Preface

Asset pricing theory plays a central role in finance theory and applications. Every asset, liability or cash flow has a value but an essential problem is, how to price it. In recent years, people even started talking about pricing an idea as an intangible asset! During the past half century, there appeared several methodologies to solve this problem. The first methodology is based on various economic partial (or general) equilibrium pricing models. Among these, the best known is Lucas' consumption-based asset pricing model. It links the asset pricing problem with the dynamical macro-economic theory. However, it is challenged by the well-known equity premium puzzle as pointed out by Mehra and Prescott in 1985. The issue is unresolved to-date. The second methodology is based on the so-called First Theorem in Finance due to Ross in 1976. Specifically, it values an asset by invoking the no-arbitrage principle in a complete capital market. This methodology/approach leads to, the well-known Arrow-Debreu security, the risk-neutral pricing, the APT (arbitrage pricing theory), and the equivalent-martingale measure. Interestingly, Sharpe's CAPM model can be derived from both methodologies referred to above. The third methodology involves the production-based pricing models. It links, in the long term, the firm economic growth theory with the asset pricing problem. In recent years, there has been a trend to unify the above methodologies and views under the title of stochastic discount factor (SDF) pricing models. Cochrane (2000) has summarized this trend.

In this book, we develop a new theory, which we call the Structural Theory, for asset pricing thereby putting the SDF pricing model firmly on a mathematical foundation. It includes a series of original results. Here we list some of the important ones. We separate the problem of finding a better asset pricing model from the problem of searching for "no equity" premium puzzle. The uniqueness theorem and the dual theorem of asset pricing indicate that, given market traded prices, a necessary and sufficient condition for the pricing functional space (or the SDF space) to have a unique *correctly pricing functional* (or *correctly pricing SDF*) is that, the space is isometric to the asset payoff space. The orthogonal projection operator, introduced in the dual theorem, provides a bijective and valuation-preserving mapping between the two spaces. A new explanation for the Mehra and Prescott's puzzle can be described as follows: (1) The structure of the consumption growth power space is not rich enough to provide an SDF which is capable of pricing every portfolio in the asset space correctly (*i.e.*, the two spaces are not isometric), within feasible ranges of the economic parameters. For example, when the risk aversion is chosen to be less than 5, a big pricing error appears. (2) In order to have no pricing error, given insufficient structure of the SDF space, the estimated parameter has to be exaggerated to an unreasonable level. For example, the risk aversion must be beyond 50 for the U.S market. The structure theory indicates that the appearance of the equity premium puzzle is *relative* and it depends on a matching (or rather a valuation-preserving isometric mapping) between an SDF space and an asset space. For matching pairs of the two spaces, there exists a unique SDF to price every portfolio in the asset space correctly. If the correctly pricing SDF is with sensible economic parameters, then there is no puzzle. However, if the correctly pricing SDF is with infeasible economic parameters, we say that the puzzle appears to this SDF space. Theoretically, given the asset space, we can remove this puzzle by enlarging the SDF space to one with sensible new economic parameters, for example, by adding new economic state variables to span a bigger SDF space rather than extending the range of the parameters in the original (smaller) SDF space to an unreasonable level. If the augmented SDF space is matched to the asset space, there is a new SDF that may provide the tool to price every portfolio in the asset space correctly, with the result that there is no pricing error and hence no puzzle to the asset space. Alternatively, given the SDF space, we may, by dropping some assets from the asset space, find a new correctly pricing SDF with sensible parameters to the reduced asset space, with the result that we would then see no pricing error and hence no puzzle to the reduced asset space. In general, mis-matching an SDF space with an asset space will definitely create some pricing error. The puzzle is in fact the result of an improper attempt to remove the pricing error. Using the above theory, it is easy to see why the Epstein-Zin model is less prone to cause the puzzle. The SDF space generated by the Epstein-Zin model is richer than the CRRA (constant-relative-risk aversion) based SDF space used by Mehra and Prescott in 1985. The Epstein-Zin SDF space is spanned by two state variables, namely the consumption growth and the market return. But the CRRA (constant-relative-risk aversion) based SDF space is only spanned by the single consumption growth state variable. So, within a relatively reasonable range of the parameters, the Epstein-Zin based model is capable of achieving a smaller pricing error for the same asset space.

The symmetric theorem of asset pricing provides a way to value non-tradable factors, such as economic indices, by reflexively using market assets and their corresponding market prices. The expanding theorem of asset pricing provides a bottom-up way to construct a correctly pricing SDF for an asset space. Based on correctly pricing SDFs for subspaces of the asset space and other covariance information, we can find a unique SDF, in a minimum complete expansion of the sum of SDF subspaces, to price whole portfolios in the asset space correctly. The compression theorem of asset pricing provides a top-down way to construct asset pricing models. To price well-diversified asset portfolios with K-factor structures correctly, a necessary and sufficient condition for an SDF space to have a unique correctly pricing SDF is that the SDF space possesses a K-factor structure as well. In other words, both spaces have no idiosyncratic risk and only K factor risks are left to be considered. Cochrane (2000) has used this fact without giving a rigorous proof. A combination of the expanding theorem and the compression theorem can provide a routine way to value portfolios at different levels. Based on the theory of corporate finance, the theory of interest rate and the theory of derivative pricing, the valuation of an individual security has been well developed. However, portfolio valuation, in particular, risk arbitrage portfolio valuation, well-diversified portfolio valuation or index valuation, is less well developed. The pricing error theorem of asset pricing indicates a way of measuring how well a given SDF does the pricing job, by first projecting it and the unknown correctly pricing SDF into the asset space, and then measuring the closeness between the two projected proxies by using, for example, the Hansen-Jagannathan distance. There are three possible sources of pricing errors: the difference in the means of the proxies, the difference in the volatilities of the two proxies, and the imperfect correlation between the two proxies. Empirical results suggest that the main contribution to the pricing error is the difference in volatility.

In a multi-period framework, we propose to link CPPI (constant proportion portfolio insurance) with Merton's consumption pricing model with minimal constraint on consumption.

Throughout this book, various real examples are used to illustrate ideas and applications in practice.

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