Preface

"This is mission impossible", said my friends, when they heard that I was writing a math book for the enjoyment of non-mathematicians and those who "hated math in school". They are right. Even Stephen Hawking and Roger Penrose, two of the most famous mathematicians in the 21st century, alluded in their books to the common belief that each mathematical equation in a book would halve its sales and readership. Still I persisted.

This book is written for the young and the young-at-heart. They respond more readily to beauty, patterns and symmetry, with awe and amazement. The young are always interested in what they see around them in the world. They wonder about how things can be the way they are. And they always ask the question "why?"

This book began life as a booklet, some twenty, thirty pages long, and was meant only for my granddaughters. Then it took on a life of its own, and grew and grew, as more and more equations were added.

The genesis of the book was prompted by my two granddaughters, Rebecca, 6 and Kathryn, 4. They love playing computer games. After short lessons with me, they would play the games again and again, until they master them. Some of the games require fairly complex heuristic algorithms (game plans), executed in logical systematic sequences. And yet the kids could master them fairly effortlessly. As I watched them playing enthusiastically, it occurred to me that young

children might be able to master complex mathematical algorithms too, if they were similarly motivated.

Therefore I decided to teach Rebecca some mathematics. I mentioned π, explained what it is, and gave her the value of π up to 10 decimal places. A few days later, I asked her what π was. She gave me the answer, with its value correct for 9 digits out of 10, and in the right sequence. I then taught her the Liebniz-Gregory infinite series for $\frac{\pi}{4}$, and Euler's series for $\frac{\pi^2}{6}$. After the first four terms, she saw the underlying patterns, and could continue the series for the next many terms.

So the definition of π, π to 10 decimals and the Liebniz-Gregory and Euler series became the "passwords" that she had to write down, to gain access to the computer for her games. Being only 4, Kathryn had it easier — all she had to write down was the infinite series for "e". She was especially proud of the fact that she learnt the meaning of "factorial" (!) before her elder sister did.

The effortless encounters of Rebecca and Kathryn with the infinite series for π and e, suggested to me the possibility that many people could also enjoy an easy introduction to the pleasures and beauty of mathematics the painless way. For far too long "mathematics" has been synonymous with "boredom", "pain", and "total incomprehension" for far too many people. If little children can appreciate and enjoy the patterns of infinite series for π and e, so can everyone else.

Therefore I decided to write this book as a gift to my granddaughters so that as they grow up, and at the appropriate time, they may be initiated into the pleasures of "pure mathematics" — one of the most beautiful subjects in the world, according to the eminent former Cambridge mathematician, Professor G.H. Hardy.

This book consists of two short sections. Section I is largely a visual treat, a feast for the eyes, with equations which can be read and enjoyed by almost everyone who has heard of π, and know the simple integers and fractions. The series have been selected for their beauty, elegance and simplicity from the huge domain of

mathematics. There are literally an infinite number of such series in mathematics to choose from. Section I is best read at one sitting, just as one would read a book of art, full of beautiful pictures, with short commentaries. That way, the beauty and mystery of the patterns and the rhythms of many of the equations will fill the reader, or perhaps for those seeing them for the first time, with a sense of awe and amazement. It should set them wondering why they had never encountered such beauty in the math that they had studied in school. Some of the series are so simple and elegant in their symmetry, that even Kathryn, a four-year-old child, could continue the series after the first few terms. If non-mathematicians and those who "hated math in school" could finish Section I, the effort in writing this book would have been worthwhile.

Section II of the book is largely a feast for the mind, and requires a bit more effort and some background in mathematics, preferably up to high school level. Here in the most systematic way possible, I have chosen the simplest proofs for the beautiful equations given in Section I, so that readers will be able to derive the beautiful equations themselves. Reading and enjoying Section I alone would be pleasure enough. But there is far greater pleasure and a deeper sense of intellectual satisfaction to know that, with an extra bit of effort and some patience, one could follow in the footsteps, and think the same thoughts, as some of the greatest mathematics geniuses in the world, such as Newton, Leibniz, and Euler.

Albert Einstein, the greatest 20th century physicist, said:

> Everything should be made as simple as possible
> But not simpler.

So Section II begins with the "Easy Proofs" — proofs so simple that Rebecca could do some of them. These are then followed by the "Less Easy Proofs" and finally, the "Not-So-Easy Proofs".

Section II of the book is best savoured slowly like fine wine, preferably not more than one or two proofs at a time — unless you

are a mathematician, in which case, the proofs would be effortless and plain sailing for you.

John Keats, the young 19th century romantic poet, said:

> A thing of beauty is a joy forever
> Its loveliness increases
> It will never pass into nothingness.

In this book, you will see that many of the terms in the equations of great beauty do "pass into nothingness" as the series tends to infinity!

The mathematical ideas, equations, formulas, proofs, etc. in this book are about 200–400 years old. Thousands of mathematicians had written about them over the centuries.

What is original in this book is the method of presentation of the math, with the primary purpose of illustrating the beauty of pure math via the vehicle of infinite series. After I had finished writing the book, I went through the books in the math section of Borders in Singapore and San Francisco, and Waterstone's, Blackwell's and Foyle's in London. I did not find a single book that presented math in the way that I had. My hope is that readers of the book (especially of Section I) will find it entertaining and pleasurable. If readers also find it educational (especially readers of Section II), it would be a happy bonus.

So enjoy, and take pleasure in the infinite series of π, e and other interesting numbers.