would have been inconceivable to our ancestors, and even to us it is a wonder. In ancient times, Earth was believed to be the center of the world — that Sun as well as Moon and other planets revolved about us (Figure 1.10). These beliefs, whose lineage date to Aristotle, were even elevated to the level of (Roman) Church doctrine. Just such a doctrine — the centrality of the Earth — was contrary to the visual observations made by Copernicus. He saw that Earth and the other planets revolve about the Sun and believed that Earth rotates about its own axis, which he asserted was responsible for day and night. Galileo was persuaded and his persistence in the belief that Earth was not the center of the world brought him into serious conflict with the Roman Ecclesiastical Inquisition (see Section 8.5). Now we know a greater wonder about our universe; *there is no center.*

1.6. Karl Schwarzschild and Black Holes

A German astrophysicist, Karl Schwarzschild (Figure 1.11) of considerable renown, volunteered for military service on the outbreak of war in August 1914 (called the First World War). While on the Russian front, he obtained the first exact solution of Einstein's General Relativity.

Fig. 1.11 Karl Schwarzschild obtained the exact solution of Einstein's General Relativity for the spacetime outside spherical stars including black holes. Einstein was surprised to learn that an exact solution to his theory could be obtained.

Before he had enlisted, he was, and remains, a prominent figure in astronomy. He made and explained many effects, which even today are important and bear his name. Schwarzschild mastered celestial mechanics by the age of sixteen, and wrote his first two papers on the theory of orbits of double stars while still at the Frankfurt Gymnasium (high school). The papers were published in *Astronomische Nachrichten* in 1890. At a meeting of the German Astronomical Society in Heidelberg in 1900, Schwarzschild had the foresight to discuss the possibility that space was non-Euclidean.

From 1901 until 1909 Schwarzschild was professor at Göttingen and also director of the Observatory. In Göttingen he collaborated with Klein, Hilbert and Minkowski; all are renowned in the world of science. Near the end of 1909, Schwarzschild left Göttingen to take up an appointment as director of the Astrophysical Observatory in Potsdam, the most prestigious post for an astronomer in Germany.

Schwarzschild volunteered for military service in the First World War and served in Belgium where he was put in charge of a weather station, France where he was assigned to an artillery unit and given the task of calculating missile trajectories, and then Russia.

While in Russia he wrote two papers on Einstein's theory of General Relativity. Schwarzschild's papers give the first exact solution of General Relativity for the spacetime outside a spherical star, which reveals the possible existence of a star, now known as a black hole, from which not even light could escape if the mass were too large for the size of the star. He sent the paper to Einstein who replied: "I had not expected that one could formulate the exact solution of the problem in such a simple way."

Karl Schwarzschild contracted an illness while on the Russian front, a rare blistering disease of the skin. In Schwarzschild's time there was no known treatment and, after being invalided home in March 1916, he died two months later. But his name lives on as the first to realize that General Relativity implies the possible existence of black holes, so named because not even light can escape from within. They are sometimes referred to as "Schwarzschild black holes."

1.7. Questions

1. *How is the mass of the Galaxy measured?*

   According to Kepler's law, $M_{\text{Galaxy}} + M_{\text{Sun}} = (2\pi)^2 R^3/(GP^2)$ where P is the period or time for one full orbit of the Sun about the galactic center, namely $P \approx 250$ million years, R is the radius of the orbit, and G is Newton's constant. The Sun has made about 18 circuits of the Milky Way since its birth 4.5 billion years ago. The Sun's mass can be neglected in comparison with the Galaxy mass in the formula for the mass.

2. *What is a photon? What is a baryon?*

   Light has a dual nature, with particle-like and wave-like properties. The photon is the particle-like unit of light whose energy is related to the frequency, or inversely to the wavelength of its wave-like nature. A baryon is a particle such as the proton and neutron; the nucleus of an atom is composed of these two particles. The number of protons in a nucleus is what characterizes one element from another. The proton carries a unit of positive charge. The number of neutrons, equals or exceeds the number of protons in the nucleus. They carry no electric charge. The positive charge of the nucleus is neutralized in an atom by the electrons surrounding the nucleus. The number of neutrons in a nucleus effect its atomic weight but not its chemical properties.

3. *Why did radiation and then mass fade in importance in their gravitational attraction as the universe expanded, and therefore their tendency to slow the expansion?*

   As the universe expanded, number densities of photons and baryons decreased inversely to the volume ($n \sim 1/R^3$). The wavelength of radiation expanded proportionately, and hence the energy of each photon decreased as $1/R$. So the energy density of radiation decreased as $1/R^4$. In contrast, because baryons do not change in mass, matter density decreased as their number density, $1/R^3$. As a consequence, radiation dominated the early universe, and matter later in time (see Box 2). Eventually, the small cosmological constant that Einstein introduced into his theory came to dominate the expansion.

4. *If the cosmological constant is causing the universe to accelerate its expansion in the present epoch, why was it not always so?*

   The cosmological constant was outweighed in importance in earlier epochs by gravity acting on radiation and mass. However, as radiation, first, and then matter were diluted by the cosmic expansion, the cosmological constant began to dominate. This occurred about two billion years ago and will continue into the indefinite future. (This raises the interesting anthropological question: why did the switch in importance take place so close to the present epoch? See Martin Reese: *Just Six Numbers* and the suggestion of multi-universes as a rationalization of such "coincidences".)

5. *To understand Hubble's observation that the velocity of distant galaxies is proportional to their distance $v = Hd$ make a one-dimensional model.*

Imagine an infinitely long elastic string on which buttons representing galaxies (which being gravitationally bound, do not expand with the universal expansion) are fixed at equal intervals. Now stretch the string at uniform speed. Note that in the time that the distance between buttons 5 and 6 doubled, the distance between 5 and 7 quadrupled, and so on for any set of consecutive three buttons. Make a diagram that illustrates this for some section of the string.

6. *Concerning the above situation, would an observer sitting on any one of the buttons be justified in saying that he was at the center of the expansion?*

No. The expansion would appear the same to any observer.

7. *How are astronomers able to measure the distance to remote objects?*

The wavelength of light emitted by stars or galaxies moving away from us is redshifted, analogous to the change of pitch of a moving train as it passes (usually called the Doppler shift in this context). The redshift occurs because, as discovered by Edwin Hubble, the more distant a galaxy, the greater its speed of recession. The amount of the redshift, denoted by z, of a distant object gives a measure of the relative change in scale, R, of the universe between the time of emission of light and our time of detection: $1 + z = \lambda_0/\lambda = R_0/R > 1$. (See Box 5.) Here, R_0 denotes the scale of the universe in our time, and R the smaller scale at the earlier time of emission, and similarly, λ_0 is the redshifted wavelength of the distant receding object as measured in the laboratory, and λ the wavelength of emitted light.

8. *Why is z measured instead of time?*

Time past is not measurable, whereas redshift is. The way that time evolves in three eras of the universe is given in Box 8 according to a model of the universe which is assumed to be isotropic and homogeneous.

9. *How is z measured?*

From a comparison of spectral lines from an identifiable spectrum, with a laboratory standard, according to the formula, $z = (\lambda_0 - \lambda)/\lambda$.

10. *What is a standard candle?*

In astronomy there are certain celestial objects that are referred to as *standard candles.* The meaning of this is that wherever located in the universe, a standard candle of given type emits the same amount of light in unit time as any other of that type. The Cepheid variable stars are such a standard; they are close enough that the distance to them can be calibrated through their parallax. And in this way, the faintness or brightness of one of them can be related to its distance. Knowing that the intensity of light falls off with distance as $1/r^2$, the brightness and distance of these stars can be tabulated. So it is for any other objects that are standard candles. A particular kind of supernova called type Ia provides another standard candle. They all have (approximately) the same intrinsic brightness at their own location.

11. *What is a type Ia supernova?*

These supernova are thermonuclear explosions on a white dwarf whose surface has been heated by the energy imparted to it by matter that the dwarf is accreting from a companion luminous and therefore much less dense star.

12. *Do all type Ia supernova have the same intrinsic brightness?*

No. Their brightness increases shortly after the explosion, and then fades slowly over weeks (as viewed from Earth). But by successive observations over a period of a week or so, the brightness curve can be calibrated and the brightness of various events can thus be compared. They have roughly the same brightness curve, but, as might be expected, they do have some individuality.

13. *What type of observations were made that revealed cosmic acceleration?*

The cosmological constant introduced by Einstein into his theory and the global curvature of the universe have been determined in the last

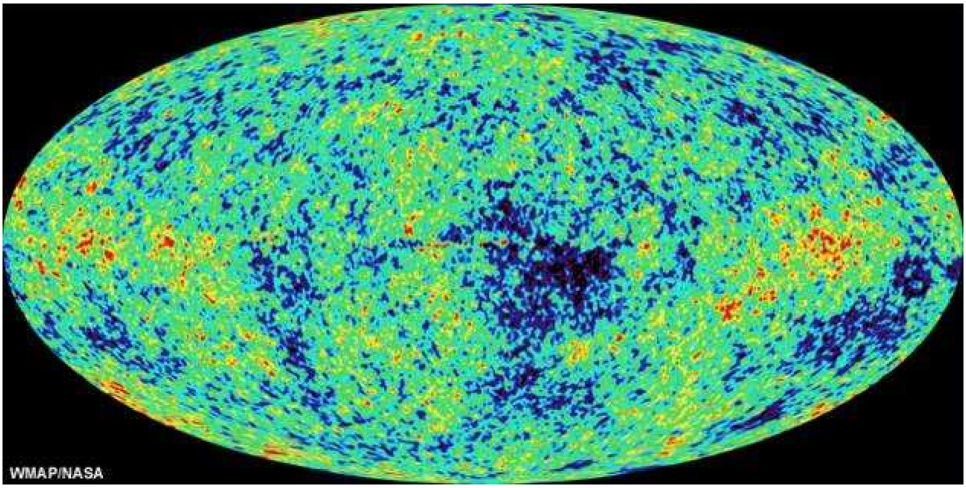

Fig. 1.12 The snapshot shows the state of the universe about 380,000 years after the beginning. No stars were yet present. The radiation present at that time is not perfectly smooth, but lumpy. This lumpiness caused a corresponding lumpiness in the distribution of matter. These clumps formed the seeds of galaxies. The universe is composed of 4% ordinary matter, 23% of an unknown type of dark matter, and 73% of a mysterious dark energy. This is a confirmation of the so-called concordance lambda-CDM model. The study of this cosmic microwave background radiation (CMBR) was made using NASA's space-based Microwave Anisotropy Probe (MAP) observatory. Credit: NASA/WMAP Science Team.

few years by measurements of the distance and age of far off (type Ia) supernovae; these measurements indicate acceleration and a flat ($k = 0$) universe. (See S. Perlmutter, *Physics Today*, April 2003.)

14. How could astronomers learn that the expansion of the universe is accelerating?

The expansion of the universe is affected by how much mass density is in it (like stars and galaxies). The more mass density, the more gravity will slow the expansion. In the simplest cosmological model (no mass, no cosmological constant) the expansion would proceed at a steady rate. But there is mass — the galaxies have mass. So the rate of expansion at the location of distant galaxies would be, by this expectation, smaller. It is not: it is *larger*! This implies that the expansion at some time in the past began to accelerate. This is the evidence for a positive cosmological constant Λ.

15. *What actually is measured in the determination of cosmic acceleration?*

It is based on a measure of redshift z. The wavelength of spectral lines coming from the supernova originating from specific atomic transitions are compared with those of a laboratory spectrum. In this way the redshift $z \equiv (\lambda_0 - \lambda)/\lambda$ can be measured. (The subscript 0 stands for the measurement made by the observer on Earth.)

16. *How old is the universe and how is this learned?*

By looking deep into space one is looking back in time because of the time it takes light to travel to us. The look-back-time is related *approximately* to the Hubble constant and the redshift by $\Delta t \approx z/H$ (see Box 5). New data (2003) from measurements of radiation emitted before there were any stars show the universe to be about 13.7 billion years old, to within one percent (±137 million years).

17. *Why is there only an approximate relationship for time, as in the above answer?*

Because times past cannot be directly measured. They depend on how fast the universe expanded, which has *varied* over time. But the amount by which light from a distant galaxy is redshifted depends uniquely on the amount by which the universe has expanded in the interim. And the redshift *is* measurable. So $z \equiv R_0/R - 1$ is an exact way of referring to the past (R denotes the scale of the universe at an earlier time and R_0 the scale now). For example, if the measured cosmological redshift of a distant object is $z = \frac{2}{3}$, then the scale of the universe was $R = \frac{3}{5}R_0$ of the present scale (size). If $z = 1.5$ then $R = \frac{2}{5}R_0$. What fraction of the present size was the universe for a group of galaxies at $z = 2$?

18. *Is R a measure of the size of the universe?*

It is a relative measure by which two eras of the universe can be compared by their respective scales.

19. *What is the largest presently measured redshift of an object?*

In 1994 the redshift of a galaxy 8C1435+635 was measured with an approximate value of 4.25. The universe then was $\frac{4}{17}$ of its present size. Two emission lines of ionized carbon and hydrogen were measured to obtain the red shift.

20. *How is the redshift of far-off galaxies measured?*

Light emitted by atoms at the periphery of stars in a galaxy can be detected by astronomers using telescopes: The light is emitted at unique energies or *wavelengths* corresponding to transitions of electrons between energy levels in the atoms. Each type of atom has its unique signature. The galaxy is receding because of the cosmic expansion, so the wavelength (λ) corresponding to each transition taking place in the receding galaxy is redshifted from that of a stationary atom (λ_0) in the observer's laboratory (i.e. $\lambda_0 > \lambda$) . The fractional change in wavelength is directly related to the redshift also called the Doppler shift (see also Box 5) by $z = (\lambda_0 - \lambda)/\lambda$.

21. *What is a black hole?*

From General Relativity, it is found that if an (non-rotating) object of mass M lies within a sphere of radius $2MG/c^2$, where G is Newton's constant and c the speed of light, then it is a black hole — an object from which not even light can escape. (If the object is rotating, the relationship quoted is more complex.) Oppenheimer and Volkoff discovered the relationship in modern times. Though not precisely written down, it was discovered in 1783 by the Rev. John Michell and published in the *Proceedings of the Royal Society, Vol LXXIV.* The term "black hole" was not introduced until much after the theoretical investigation of these strange objects by J. R. Oppenheimer and H. Snyder in 1939. The name "black hole" was coined a few years later by John A. Wheeler, who earlier did not think they could exist — that some process would intervene to evade the collapse.

22. *Why do astronomers believe that a gigantic black hole occupies the center of the Milky Way?*

Near the center of the Milky Way, astronomers have observed stars that are moving in tight orbits at very high speeds (Figures 1.13 and 1.14). If they were not attracted toward the center by a very great mass, they would fly off into space. Using the measured velocities for these stars one can calculate, by Kepler's law, how much mass at the center is required to hold these visible stars. The massive object at the center of our galaxy has a mass of at least 300 million times the mass of our Sun, and its radius is too small to be other than a black hole.

Fig. 1.13 Image created in x-rays of the region of the Milky Way's central black hole which is called Sagittarius A*. The black hole itself cannot be seen; neither light nor anything else can escape it. Its location is near the center in the bright region of the multi-million degree gas, which is what is seen in this figure. Credit: Rainer Schödel (MPE) *et al.*, NAOS-CONICA, ESO.

23. *How could the radius of the central black hole in our galaxy be measured?*

Half of the angular separation of the two extremes (the approaching and receding stars) in Figure 1.14 and the distance from us to the Galaxy center provide R, the radius of the orbiting stars. The Doppler shift

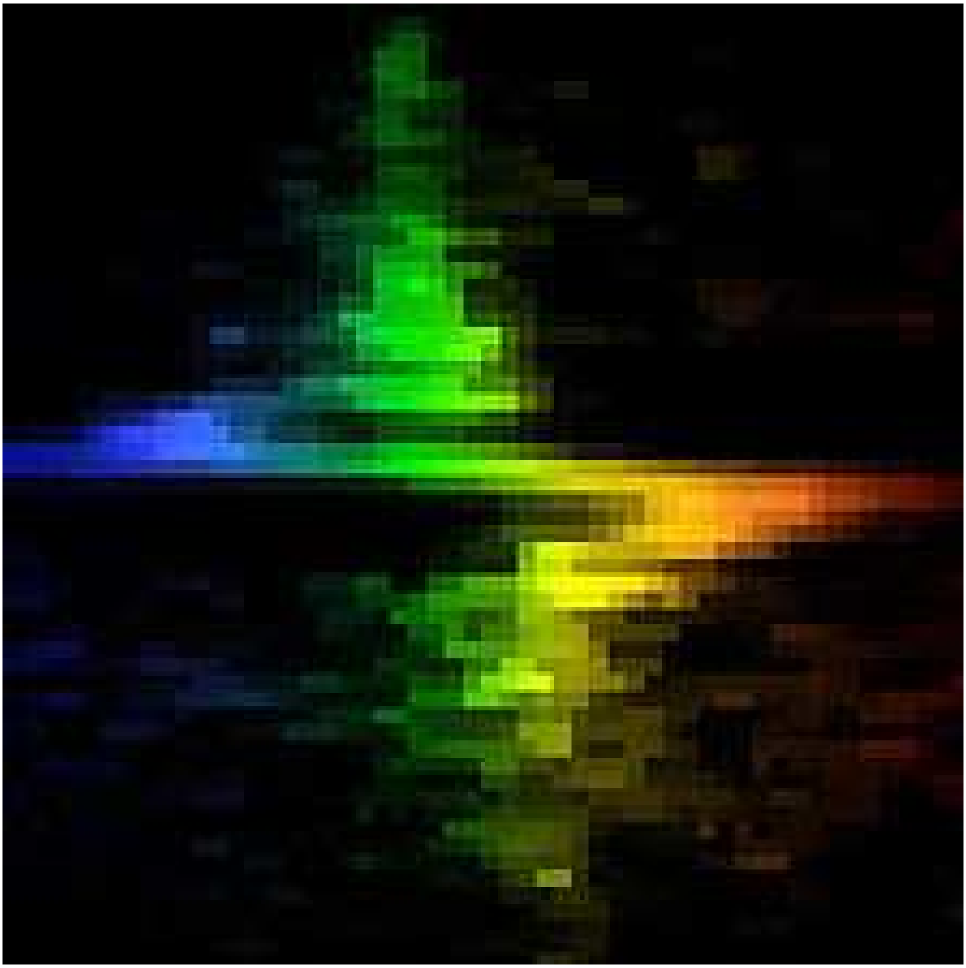

Fig. 1.14 This figure shows observations made on the center of the Milky Way that reveal the presence of a massive black hole. What is recorded is the Doppler shift of light emitted by ionized atoms in the material that is swirling around the black hole. The excursions about the mean position in the figure reveal the Doppler shift as the material approaches the observer (blue), then passes in front of the black hole, and then recedes (red-orange) from the observer. Credit: Gary Bower, Richard Green (NOAO), the STIS Instrument Definition Team, and NASA.

of light from the circulating stars provide v/c (see page 33). Hence the period $P = 2\pi R/v$ can be calculated. Put these in the Kepler formula $M_{\mathrm{BH}} + M_{\mathrm{stars}} = (2\pi)^2 R^3/(GP^2)$. The mass of the stars can be ignored in comparison with the much bigger black hole mass.

24. *Why do astronomers believe that very massive stars existed briefly at very early time (several hundred million years)?*

Observations made with the Hubble Space Telescope (HST) have discovered that small amounts of iron are present in very distant — therefore ancient — quasars. The only elements that could have formed in the very early universe — long before there were stars — were the light ones, mostly helium; the universe expanded and cooled too rapidly for heavier elements to have formed. Rather, the heavier ones must be made in and around stars. The existence of iron in the ancient quasars indicates that the first stars formed as little as 200 million years after the beginning.

25. *What is a quasar?*

Quasar is short for *quasi-stellar radio source.* They are very distant objects that are exceedingly luminous for their angular size. They are now believed to be the central region of a galaxy that is referred to as an *active galactic nucleus.* The source of power is thought to be a supermassive black hole whose gravity is tearing apart nearby stars as they spiral into the hole. The rapid variability in the emitted power indicates that the powerhouse of this activity is a very small region of space.

26. *How do astronomers know what is the density of matter in the universe today?*

This is a long and complicated story. Light elements like deuterium and hydrogen were forged from nucleons in the brief intense fire and high density of the first few minutes. Such conditions have never again existed. The amount of deuterium produced *then* depended sensitively on the ratio of the number of baryons to photons because deuterons are made from a neutron and proton, but are destroyed if the number of photons is too great. (Deuterons are also made in stars, but then destroyed to make helium.) The measured deuteron abundance provides the value of this baryon to photon ratio because the number of neither baryons nor photons has since changed; the baryon number has not changed because of a conservation law, and photons, because they so overwhelmingly outnumber baryons that a typical photon has not encountered matter since that early time when the universe was dense. From the present temperature of the cosmic background radiation, 2.7 K, measured first by Penzias and Wilson, the *number of photons in any specified volume* (often called the photon number density) can be computed. Therefore the present baryon density can be obtained because their ratio is known. (See Box 9.)

27. *Why is gravity often referred to as a universal force as in universal gravity?*

Gravity is a very long-range force that falls off with distance d only as $1/d^2$. Unlike another long-range force, the Coulomb, or electric force, there is no object having the opposite effect of attraction, whereas there are both positive and negative electric charges which attract each other and neutralize themselves. Gravity is unique in having a long-range unshielded effect.

28. *Scientists refer to some theories as beautiful. What is it that to their minds makes a theory beautiful?*

The great Indian astrophysicist, Chandrasekhar, answered this question in a wonderful little book called *Truth and Beauty* (1987). I try to capture the essence of his essay as follows: It is the brevity — the economy both of words and of mathematical statement — that can make a theory beautiful. More than that — a beautiful theory has an element of strangeness to it. Most physicists agree that Einstein's General Relativity is a beautiful theory, in fact the most beautiful theory that we know. One equation says it all; the equation is $G_{\mu\nu} = -8\pi T_{\mu\nu}$. (It is granted that the notation — a mathematics that took Einstein years to uncover — represents much more than is apparent.) On each side is a mathematical object called a tensor. A tensor is a generalization of a vector, which as we know, is a pointer in space, or in spacetime. The left side of the equation expresses the curvature of spacetime and the right side carries information about the distribution and motion of matter and energy. The element of strangeness about it is the juxtaposition of two previously unrelated concepts — the geometry of spacetime on one side *and* matter and its motion on the other.

29. *Einstein is reputed to have said that his greatest mistake was in adding to his gravitational equation what is known as the cosmological term symbolized by* Λ *so that his theory reads;* $G_{\mu\nu} = -8\pi T_{\mu\nu} + \Lambda g_{\mu\nu}$. *Why did he add it? Is it now relevant?*

Einstein added the cosmological term with a value chosen so that his theory would describe an *unchanging* universe that would neither collapse eventually under its own gravity, nor coast forever in expansion.[c]

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[c] Einstein was mistaken in thinking that an equilibrium could be established so that the universe would neither expand or contract. With the cosmological constant, only a temporary equilibrium would exist. The least perturbation would cause it to do one or the other.

Permanence and an everlasting universe were fixed beliefs in Western philosophy at the time (1915). Hubble later (1929) discovered that the universe is expanding and Perlmutter *et al.* (1999) discovered that despite gravity, the expansion is *accelerating*, that the cosmological constant, Λ, *is* needed and whatever it represents, its effect — if larger than a critical value — is to accelerate the expansion. In fact, the stage that the universe is in at the present time (and indeed has been for the past several billion years) is *accelerating expansion*, possibly forever.

1.8. Boxes 1–9

1 Units and Astronomical Data

$c \approx 3 \times 10^5$ km/s (light velocity)
$G \approx 6.7 \times 10^{-8}$ cm^3/(g s^2) (Newton's constant)
year $\approx 3.2 \times 10^7$ s
distance: l-yr (light-year) $\approx 9.5 \times 10^{12}$ km
distance: mile = 1.6 km
distance: pc (parsec) ≈ 3.3 l-y $= 3.1 \times 10^{13}$ km
Hubble constant: $H_0 \approx 20$ km/s $\cdot\, 1/(10^6$ l-y)
$1/H_0 \approx 15 \times 10^9$ yr $\approx 4.6 \times 10^{17}$ s
parsec ≈ 3.3 l-y $\approx 3.1 \times 10^{13}$ km
Universe age $\approx 1/H_0$
Earth age $\approx 4.5 \times 10^9$ yr
Earth mass $\approx 6 \times 10^{27}$ g
Sun age $\approx 4.5 \times 10^9$ yr
Sun mass $M_\odot \approx 2.0 \times 10^{33}$ g
Milky Way age $\approx 10^{10}$ yr
Milky Way mass $\leq 10^{12} M_\odot$
life of a 10 $M_\odot$ star $\approx 10^7$ yr
time since dinosaurs $\approx 7 \times 10^7$ yr
" $\approx$ " means approximately

2 Friedmann–Lemaître Equations

For the Robertson–Walker line element corresponding to a *homogeneous and isotropic* universe, only two of Einstein's (10 independent) field equations, $G_{\mu\nu} = -8\pi T_{\mu\nu} + \Lambda g_{\mu\nu}$, are independent [Narlikar (2002); Glendenning (2004)]. They can be taken as

$$\dot{R}^2 + kc^2 = (1/3)(\Lambda + 8\pi G\rho)R^2\,,$$

and

$$\ddot{R} = (1/3)[\Lambda - 4\pi G(\rho + 3p/c^2)]R\,.$$

Here, R is the scale of the universe in arbitrary units, $\rho = \epsilon/c^2$ is the sum of mass density of matter ρ_m and of radiation ρ_r, ϵ their total energy density, p the pressure, Λ Einstein's cosmological constant, and k is the curvature parameter.

Take the derivative of the first of the above pair, multiply the second by $\dot{R}$ and eliminate the Λ term from the resulting pair to obtain the conservation law implicit in the Einstein equations (divergenceless stress-energy tensor),

$$\dot{\rho} = -3(p/c^2 + \rho)(\dot{R}/R)\,.$$

This equation can also be written in two different ways:

$$d/dt(\rho c^2 R^3) = -p\,dR^3/dt$$

or

$$d\rho/dR = -3(p/c^2 + \rho)/R\,.$$

We can derive rigorously the behavior of radiation and matter densities as the universe expands. The equation of state for radiation is $p = (1/3)\rho_r c^2$. Therefore

$$d\rho/\rho = -4dR/R\,.$$

This yields the conservation equation for the equivalent mass density of *radiation*

$$\rho/\rho_0 = (R_0/R)^4,\ \text{radiation.}$$

However, for *matter*, $p \ll \rho_m/c^2$, and we obtain instead

$$\rho/\rho_0 = (R_0/R)^3,\ \text{matter.}$$

Thus radiation dominates early in the history of the universe, matter later, and finally after these have been diluted by the universal expansion, the cosmological constant dominates.

3 Einstein's Stationary Universe

At the time that Einstein applied his General Theory of Relativity to the universe, he, and most everyone else, thought that the world was stationary. Hubble had not yet discovered the expansion of the universe. To obtain a stationary solution, put $\dot{R} = \ddot{R} = 0$ into the two Friedmann equations (Box 2). We then obtain,

$$3kc^2/{R_0}^2 = \Lambda + 8\pi G\rho_0 \,,$$

and

$$\Lambda = 4\pi G\rho_0 \,.$$

Rearrange these to find,

$$\Lambda = kc^2/{R_0}^2.$$

Take the size of the universe to be its age, 13.5×10^9 years times the speed of light. Then, for $k = 1$,

$$\Lambda = 5 \times 10^{-36} (1/\mathrm{s}^2) \,,$$

and

$$\rho_0 = \Lambda/(4\pi G) = 6 \times 10^{-30} \ \mathrm{g/cm}^3 \,.$$

Such a cosmological constant with such an average density would produce a stationary universe.

4 Redshift in Cosmology

In cosmology, a past event is usually referenced by the value of the redshift z because that is what can be measured, whereas the time at which it occurred cannot. An approximate time can be referenced only with an assumption of how the universe actually evolved with time, which of course is not measurable. The best one can do is to use a model of the expansion by reference to a particular scenario, say the Friedmann and Lemaître equation with definite assumptions about the cosmological parameters that appear in it. These parameters k, Λ, and the densities of matter and radiation ρ are known only within errors and there are three of them against one, the redshift z.

5 Redshift

The cosmological redshift of light emitted by a receding source, say a galaxy, is the fractional change in wavelength between that received by an observer λ_0 and that emitted by the source λ,

$$z \equiv (\lambda_0 - \lambda)/\lambda \geq 0\,.$$

Let R_0 denote the scale of the universe in the era of the observer, and R the scale of the universe at the earlier era when the light was emitted by the source. Then $R_0 > R$. (R is referred to as the scale factor of the universe, and represents size in relative units.) We note that in an expanding universe, wavelength of radiation is stretched in proportion to the expansion so that, $\lambda/\lambda_0 = R/R_0$. The very important connection between cosmological redshift z and the scale of the universe R follows from this:

$$z + 1 = R_0/R\,.$$

This tells us that for a distant object (seen as it was in the past because of light travel time) at a redshift of z, the scale of the universe now R_0, is larger than the scale R at that time in the past, by the factor $z + 1$. It follows that

$$\begin{aligned} z &= (R_0 - R)/R \equiv \Delta R/R = [(\Delta R/(\Delta t R)]\Delta t \\ &\approx (\dot{R}/R)\Delta t \equiv H\Delta t\,, \end{aligned}$$

where $\dot{R} \equiv dR/dt$. Here we have introduced the *Hubble constant*, $H \equiv \dot{R}/R$, (with the unit of inverse time) which expresses his discovery that the distant galaxies are receding from us at a velocity proportional to their distance,

$$v \equiv \dot{R} = HR\,.$$

The distance to the source (distant galaxy) is $R = c\Delta t$, where Δt is the time taken for light to travel from the source to the observer. Therefore, from above we have,

$$z \approx H\Delta t = (v/R)(R/c) = v/c \quad \text{(for small } z)\,.$$

This shows that the redshift of a not-to-distant galaxy is approximately equal to its velocity in units of light velocity if $v \ll c$.

6 Why Cosmological Redshift Rather than Doppler Shift

Note that the cosmological redshift cannot be properly called the Doppler shift. The latter refers to recession of an observed object in a fixed space, whereas the cosmological redshift refers to recession because space *itself* is expanding and the observed object is co-moving with the expansion. This is the case for the universe.

7 Three Epochs of the Universe

Einstein's equations describe, among other things, how gravity controls the expansion of the universe. For a uniform homogeneous universe they take the simple form, the Friedmann–Lemaȋtre equation, which governs expansion and the continuity equation for energy conservation (see Box 2). During the radiation epoch (when the density of radiation dominated that of matter and the cosmological constant) the expansion equation takes the form;

$$\dot{R}^2 = 8\pi G\rho_0 R_0^4/3R^2$$

where ρ_0 and R_0 are the values of the density and scale factor at any convenient reference time (for example the present). We learn that the size of the visible universe increases with time in proportion to the square root of time, $R \sim \sqrt{t}$, but ever more slowly.

The mass densities of radiation and matter became equal at about a million years when the temperature was about 2000 degrees Kelvin; radiation slowly faded thereafter. When the matter density ρ_m is the dominant term of those on the right of the Friedmann–Lemaȋtre equation, the universe entered the matter dominated epoch. The universal expansion is then controlled according to the equation

$$\dot{R}^2 = 8\pi G\rho_0 R_0^3/3R\,.$$

In this epoch, the expansion increases with time as $R \sim t^{2/3}$; the speed of expansion continues to decelerate because of the gravitational attraction of matter and radiation.

There is a *third epoch* when density has diluted and the dark energy term Λ dominates. The Friedmann–Lemaȋtre equation becomes with time

$$\dot{R}^2 = \Lambda R^2/3\,.$$

The solution to the expansion equation for positive Λ, the dark energy, is $R \sim \exp\sqrt{\Lambda/3}\,t$. We can also note that $\ddot{R} = (\Lambda/3)R$, so that the universal expansion *accelerates* in the third era.

8 Redshift Correspondence with Time

The connection between redshift and time depends on the way the universe evolves. We do not know the time dependence of the expansion. The expansion proceeded differently according to the dominant contents of the universe. The Friedmann–Lemaître equations can be solved for the three epochs corresponding to radiation, matter and cosmological constant dominance. The connection is (where t_0 is the present time and t an earlier one, as measured from the beginning of the expansion):

$$1+z=\frac{R_0}{R}=\begin{cases}(t_0/t)^{1/2} & \text{radiation era}\\ (t_0/t)^{2/3} & \text{matter era}\\ \exp[\sqrt{\Lambda/3}\,(t_0-t)] & \text{dark energy era}\end{cases}$$

It is worth noting that the temperature of radiation falls as the universe expands because the wavelength of radiation stretches with the expansion;

$$\frac{T}{T_0}=\frac{R_0}{R}\,.$$

In the first two ages, the expansion decelerates because of gravity acting on the equivalent mass density of radiation at first and when that fades in importance, then on the mass. In the third, the cosmological term dominates and the cosmic expansion accelerates. The acceleration in the three ages is summarized as;

$$a\sim\begin{cases}-1/(t)^{3/2} & \text{radiation era}\\ -1/(t)^{4/3} & \text{matter era}\\ \Lambda/3\exp[\sqrt{\Lambda/3}\,t] & \text{dark energy era}\end{cases}$$

These connections can be derived as limiting cases from Friedmann's equation.

9 Present Baryon and Photon Density

From the present temperature of the radiation background, $T_0 = 2.728\,\mathrm{K}$, the *present* mass density of radiation can be calculated from the Steffan–Boltzmann law,

$$\rho_r(t_0) = aT_0^4/c^2 = 4.66 \times 10^{-34}\ \mathrm{g/cm^3}\,.$$

The present *number* density of photons (i.e. the number per cubic centimeter) can be found from Planck's law for black-body radiation and the measured cosmic background temperature. The number density is

$$n_\gamma(t_0) = 0.244(2\pi kT_0/hc)^3 = 413\ \mathrm{photons/cm^3}\,.$$

The baryon to photon ratio can be determined from primordial abundances (cf. [Glendenning (2004)], Section 5.4.4); it is $n_\mathrm{B}/n_\gamma = 5 \times 10^{-10}$. Hence, we can calculate the *present* baryon *number* density as

$$n_\mathrm{B} = 2 \times 10^{-7}/\mathrm{cm^3}$$

and the mass density is then found to be

$$\rho_\mathrm{B} = n_\mathrm{B} m_N = 3.5 \times 10^{-31}\ \mathrm{g/cm^3}\,.$$

As an added note we can emphasize that the number of photons vastly outnumbers the number of baryons,

$$n_\gamma = 2 \times 10^9\, n_\mathrm{B}.$$

As a result of this, photons effectively *did not* encounter baryonic matter or electrons from a very early time in the history of the universe.