Preface

One of us (Balachandran) gave a course of lectures on "Fuzzy Physics" during spring, 2002 for students of Syracuse and Brown Universities. The course which used video conferencing technology was also put on the websites [1]. Subsequently A.P. Balachandran, S. Kürkçüoğlu and S.Vaidya decided to edit the material and publish them as lecture notes. The present book is the outcome of that effort.

The recent interest in fuzzy physics begins from the work of Madore [2, 3] even though the basic physical and mathematical ideas are older and go back to Hoppe [4]. Many of the mathematical developments are based on the works of Kostant and Kirillov [5] and Berezin [6]. They emerge from the fundamental observation that coadjoint orbits of Lie groups are symplectic manifolds which can therefore be quantized under favorable circumstances. When that can be done, we get a quantum representation of the manifold. It is the fuzzy manifold for the underlying "classical manifold". It is fuzzy because no precise localization of points thereon is possible. The fuzzy manifold approaches its classical version when the effective Planck's constant of quantization goes to zero.

Our interest will be in compact simple and semi-simple Lie groups for which coadjoint and adjoint orbits can be identified and are compact as well. In such a case these fuzzy manifold is a finite-dimensional matrix algebra on which the Lie group acts in simple ways. Such fuzzy spaces are therefore very simple and also retain the symmetries of their classical spaces. These are some of the reasons for their attraction.

There are several reasons to study fuzzy manifolds. Our interest has its roots in quantum field theory (qft). Qft's require regularization and the conventional nonperturbative regularization is lattice regularization. It has been extensively studied for over thirty years. It fails to preserve space-time symmetries of quantum fields. It also has problems in dealing with topological subtleties like instantons, and can deal with index theory and axial anomaly only approximately. Instead fuzzy physics does not have these problems. So it merits investigation as an alternative tool to investigate qft's.

A related positive feature of fuzzy physics, is its ability to deal with supersymmetry(SUSY) in a precise manner [7–10]. (See however,[11]). Fuzzy SUSY models are also finite-dimensional matrix models amenable to numerical work, so this is another reason for our attraction to this field.

Interest in fuzzy physics need not just be utilitarian. Physicists have long speculated that space-time in the small has a discrete structure. Fuzzy space-time gives a very concrete and interesting method to model this speculation and test its consequences. There are many generic consequences of discrete space-time, like CPT and causality violations, and distortions of the Planck spectrum. Among these must be characteristic signals for fuzzy physics, but they remain to be identified.

We are a part of a collaboration on fuzzy physics and noncommutative geometry. These lecture notes have been strongly influenced by our interactions with our colleagues in this collaboration. We thank them for discussions and ideas. The work of APB has been supported in part by DOE under contract number DE-FG02-85ER40231. The work of SK is supported by Irish Research Council for Science Engineering and Technology(IRCSET).

These lecture notes are not exhaustive, and reflect the research interests of the authors. It is our hope that the interested reader will be able to learn about the topics we have not covered with the help of our citations.