

Contents

Preface

v

1. <i>Elements</i> II and Babylonian Metric Algebra	1
1.1. Greek Lettered Diagrams vs. OB Metric Algebra Diagrams	2
1.2. <i>El.</i> II.2-3 and the Three Basic Quadratic Equations	7
1.3. <i>El.</i> II.4, II.7 and the Two Basic Additive Quadratic-Linear Systems of Equations	10
1.4. <i>El.</i> II.5-6 and the Two Basic Rectangular-Linear Systems of Equations	12
1.5. <i>El.</i> II.8 and the Two Basic Subtractive Quadratic-Linear Systems of Equations	14
1.6. <i>El.</i> II.9-10, Constructive Counterparts to <i>El.</i> II.4 and II.7	16
1.7. <i>El.</i> II.11* and II.14*, Constructive Counterparts to <i>El.</i> II.5-6	18
1.8. <i>El.</i> II.12-13, Constructive Counterparts to <i>El.</i> II.8	22
1.9. Summary. The Three Parts of <i>Elements</i> II	24
1.10. An Old Babylonian Catalog Text with Metric Algebra Problems	27
1.11. A Large Old Babylonian Catalog Text of a Similar Kind	29
1.12. Old Babylonian Solutions to Metric Algebra Problems	35
1.12 a. Old Babylonian problems for rectangles and squares	35
1.12 b. Old Babylonian problems for circles and chords	42
1.12 c. Old Babylonian problems for non-symmetric trapezoids	48
1.13. Late Babylonian Solutions to Metric Algebra Problems	50
1.13 a. Problems for rectangles and squares	50
The seed measure of a hundred-cubit-square. Metric squaring	51
A rectangle of given front and seed measure. Metric division	53
A square of given seed measure. Metric square side computation	54
A rectangle of given side-sum and seed measure. Basic problem of type B1a	55
A rectangle of given side-difference and seed measure. Type B1b.	57
A square band of given width and seed measure. Type B3b	58
1.13 b. Problems for circles	59
A circle of given seed measure divided into five bands of equal width	59
A circle of given circumference divided into five bands of equal width	61
A Seleucid pole-against-a-wall problem	64
Seleucid parallels to <i>El.</i> II.14* (systems of equations of type B1a)	66
1.14. Old Akkadian Square Expansion and Square Contraction Rules	68
1.15. The Long History of Metric Algebra in Mesopotamia	69

2. <i>El.</i> I.47 and the Old Babylonian Diagonal Rule	73
2.1. Euclid's Proof of <i>El.</i> I.47	73
2.2. Pappus' Proof of a Generalization of <i>El.</i> I.47	74
2.3. The Original Discovery of the OB Diagonal Rule for Rectangles	76
2.4. Chains of Triangles, Trapezoids, or Rectangles	79
3. Lemma <i>El.</i> X.28/29 1a, Plimpton 322, and Babylonian <i>igi-igi.bi</i> Problems	83
3.1. Greek Generating Rules for Diagonal Triples of Numbers	83
Euclid's Generating Rule in the Lemma <i>El.</i> X.28/29 1a	83
The Generating Rules Attributed to Pythagoras and Plato	84
Metric Algebra Derivations of the Greek Generating Rules	85
3.2. Old Babylonian <i>igi-igi.bi</i> Problems	86
3.3. Plimpton 322: A Table of Parameters for <i>igi-igi.bi</i> Problems	88
4. Lemma <i>El.</i> X.32/33 and an Old Babylonian Geometric Progression	95
4.1. Division of a right triangle into a pair of right sub-triangles	95
4.2. A Metric Algebra Proof of Lemma <i>El.</i> X.32/33	96
4.3. An Old Babylonian Chain of Right Sub-Triangles	97
5. <i>Elements</i> X and Babylonian Metric Algebra	101
5.1. The Pivotal Propositions and Lemmas in <i>Elements</i> X	101
A Concise Outline of the Contents of <i>Elements</i> X	102
5.2. Binomials and Apotomes, Majors and Minors	103
5.3. Euclid's Application of Areas and Babylonian Metric Division	113
5.4. Quadratic-Rectangular Systems of Equations of Type B5	116
6. <i>Elements</i> IV and Old Babylonian Figures Within Figures	123
6.1. <i>Elements</i> IV, a Well Organized Geometric Theme Text	123
An Outline of the Contents of <i>Elements</i> IV	123
6.2. Figures Within Figures in Mesopotamian Mathematics	125
7. <i>El.</i> VI.30, XIII.1-12, and Regular Polygons in Babylonian Mathematics	141
7.1. <i>El.</i> VI.30: Cutting a Straight Line in Extreme and Mean Ratio	141
7.2. Regular Pentagons and Equilateral Triangles in <i>Elements</i> XIII	142
An Outline of the Contents of <i>El.</i> XIII.1-12	142
7.3. An Extension of the Result in <i>El.</i> XIII.11	146
7.4. An Alternative Proof of the Crucial Proposition <i>El.</i> XIII.8	149
7.5. Metric Analysis of the Regular Pentagon in Terms of its Side	151
7.6. Metric Analysis of the Regular Octagon	155
7.7. Equilateral Triangles in Babylonian Mathematics	159

7.8. Regular Polygons in Babylonian Mathematics	160
7.9. Geometric Constructions in Mesopotamian Decorative Art	164
8. <i>El.</i> XIII.13-18 and Regular Polyhedrons in Babylonian Mathematics	171
8.1. Regular Polyhedrons in <i>Elements</i> XIII	171
An Outline of the Contents of <i>El.</i> XIII.13-18	171
Conclusion.	181
8.2. MS 3049 § 5. The Inner Diagonal of a Gate	181
8.3. The Weight of an Old Babylonian Colossal Copper Icosahedron	184
9. <i>Elements</i> XII and Pyramids and Cones in Babylonian Mathematics	189
9.1. Circles, Pyramids, Cones, and Spheres in <i>Elements</i> XII	189
9.2. Pre-literate Plain Number Tokens from the Middle East in the Form of Circular Lenses, Pyramids, Cylinders, Cones, and Spheres	192
9.3. Pyramids and Cones in OB Mathematical Cuneiform Texts	195
9.3 a. The volume and grain measure of a ridge pyramid	196
9.3 b. The grain measure of a ridge pyramid truncated at mid-height	200
9.3 c. Problems for cones and truncated cones	202
9.4. Pyramids and Cones in Ancient Chinese Mathematical Texts	202
9.4 a. The fifth chapter in <i>Jiu Zhang Suan Shu</i>	202
9.4 b. Liu Hui's commentary to <i>Jiu Zhang Suan Shu</i> , Chapter V.	206
9.5. A Possible Babylonian Derivation of the Volume of a Pyramid	207
10. <i>El.</i> I.43-44, <i>El.</i> VI.24-29, <i>Data</i> 57-59, 84-86, and Metric Algebra	211
10.1. <i>El.</i> I.43-44 & <i>Data</i> 57: Parabolic Applications of Parallelograms	212
10.2. <i>El.</i> VI. 28 & <i>Data</i> 58. Elliptic Applications of Parallelograms	217
10.3. <i>El.</i> VI. 29 & <i>Data</i> 59. Hyperbolic Applications of Parallelograms	219
10.4. <i>El.</i> VI.25 and <i>Data</i> 55	220
10.5. <i>Data</i> 84-85. Rectangular-Linear Systems of Equations	225
10.6. <i>Data</i> 86. A Quadratic-Rectangular System of Equations of Type B6.	227
10.7. Zeuthen's Conjecture: Intersecting Hyperbolas	232
10.8. A Kassite Series Text with Modified Systems of Types B5 and B6	233
11. Euclid's Lost Book <i>On Divisions</i> and Babylonian Striped Figures	235
11.1. Selected Division Problems in <i>On Divisions</i>	236
OD 1-2, 30-31. To divide a triangle by lines parallel to the base	236
OD 3. To bisect a triangle by a line through a point on a side	237
OD 4-5. To divide a trapezoid by lines parallel to the base.	237
OD 8, 12. To bisect a trapezoid by a line through a point on a side	238
OD 19-20. To divide a triangle by a line through an interior point.	239

OD 32. To divide a trapezoid by a parallel in a given ratio 242

11.2. Old Babylonian Problems for Striped Triangles 244

 11.2 a. Str. 364 § 2. A model problem for a 3-striped triangle 244

 11.2 b. Str. 364 § 3. A quadratic equation for a 2-striped triangle 244

 11.2 c. Str. 364 §§ 4-7. Quadratic equations for 2-striped triangles 249

 11.2 d. Str. 364 § 8. Problems for 5-striped triangles 252

 11.2 e. *TMS* 18. A cleverly designed problem for a 2-striped triangle 255

 11.2 f. *MLC* 1950. An elegant solution procedure 258

 11.2 g. *VAT* 8512. Another cleverly designed problem 259

 11.2 h. *YBC* 4696. A series of problems for a 2-striped triangle 261

 11.2 i. *MAH* 16055. A table of diagrams for 3-striped triangles 264

 11.2 j. *IM* 43996. A 3-striped triangle divided in given ratios 267

11.3. Old Babylonian Problems for 2-Striped Trapezoids 269

 11.3 a. *IM* 58045, an Old Akkadian problem for a bisected trapezoid 269

 11.3 b. *VAT* 8512, interpreted as a problem for a bisected trapezoid 271

 11.3 c. *YBC* 4675. A problem for a bisected quadrilateral 272

 11.3 d. *YBC* 4608. A 2-striped trapezoid divided in the ratio 1: 3 274

 11.3 e. Str. 367. A 2-striped trapezoid divided in the ratio 29 : 51 277

 11.3 f. *Ist. Si.* 269. Five 2-striped trapezoids divided in the ratio 60 : 1 279

 11.3 g. The Bloom of Thymaridas and its relation to
 Old Babylonian generating equations for transversal triples 282

 11.3 h. Relations between diagonal triples and transversal triples 283

11.4. Old Babylonian Problems for 3-and 5-Striped Trapezoids 285

11.5. *Erm.* 15189. Diagrams for Ten Double Bisected Trapezoids 287

11.6. *AO* 17264. A Problem for a Chain of 3 Bisected Quadrilaterals 292

11.7. *VAT* 7621 # 1. A 2 · 9-striped trapezoid 296

11.8. *VAT* 7531. Cross-wise striped trapezoids. 297

11.9. *TMS* 23. Confluent Quadrilateral Bisections in Two Directions 299

11.10. *Erm.* 15073. Divided Trapezoids in a Recombination Text 304

12. Hippocrates’ Lunes and Babylonian Figures with Curved Boundaries 309

 12.1. Hippocrates’ Lunes According to Alexander 309

 12.2. Hippocrates’ Lunes According to Eudemus 311

 12.3. Some Geometric Figures in the *OB* Table of Constants *BR* 316

 12.3 a. *BR* 10-12. The ‘bow field’ 316

 12.3 b. *BR* 13-15. The ‘boat field’ 317

 12.3 c. *BR* 16-18. The ‘barleycorn field’ 318

 12.3 d. *BR* 19-21. The ‘ox-eye’ 319

 12.3 e. *BR* 22-24. The ‘lyre-window’ 319

 12.3 f. *BR* 25. The ‘lyre-window of 3’ 320

 12.4. *W* 23291-x § 1. A Late Babylonian Double Segment and Lune 321

12.5. A Remark by Neugebauer Concerning BM 15285 # 33	326
13. Traces of Babylonian Metric Algebra in the <i>Arithmetica</i> of Diophantus	327
13.1. Determinate Problems in Book I of Diophantus' <i>Arithmetica</i>	328
13.1. Four Basic Examples in Book II of Diophantus' <i>Arithmetica</i>	332
13.2 a. <i>Ar.</i> II.8 (Sesiano, <i>GA</i> (1990), 84).	332
13.2 b. <i>Ar.</i> II.9 (Sesiano, <i>GA</i> (1990), 85).	334
13.2 c. <i>Ar.</i> II.10 (Sesiano, <i>GA</i> (1990), 86).	336
13.2 d. <i>Ar.</i> II.19 (Sesiano, <i>GA</i> (1990), 86).	337
13.2. <i>Ar.</i> "V".9. Diophantus' Method of Approximation to Limits	338
13.3. <i>Ar.</i> III.19. A Square Number Equal to a Sum of Two Squares in Four Different Ways	341
Everywhere rational cyclic quadrilaterals	343
Diophantus' <i>Ar.</i> III.19, Birectangles, and the OB Composition Rule	345
13.4. <i>Ar.</i> "V".30. An Applied Problem and Quadratic Inequalities	349
An indeterminate combined price problem	349
13.5. <i>Ar.</i> "VI". A Theme Text with Equations for Right Triangles	352
<i>Ar.</i> "VI".16. A right triangle with a rational bisector	357
13.6. <i>Ar.</i> V.7-12. A Section of a Theme Text with Cubic Problems	358
13.7. <i>Ar.</i> IV.17. Another Appearance of the Term 'Representable'	360
14. Heron's, Ptolemy's, and Brahmagupta's Area and Diagonal Rules	361
14.1. <i>Metrica</i> I.8 / <i>Dioptra</i> 31. Heron's Triangle Area Rule	361
14.2. Two Simple Metric Algebra Proofs of the Triangle Area Rule	363
14.3. Simple Proofs of Special Cases of Brahmagupta's Area Rule	365
14.4. Simple Proofs of Special Cases of Ptolemy's Diagonal Rule	368
14.5. Simple Proofs of Special Cases of Brahmagupta's Diagonal Rule	370
14.6. A Proof of Brahmagupta's Diagonal Rule in the General Case	370
15. Theon of Smyrna's Side and Diagonal Numbers and Ascending Infinite Chains of Birectangles	373
15.1. The Greek Side and Diagonal Numbers Algorithm	375
15.2. MLC 2078. The Old Babylonian Spiral Chain Algorithm	377
15.3. Side and Diagonal Numbers When $\text{Sq. } p = \text{Sq. } q \cdot D - 1$	381
15.4. Side and Diagonal Numbers When $\text{Sq. } p = \text{Sq. } q \cdot D + 1$	382
16. Greek and Babylonian Square Side Approximations	385
16.1. <i>Metrica</i> I.8 b. Heron's Square Side Rule	385
16.2. Heronic Square Side Approximations	386
16.3. A New Explanation of Heron's Accurate Square Side Rule	387

16.4. Third Approximations in Ptolemy's <i>Syntaxis</i> I.10	390
16.5. The General Case of Formal Multiplications	391
16.6. A New Explanation of the Archimedian Estimates for Sqs. 3	392
16.7. Examples of Babylonian Square Side Approximations	394
The additive and subtractive square side rules	394
Late and Old Babylonian approximations to sqs. 2	395
Late and Old Babylonian approximations to sqs. 3	397
A Late Babylonian approximation to sqs. 5	399
Late and Old Babylonian exact computations of square sides	399
17. Theodorus of Cyrene's Irrationality Proof and Descending Infinite Chains of Birectangles	405
17.1. <i>Theaetetus</i> 147 C-D. Theodorus' Metric Algebra Lesson	405
17.2. A Number-Theoretical Explanation of Theodorus' Method	406
17.3. An Anthyphairctic Explanation of Theodorus' Method	407
17.4. A Metric Algebra Explanation of Theodorus' Method	409
18. The Pseudo-Heronian <i>Geometrica</i>	415
18.1. <i>Geometrica</i> as a Compilation of Various Sources	415
18.2. <i>Geometrica</i> mss AC	417
18.3. <i>Geometrica</i> ms S 24	420
18.4. <i>Metrica</i> 3.4. A Division of Figures Problem	429
Appendix 1. A Chain of Trapezoids with Fixed Diagonals	431
A.1.1. VAT 8393. A New Old Babylonian Single Problem Text	431
A.1.2. VAT 8393. About the Clay Tablet	440
Appendix 2. A Catalog of Babylonian Geometric Figures	443
Index of Texts, Propositions, and Lemmas	447
Index of Subjects	453
Bibliography	463
Comparative Mesopotamian, Egyptian, and Babylonian Timelines	476