

Preface

This little book is the outcome of a course I have given over the last ten years at the Technical University Darmstadt for students of Mathematics and Computer Science. The aim of this course is to provide a solid basis for students who want to write their Masters Thesis in the field of Denotational Semantics or want to start a PhD in this field. For the latter purpose it has been used successfully also at the Univ. of Birmingham (UK) by the students of Martin Escardó.

Thus I think this booklet serves well the purpose of filling the gap between introductory textbooks like e.g. [Winskel 1993] and the many research articles in the area of Denotational Semantics. Intentionally I have concentrated on denotational semantics based on *Domain Theory* and neglected the more recent and flourishing field of *Game Semantics* (see [Hyland and Ong 2000; Abramsky et.al. 2000]) which in a sense is located in between Operational and Denotational Semantics. The reason for this choice is that on the one hand Game Semantics is covered well in [McCusker 1998] and on the other hand I find domain based semantics mathematically simpler than competing approaches since its nature is more abstract and less combinatorial. Certainly this preference is somewhat subjective but my excuse is that I think one should write books rather about subjects which one knows quite well than about subjects with which one is less familiar.

We develop our subject by studying the properties of the well known functional kernel language PCF introduced by D. Scott in the late 1960ies. The scene is set in Chapters 2 and 3 where we introduce the operational and domain semantics of PCF, respectively. Subsequently we concentrate on studying the relation between operational and domain semantics employing more and more refined *logical relation* techniques culminating in the construction of the fully abstract model for PCF in Chapters 11 and

12. I think that our construction of the fully abstract model is more elegant and more concise than the accounts which can be found in the literature though, of course, it is heavily based on them. Somewhat off this main thread we show also how to interpret recursive types (Chapter 9) and give a self contained account of computability in Scott domains (Chapter 13) where we prove the classical theorem of [Plotkin 1977] characterizing the computable elements of the Scott model of PCF as those elements definable in PCF extended by two parallel constructs `por` (“parallel or”) and \exists (Plotkin’s “continuous existential quantifier”) providing an extensional variant of the *dove tailing* technique known from basic recursion theory.

Besides basic techniques like naive set theory, induction and recursion (as covered e.g. by [Winskel 1993]) we assume knowledge of basic category theory (as covered by [Barr and Wells 1990] or the first chapters of [MacLane 1998]) from Chapter 9 onwards and knowledge of basic recursion theory only in the final Chapter 13. Except these few prerequisites this little book is essentially self contained. However, the pace of exposition is not very slow and most straightforward verifications—in particular at the beginning—are left to the reader. We recommend the reader to solve the many exercises indicated in the text whenever they show up. Most of them are straightforward and in case they are not we give some hints.

I want to express my gratitude to all the colleagues who over the years have helped me a lot by countless discussions, providing preprints etc. Obviously, this little book would have been impossible without the seminal work of Dana Scott and Gordon Plotkin. The many other researchers in the field of domain theoretic semantics who have helped me are too numerous to be listed here. I mention explicitly just Klaus Keimel and Martin Escardó, the former because he was and still is the soul of our little working group on domain theory in Darmstadt, the latter because his successful use of my course notes for his own teaching brought me to think that it might be worthwhile to publish them. Besides for many comments on the text I am grateful to Martin also for helping me a lot with $\text{T}_{\text{E}}\text{X}$ nical matters. I acknowledge the use of Paul Taylor’s diagram and proof tree macros which were essential for type setting.

Finally I want to thank the staff of IC press for continuous aid and patience with me during the process of preparing this book. I have experienced collaboration with them as most delightful in all phases of the work.