

*Errata and addenda to*

## Equilibrium and Non-equilibrium Statistical Mechanics

by

Carolyn M. Van Vliet, Ph.D.

June 2008, World Scientific Publishing Co. Pte. Ltd., Singapore

À propos : *Everyone who ever published an article has faced the conundrum that errors tend to only become visible when the paper appears in print; it is not any different with publishing a book, particularly one of this length. Since this book was prepared in camera-ready form, the responsibility rests solely with the author. We apologise for inconveniences that ‘essential errors’ may have caused; also for the many instances of wrongly referenced equations. Our main solace: the Publisher will incorporate the changes noted below in a second, revised printing.*

Errors, noticed while we read, re-read, and taught graduate courses from this textbook, along with various improvements that presented themselves upon renewed reflection, are listed by *chapter, page, alinea and ‘line’*. *Lines are counted from the left margin by paragraph, irrespective of disruption by display equations.*

### Chapter 1

- 24, Eq. (1.6-30): Last factor on lhs should read  $(N - N_1 - \dots - N_{K-2})! / N_{K-1}!(N - \sum N_j)!$   
27, §2, 2: Substituting  $y^2 = \alpha x^2$ .

### Chapter II

- 54, §2, 3: Section 1.5  
55, §3, 6:  $\Delta\Omega = h^{fN} N!$   
57, §2, 4: Comparing (2.5-3) and (2.5-3')  
64, Problem 2.6: where  $\mathcal{L}$  is the *superoperator* ... defined by  $\mathcal{L}K = (1/\hbar)[\mathcal{H}, K]$ .

### Chapter III

- 73, §3, 6: cf. (2.4-15).  
76, Eq. (3.5-15): lhs should read  $S(T, N) / k_B$   
§3, 3: where  $\alpha = \hbar\omega / k_B T$ .  
79, Eq. (3.5-30): add two minus signs,  $-(\partial\zeta / \partial T)_{V, N}$  and  $-(\partial\zeta / \partial V)_{T, N}$   
83, Eq. (3.6-14):  $(\partial^2 G / \partial T^2)_{E, N, \{A_i\}} = \dots$   
§2, 4: of gas and liquid balance, ... (omit non-applicable symbols)  
86, §1, 4: where on the rhs, strictly speaking,  $\mathbf{a} = \langle \eta | \mathbf{a} | \eta \rangle$ .

## Chapter IV

94, Eqs. (4.1-1) and (4.1-2) should have  $\mathcal{E}^c$  instead of  $\mathcal{E}_c$ .

118, §1, 3: after  $k \ll N$  add footnote <sup>18c</sup>:

$$^{18c} \text{ For the general term we have: } \frac{1}{k!} \binom{N}{2}^k \left( \frac{2b_2}{V} \right)^k \approx \frac{1}{k!} N^k \left( \frac{Nb_2}{V} \right)^k \approx \binom{N}{k} \left( \frac{Nb_2}{V} \right)^k. \quad (4.4-51')$$

123, Eq. (4.5-15):  $= -k_B T [(1-na)\zeta_0 + n \ln K(\zeta_0)]$ .

130, §2, 8: add footnote <sup>31a</sup>:

$$^{31a} f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z-z_0)^{n+1}} dz; \text{ for a MacLaurin series } f(z) = \sum_n a_n z^n, \quad a_n = \frac{1}{2\pi i} \oint \frac{f(z)}{z^{n+1}} dz.$$

$$130, \text{ Eq. (4.7-14): } l^2 b_l = \frac{1}{2\pi i} \oint \frac{\xi(dw/d\xi)}{\xi^{l+1}} d\xi = \frac{1}{2\pi i} \oint \frac{\exp(l \sum_k \beta_k w^k)}{w^l} dw. \quad (4.7-14)$$

131, §1, 7: add, below, *depicting two-body, three-body, four-body interactions, etc.*

137, Fig. 4-13: the lettering of the straight lines should read: *slope* =  $k_B T_2 / pJ_0$ , etc.

146, Fig. 4-17: in the ordinate replace  $F$  by  $\Phi$ .

168, §1, 1: plane except at the poles  $z = 0, -1, -2, \dots$

## Chapter V

171, Fig. 5-1: lettering between the boxes,  $\mathcal{E} = A_0$

## Chapter VI

207, first Eq. (6.3-8): add  $d\omega$  before the slash.

second Eq. (6.3-8): relabel (6.3-9); Eq. (6.3-9), relabel (6.3-9')

211, Eq. (6.5-3): add  $d^3q'$  in  $[\ ]^{-1}$

215, Fig. 6-3(b): the  $\xi$  most to the right should be replaced by  $\zeta$

217, §3, 7-8: replace by a new paragraph,

We shall introduce the characteristic temperature  $\theta_{rot}$  by  $k_B \theta_{rot} = \hbar^2 / 2I$  and the parameter  $\sigma$  by  $\sigma = \theta_{rot} / T$ . For very small  $\sigma$  the series (6.7-14) is rapidly convergent. For moderately small values,  $\sigma (\leq 1/2)$ , the convergence is poor; we will find another series to be more useful. Generally, rotator partition functions can be expressed in ...

227, Problem 6.7, part (d): replace by

(d) Let  $L$  be large; find again  $P/T$  and confirm the barometric height formula.

## Chapter VII

255, Eq. (7.8-1): replace  $H$  by  $\mathcal{H}$ .

### Chapter VIII

- 264, Fig. 8-1: A small tangent segment  $dS$  should be drawn  $\perp$  to the dashed line ---  
 266, Eq. (8.9-2): last factor  $(n_x^2 + n_y^2 + n_z^2)$   
 274, §1, 8: rather than  $\propto T^{3/2}$ . Omit line 9.  
 287, Fig. 8-12: the three dashed lines emanating from point O should be parabolically curved.  
 289, Eq. (8.6-2):  $e_{\alpha\beta} = \partial u_\beta / \partial x_\alpha \equiv \partial_\alpha u_\beta$ , or  $\mathbf{e} = \text{Grad } \mathbf{u}$ . (8.6-2)  
 290, §2, 3:  $\mathbf{P}_{\mathbf{q},\mathbf{b}}$  and  $\mathbf{Q}_{\mathbf{q},\mathbf{b}}$  (bold)  
 294, Eq. (8.6-29): remove the exponent  $1/2$  in the first bracket

### Chapter IX

- 305, §2, 1-7: replace by

We will now find the response function  $\varphi(\mathbf{r}) \equiv \delta m(\mathbf{r}) / \delta h(\mathbf{r})$  and the generalized susceptibility  $\chi = \int d^3r \varphi(\mathbf{r})$ . Since there is no integral equation for  $m(\mathbf{r})$  in terms of  $h(\mathbf{r})$ , we must do the functional differentiation in (9.2-13) for this case *implicitly*. This is carried out by assuming that we have a small perturbation centred on  $\mathbf{r}'$ , i.e. we set  $h(\mathbf{r}) = h_0(\mathbf{r}) + h_1 \delta(\mathbf{r} - \mathbf{r}')$ , with response  $m(\mathbf{r}) = m_0(\mathbf{r}) + h_1 \varphi(\mathbf{r})$ . Substituting into (9.2-13) and retaining only terms linear in  $h_1$ , one obtains

$$\underbrace{h_0(\mathbf{r})}_{ss} + h_1 \delta(\mathbf{r} - \mathbf{r}') = \underbrace{a_2 m_0(\mathbf{r})}_{ss} + a_2 h_1 \varphi(\mathbf{r}) + \underbrace{a_4 [m_0(\mathbf{r})]^3}_{ss} + 3a_4 [m_0(\mathbf{r})]^2 h_1 \varphi(\mathbf{r}) + \dots - \underbrace{b \nabla^2 m_0(\mathbf{r})}_{ss} - b h_1 \nabla^2 \varphi(\mathbf{r}). \quad (9.2-13')$$

Cancelling the steady-state terms and dividing out  $h_1$ , we find for  $\varphi(\mathbf{r})$ :

$$\nabla^2 \varphi - \xi^{-2} \varphi = -b^{-1} \delta(\mathbf{r} - \mathbf{r}'), \quad (9.2-14)$$

with

$$\xi = \sqrt{\frac{b}{a_2 + 3a_4 m_0^2(\mathbf{r})}} = \begin{cases} \sqrt{b / a_{20}(T - T_c)}, & T > T_c, \\ \sqrt{b / 2a_{20}(T_c - T)}, & T < T_c. \end{cases} \quad (9.2-14')$$

- 316, Eq. (9.4-22): upper line should read

$$C_V = x_g C_{V_g} + x_\ell C_{V_\ell} + x_g \left( \frac{\partial \tilde{V}_g}{\partial T} \right)_{coex} \left[ T \left( \frac{\partial \tilde{S}_g}{\partial \tilde{V}_g} \right)_T - T \frac{dP}{dT} \right]$$

- 317, Eq. (9.4-26), second part should be  $+x_\ell \left[ C_{V_\ell} - T \left( \frac{\partial \tilde{V}_\ell}{\partial T} \right)_{coex}^2 \left( \frac{\partial P}{\partial \tilde{V}_\ell} \right)_T \right]$ . (9.4-26)

## Chapter X

- 340, §1, 3: result is  $\text{tr}[\dots]^N$ , where ‘tr’ denotes the matrix trace.<sup>9a</sup> Add footnote<sup>9a</sup>:  
<sup>9a</sup> Henceforth tr denotes the trace of a matrix, while Tr will be reserved for the trace of an operator.
- 346, §1, 28-29: ..be  $J_1/k_B T$ ; the points to the left of the line indicate high temperatures (disordered state) and those to the right low temperatures (ordered state).

## Chapter XI

- 380, Eq. (11.2-25):  $c_{M+1} = -c_1$ ,  $c_{M+1}^\dagger = -c_1^\dagger$ , (anti-periodic) for  $n$  even ,  
 $c_{M+1} = c_1$ ,  $c_{M+1}^\dagger = c_1^\dagger$ , (periodic) for  $n$  odd .
- 383, Eq. (11.2-42), last line:  $V_{\bar{q}} = (V_{1\bar{q}})^{1/2} V_{2\bar{q}} (V_{1\bar{q}})^{1/2}$ ,  $V_{0 \text{ or } \pi} = V_{10 \text{ or } 1\pi} V_{20 \text{ or } 2\pi}$  .
- 383, Eq. (11.2-40): there is a minus sign in front of  $2K_1^*$   
 $V_{10} = \exp[-2K_1^*(c_0^\dagger c_0 - \frac{1}{2})]$ ,  $V_{1\pi} = \exp[-2K_1^*(c_\pi^\dagger c_\pi - \frac{1}{2})]$ ,  
 $V_{20} = \exp(2K_2 c_0^\dagger c_0)$ ,  $V_{2\pi} = \exp(-2K_2 c_\pi^\dagger c_\pi)$ . (11.2-40)
- 384, Eq. (11.2-45)  $\rightarrow$  (11.2-45a):  $c_q^\dagger c_q + c_{-q}^\dagger c_{-q} = 2b_q^\dagger b_q \rightarrow 2\beta_q^+ \beta_q^- \equiv \beta_{q,z} + \mathbf{1}$ ,  
 §2, 12:  $(\bar{\beta}_q)^2 = \mathbf{1}$  since...; label the last two equation as (11.2-45b), (11.2-46)
- 385, Eq. (11.2-47), first line  $V_{2q} = \exp\{2K_2[(\beta_{q,z} + \mathbf{1})\cos q + \beta_{qx} \sin q]\}$
- 388, §2, 3-4, Eq. (11.3-3): We now turn to the case of odd total fermion occupancy. We need the eigenvalues for the operators in (11.2-40); remembering that the 0-state is occupied and the  $\pi$ -state is empty, we find

$$\begin{cases} V_{10} = e^{-K_1^*}, \\ V_{20} = e^{2K_2}, \end{cases} \quad \text{and} \quad \begin{cases} V_{1\pi} = e^{K_1^*}, \\ V_{2\pi} = 1 \end{cases} . \quad (11.3-3)$$

- 393, Eq. (11.3-27): lhs reads  $\hat{u}(T) = (d/d\beta)[\beta \hat{f}(0, T)] = \dots$
- 395, §2, 8: the sum of the squares, as well as the short-range correlations, disappear
- 396, second reference, footnote<sup>23</sup>: M. Kac, Duke Math. Journal **21**, 501-509 (1954);
- 412, Problem 11.1, Eq. (1) should be amended to read

$$V_2^{1/2} = \sqrt{A} \exp(\frac{1}{2} K^* \sigma_x) = \sqrt{A} [(\cosh \frac{1}{2} K^*) \mathbf{1} + (\sinh \frac{1}{2} K^*) \sigma_x], \quad (1)$$

$$V_1 = \exp(\beta h \sigma_z) = (\cosh \beta h) \mathbf{1} + (\sinh \beta h) \sigma_z = (\exp \beta h) \sigma_z .$$

Problem 11.3 should be replaced by:

- 11.3. Obtain the forms of the four particular diagonal operators given in Eq.(11.2-40).

While these are all the necessary and recommended changes for Chapter XI, the chapter can be improved *in presentation* if two types of changes are made additionally. On p. 382 one might do the splitting into positive and negative  $q$  somewhat later. On p. 399 the results are obtained faster if the sign of the  $q$ 's is changed for the *annihilation* operators, rather than for the creation operators. We thus offer new *optional* pages 382, 383, 384 and 399.

Hence we have, changing  $j' \rightarrow j$ ,

$$c_j = \frac{e^{-i\pi/4}}{\sqrt{M}} \sum_q c_q e^{iqj}; \quad \text{likewise} \quad c_j^\dagger = \frac{e^{i\pi/4}}{\sqrt{M}} \sum_q c_q^\dagger e^{-iqj}. \quad (11.2-32)$$

The same lemmas can be used to show that the  $c_q$ 's satisfy the fermion anticommutation rule

$$[c_q, c_{q'}^\dagger]_+ = \delta_{q,q'}; \quad (11.2-33)$$

the proof is left to the reader.

We must fix the store of  $q$ -values, such that the b.c. (11.2-25) are satisfied. We thereto set,

$$q = k\pi/M, \quad \text{with} \quad \begin{cases} k = \pm 1, \pm 3, \pm 5, \dots, \pm(M-1), & \text{for } n \text{ even,} \\ k = 0, \pm 2, \pm 4, \dots, \pm M & , \text{ for } n \text{ odd.} \end{cases} \quad (11.2-34)$$

(we assumed without undue restriction that  $M$  is even). In each case there are  $M$  values of  $q$ , like in the first Brillouin zone of a solid; note that for  $n$  odd we omitted the value  $k = -M$ , corresponding to  $q = -\pi$ .

We must now transform the operators  $V_1$  and  $V_2$ . The exponential in  $V_2$  gives

$$\begin{aligned} & \sum_j (c_j^\dagger - c_j)(c_{j+1}^\dagger + c_{j+1}) \\ &= \frac{1}{M} \sum_j \sum_{q,q'} \left( e^{i\pi/4} c_q^\dagger e^{-iqj} - e^{-i\pi/4} c_q e^{iqj} \right) \left( e^{i\pi/4} c_{q'}^\dagger e^{-iq'j} e^{-iq'} + e^{-i\pi/4} c_{q'} e^{iq'j} e^{iq'} \right) \\ &= \frac{1}{M} \sum_{q,q'} \sum_j \left\{ ic_q^\dagger c_{q'}^\dagger \underbrace{e^{-i(q+q')j}}_{M\delta_{q',-q}} e^{-iq'} + c_q^\dagger c_{q'} \underbrace{e^{-i(q-q')j}}_{M\delta_{q',q}} e^{iq'} - c_q c_{q'}^\dagger \underbrace{e^{i(q-q')j}}_{M\delta_{q',q}} e^{-iq'} + ic_q c_{q'} \underbrace{e^{i(q+q')j}}_{M\delta_{q',-q}} e^{iq'} \right\} \\ &= \sum_q \left\{ ic_q^\dagger c_{-q}^\dagger e^{iq} + c_q^\dagger c_q (e^{iq} + e^{-iq}) - e^{-iq} + ic_q c_{-q} e^{-iq} \right\}; \quad (11.2-35) \end{aligned}$$

we used here Lemma (A) and  $c_q^\dagger c_q + c_q c_q^\dagger = 1$ . Be it further noted that the sum involving  $\exp(-iq)$  is to be omitted.<sup>13</sup> We now split into two sums,  $\sum_{q>0}$  and  $\sum_{q<0}$ ; in the latter we change  $q \rightarrow -q$ . [The terms with  $q = 0$  and  $q = \pi$ , left out in this procedure for odd  $n$ , will be dealt with later.] We thus obtain

$$(11.2-35) \rightarrow \sum_{q>0} \left\{ ic_q^\dagger c_{-q}^\dagger e^{iq} + c_q^\dagger c_q (e^{iq} + e^{-iq}) + ic_q c_{-q} e^{-iq} + ic_{-q}^\dagger c_q^\dagger e^{-iq} + c_{-q}^\dagger c_{-q} (e^{iq} + e^{-iq}) + ic_{-q} c_q e^{iq} \right\}. \quad (11.2-36)$$

<sup>13</sup> By Lemma (B) we have  $\sum_{\text{all } q} \exp(iq) = 0$ , which holds for both sequences in (11.2-34). For the odd case, we have in addition  $\sum_{\text{all } q \neq 0, \pi} \exp(iq) = \sum_{\text{all } \bar{q}} \exp(i\bar{q}) = 0$ , since the  $q = 0$  and  $q = \pi$  terms cancel.

By changing the first and last term with the anticommutation rules, we then easily arrive at:

$$(11.2-36) \rightarrow 2 \sum_{q>0} \left[ \cos q \left( c_q^\dagger c_q + c_{-q}^\dagger c_{-q} \right) + \sin q \left( c_{-q}^\dagger c_q^\dagger + c_q c_{-q} \right) \right]. \quad (11.2-37)$$

We now note that operators with different wave numbers commute; this entails an enormous simplification, since the various  $q$ -contributions of the exponent can be written as a product of transfer-operator factors

$$V_2 = \prod_{q>0} V_{2q}, \text{ with} \\ V_{2q} = \exp \left\{ 2K_2 \left[ \cos q \left( c_q^\dagger c_q + c_{-q}^\dagger c_{-q} \right) + \sin q \left( c_{-q}^\dagger c_q^\dagger + c_q c_{-q} \right) \right] \right\}. \quad (11.2-38)$$

The  $c$ -operators are now uncoupled, except for the mixing of  $q$  and  $-q$ . We leave it to the reader to obtain the result for  $V_1$  by a similar procedure,

$$V_1 = (2 \sinh 2K_1)^{M/2} \prod_{q>0} V_{1q}, \text{ with} \\ V_{1q} = \exp \left\{ -2K_1^* \left( c_q^\dagger c_q + c_{-q}^\dagger c_{-q} - 1 \right) \right\}. \quad (11.2-39)$$

For the case of odd  $n$  we still also need  $V_{1q}$  and  $V_{2q}$  for  $q=0$  and  $q=\pi$ . These operators are given by (see Problem 11.3):

$$V_{10} = \exp \left[ -2K_1^* \left( c_0^\dagger c_0 - \frac{1}{2} \right) \right], \quad V_{1\pi} = \exp \left[ -2K_1^* \left( c_\pi^\dagger c_\pi - \frac{1}{2} \right) \right], \\ V_{20} = \exp \left( 2K_2 c_0^\dagger c_0 \right), \quad V_{2\pi} = \exp \left( -2K_2 c_\pi^\dagger c_\pi \right). \quad (11.2-40)$$

They are already in diagonal form and commute with each other.

To compute the partition function we need to evaluate – see Eq. (11.2-13) :

$$\mathcal{Z}_{(even)} = \text{Tr} \left\{ \left[ (2 \sinh 2K_1)^{M/2} \prod_{q>0} V_q \right]^N \right\}, \\ V_q = (V_{1q})^{1/2} V_{2q} (V_{1q})^{1/2}, \quad (11.2-41)$$

where the  $q$ 's are chosen according to the upper line of (11.2-34). For a state with odd total occupancy the effect of the diagonal operators (11.2-40) must be added on to the above product; to keep their joint occupancy odd, we assume the 0-state is occupied while the  $\pi$ -state is left empty. Moreover, the wave-vectors of the lower line of (11.2-34) will be formally assigned the wave vector  $\bar{q}$ . We thus have

$$\mathcal{Z}_{(odd)} = \text{Tr} \left\{ \left[ (2 \sinh 2K_1)^{M/2} \left( \prod_{\bar{q}>0} V_{\bar{q}} \right) V_0 V_\pi \right]^N \right\}, \\ V_{\bar{q}} = (V_{1\bar{q}})^{1/2} V_{2\bar{q}} (V_{1\bar{q}})^{1/2}, \quad V_{0 \text{ or } \pi} = V_{10 \text{ or } 1\pi} V_{20 \text{ or } 2\pi}. \quad (11.2-42)$$

For now all wave-vectors will for convenience still be denoted by 'q'. For a given q the occupation-number space involves the four basic states

$$\begin{aligned} |0_{-q}0_q\rangle, & & |0_{-q}1_q\rangle &= c_q^\dagger |0_{-q}0_q\rangle, \\ |1_{-q}0_q\rangle &= c_{-q}^\dagger |0_{-q}0_q\rangle, & |1_{-q}1_q\rangle &= c_{-q}^\dagger c_q^\dagger |0_{-q}0_q\rangle. \end{aligned} \quad (11.2-43)$$

For simplicity we omitted in the kets all q' with q' ≠ q or -q; also, without loss of generality, we assumed that the total preceding occupancy is even so that there is no factor (-1)<sup>ξ</sup> to account for and we took -q to precede q. [NB. The state |0<sub>-q</sub>0<sub>q</sub>⟩ above is *not* the ground state, which will be defined later.] Since V<sub>1q</sub> only depends on the diagonal operators n<sub>q</sub> and n<sub>-q</sub>, their action on |0<sub>-q</sub>1<sub>q</sub>⟩ and |1<sub>-q</sub>0<sub>q</sub>⟩ leaves these states unaltered except for a factor (c-number). As to V<sub>2q</sub>, the cos part is diagonal and the sin part can produce off-diagonal matrix elements only when acting on states that differ by two in fermion occupancy. These states can therefore be deleted from the set (11.2-43), leaving as basis states the *pseudo-spinor* {|1<sub>-q</sub>1<sub>q</sub>⟩ |0<sub>-q</sub>0<sub>q</sub>⟩}.<sup>14</sup>

Now we have V<sub>1q</sub><sup>1/2</sup>|1<sub>-q</sub>1<sub>q</sub>⟩ = [exp(-K<sub>1</sub><sup>\*</sup>)]|1<sub>-q</sub>1<sub>q</sub>⟩ and V<sub>1q</sub><sup>1/2</sup>|0<sub>-q</sub>0<sub>q</sub>⟩ = [exp K<sub>1</sub><sup>\*</sup>]|0<sub>-q</sub>0<sub>q</sub>⟩. Hence,

$$V_{1q}^{1/2} = \begin{pmatrix} \langle 1_{-q}1_q | V_{1q}^{1/2} | 1_{-q}1_q \rangle & \langle 1_{-q}1_q | V_{1q}^{1/2} | 0_{-q}0_q \rangle \\ \langle 0_{-q}0_q | V_{1q}^{1/2} | 1_{-q}1_q \rangle & \langle 0_{-q}0_q | V_{1q}^{1/2} | 0_{-q}0_q \rangle \end{pmatrix} = \begin{pmatrix} e^{-K_1^*} & 0 \\ 0 & e^{K_1^*} \end{pmatrix}. \quad (11.2-44)$$

To obtain the matrix for V<sub>2q</sub> we must employ more armour. Since both pseudo-spinor states involve two occupancies, Schultz et al. introduce pair operators b<sub>q</sub> = c<sub>q</sub>c<sub>-q</sub> and b<sub>q</sub><sup>†</sup> = c<sub>-q</sub><sup>†</sup>c<sub>q</sub><sup>†</sup>, similar to the Cooper-pair operators in BCS theory, discussed in the next chapter. Clearly, c<sub>-q</sub><sup>†</sup>c<sub>q</sub><sup>†</sup> + c<sub>q</sub>c<sub>-q</sub> = b<sub>q</sub><sup>†</sup> + b<sub>q</sub>. For the diagonal operators note that, as far as operations on pair states are concerned, we can set c<sub>q</sub><sup>†</sup>c<sub>q</sub> = c<sub>-q</sub><sup>†</sup>c<sub>-q</sub>c<sub>q</sub><sup>†</sup>c<sub>q</sub> = b<sub>q</sub><sup>†</sup>b<sub>q</sub> and similarly for c<sub>-q</sub><sup>†</sup>c<sub>-q</sub>. Now as indicated in connection with BCS theory [Eq. (12-6-2')], the pair operators have commutation properties identical to spin lowering and raising operators, *without* the sign-change problem of the Jordan-Wigner transformation. Thus b<sub>q</sub> → β<sub>q</sub><sup>-</sup>, b<sub>q</sub><sup>†</sup> → β<sub>q</sub><sup>+</sup>. In *this* (isomorphic) spin-space we have a representation by Pauli spin matrices such that

$$c_q^\dagger c_q + c_{-q}^\dagger c_{-q} = 2b_q^\dagger b_q \rightarrow 2\beta_q^+ \beta_q^- \equiv \beta_{q,z} + \mathbf{1}, \quad (11.2-45a)$$

$$c_{-q}^\dagger c_q^\dagger + c_q c_{-q} = b_q^\dagger + b_q \rightarrow \beta_q^+ + \beta_q^- \equiv \beta_{q,x}. \quad (11.2-45b)$$

We still need another operator

$$\bar{\beta}_q \equiv \beta_{q,z} \cos q + \beta_{q,x} \sin q, \quad (11.2-46)$$

which has the important property that (β<sub>q</sub>)<sup>2</sup> = 1 since [β<sub>q,z</sub>, β<sub>q,x</sub>]<sub>+</sub> = 0 [analogous to (11.1-13)]. For V<sub>2q</sub> this yields

<sup>14</sup> As elsewhere in this text, we use (...) for a row matrix and {...} for a column matrix to reduce space.

$$\begin{aligned}
c_{n+1} &= M^{-1/2} e^{-i\pi/4} \sum_q e^{iqn} e^{iq} (\eta_q \cos \phi_q - \eta_{-q}^\dagger \sin \phi_q), \\
c_{n+1}^\dagger &= M^{-1/2} e^{i\pi/4} \sum_q e^{-iqn} e^{-iq} (\eta_q^\dagger \cos \phi_q - \eta_{-q} \sin \phi_q). \quad (11.4-21)
\end{aligned}$$

For the annihilation operators we change the summation variables  $q' \rightarrow -q'$ ,  $q \rightarrow -q$ ; noting that  $\phi_{-q} = -\phi_q$ , we get

$$\begin{aligned}
c_m^\dagger - c_m &= M^{-1/2} e^{-i\pi/4} \sum_{q'} e^{-iq'm} (-\eta_{-q'} + i\eta_{q'}^\dagger) (\cos \phi_{q'} + i \sin \phi_{q'}), \\
c_{n+1}^\dagger + c_{n+1} &= M^{-1/2} e^{-i\pi/4} \sum_q e^{-iqn} e^{-iq} (\eta_{-q} + i\eta_q^\dagger) (\cos \phi_q - i \sin \phi_q). \quad (11.4-22)
\end{aligned}$$

Now  $(\eta_{-q} + i\eta_q^\dagger) |\bar{0}_{-q} \bar{0}_q\rangle = i |\bar{0}_{-q} \bar{1}_q\rangle$ . Next, acting on this with  $(-\eta_{-q'} + i\eta_{q'}^\dagger)$  we see that  $-q'$  must be equal to  $q$ , in order to recreate the ground state; the operator  $i\eta_{q'}^\dagger = i\eta_{-q}^\dagger$  has no effect since it creates the state  $|\bar{1}_{-q} \bar{1}_q\rangle$ , which is orthogonal to  $|\bar{0}_{-q} \bar{0}_q\rangle$ . Therefore, the result is<sup>26</sup>:

$$\begin{aligned}
\langle \Psi_0 | (c_m^\dagger - c_m) (c_{n+1}^\dagger + c_{n+1}) | \Psi_0 \rangle \\
&= M^{-1} e^{-i\pi/2} \sum_q e^{-iq(n-m)} e^{-iq} e^{-i\pi/2} (\cos \phi_q - i \sin \phi_q)^2. \\
&= M^{-1} \sum_q e^{-iq(n-m)} e^{-i(q+2\phi_q)} = a_{m,n}. \quad (11.4-23)
\end{aligned}$$

Summing all pair products with their appropriate parity we obtain the Toeplitz determinant<sup>27</sup>

$$\langle \sigma_{m,x} \sigma_{m',x} \rangle_{eq} = \begin{vmatrix} a_{m,m} & a_{m,m+1} & \cdots & a_{m,m'-1} \\ a_{m+1,m} & \cdot & & \cdot \\ \vdots & \vdots & & \vdots \\ a_{m'-1,m} & \cdot & \cdots & a_{m'-1,m'-1} \end{vmatrix} \rightarrow \begin{vmatrix} a_0 & a_1 & \cdots & a_{m'-1} \\ a_{-1} & \cdot & & \cdot \\ \vdots & \vdots & & \vdots \\ a_{-m'+1} & a_{-m'+2} & \cdots & a_0 \end{vmatrix}. \quad (11.4-24)$$

In the second form, used by MPW<sup>8</sup>, the correlation runs from the site  $m'$  to site 0, the rank of the determinant being  $m'$ . The Toeplitz determinant, denoted as  $T(F)$ , is closely related to the cyclic determinant  $C(F)$ , as will now be indicated.<sup>28</sup>

<sup>26</sup> Since  $\phi_q$  is defined modulo  $\pi/2$  [see (11.2-64)] we absorb the factor  $\exp(-i\pi)$  into  $\exp(-2i\phi_q)$ .

<sup>27</sup> In a Toeplitz determinant the element  $a_{ij}$  depends only on  $|i-j|$ .

<sup>28</sup> For a cyclic determinant  $a_{m+M, m'+M} = a_{m, m'}$ ; for a Toeplitz determinant of rank  $M$ ,  $a_{m, m'} = 0$  for  $m, m' > M$ . For an example see Mattis, Op. Cit., p. 270.

## Chapter XII

427, Eq. (12.3-4) has the creation and annihilation operators under the sum,

$$\frac{1}{2V^2} \sum_{\bar{\mathbf{k}}, \bar{\mathbf{k}}', \mathbf{k}, \mathbf{k}'} a_{\bar{\mathbf{k}}}^\dagger a_{\bar{\mathbf{k}}}^\dagger a_{\mathbf{k}'} a_{\mathbf{k}} \sum_{\mathbf{q}} V_{\mathbf{q}} \underbrace{\int d^3 r e^{-i\mathbf{r} \cdot (\bar{\mathbf{k}} - \mathbf{k} - \mathbf{q})}}_{V \delta_{\bar{\mathbf{k}}, \mathbf{k} + \mathbf{q}}} \underbrace{\int d^3 r' e^{-i\mathbf{r}' \cdot (\bar{\mathbf{k}}' - \mathbf{k}' + \mathbf{q})}}_{V \delta_{\bar{\mathbf{k}}', \mathbf{k}' - \mathbf{q}}}, \quad (12.3-4)$$

430, Eq. (12.3-18), 1<sup>st</sup> line:  $\mathcal{H}_{grand} = -\frac{1}{2} \varphi_0 \rho_0 n_0 I + \frac{1}{2} \sum_{\mathbf{k}} (\bar{\mathbf{n}}_{\mathbf{k}} + \bar{\mathbf{n}}_{-\mathbf{k}} + 1) \{ [\varepsilon(\mathbf{k}) + \varphi_0 \rho_0]$

431, Eq. (12.3-24): remove  $\langle \rangle$  brackets,  $\mathbf{N} = \mathbf{n}_0 + \sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} = n_0 I + \sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$ .

444, §1, 17:  $v_0 \rho(0) \gg 1, \dots$ ; similarly 449, §2, 6

447, §2, 3-7: we have rewritten the too terse outline in the book as follows:

‘where  $|0\rangle$  is the vacuum state. Normalization gives  $C^{-1} = \prod_{\mathbf{k}} \sqrt{1 + g_{\mathbf{k}}^2}$ . For the normal state  $g_{\mathbf{k}} = \infty$ ,  $|\mathbf{k}| < k_F$  and  $g_{\mathbf{k}} = 0$ ,  $|\mathbf{k}| > k_F$ . Indeed,

$$\prod_{\mathbf{k}} b_{\mathbf{k}}^\dagger |0\rangle = \prod_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger |0\rangle = \prod_{\mathbf{k}, \sigma} c_{\mathbf{k}, \sigma}^\dagger |0\rangle \equiv |O\rangle. \quad (12.6-6)$$

For the condensed state, the variational principle requires that upon variation of  $g_{\mathbf{k}}$

$$W \equiv \langle \Psi_0 | \mathcal{H}_{gr, red} | \Psi_0 \rangle \quad (12.6-7)$$

be a minimum. Substituting (12.6-4) and (12.6-5), one finds that the vacuum state is reproduced *iff*  $W$  takes the form

$$W = 2 \sum_{\mathbf{k}} [\varepsilon(\mathbf{k}) - \mu] \frac{g_{\mathbf{k}}^2}{1 + g_{\mathbf{k}}^2} + \sum_{\mathbf{k}, \mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{g_{\mathbf{k}} g_{\mathbf{k}'}}{(1 + g_{\mathbf{k}}^2)(1 + g_{\mathbf{k}'}^2)}. \quad (12.6-8)$$

The BCS theory is most easily handled if the following quantities are introduced

$$u_{\mathbf{k}} = 1 / \sqrt{1 + g_{\mathbf{k}}^2}, \quad v_{\mathbf{k}} = g_{\mathbf{k}} / \sqrt{1 + g_{\mathbf{k}}^2}, \quad (12.6-9)$$

with  $u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 = 1$ . This means that we must minimize the variational quantity  $W$ :

$$W = 2 \sum_{\mathbf{k}} [\varepsilon(\mathbf{k}) - \mu] v_{\mathbf{k}}^2 + \sum_{\mathbf{k}, \mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}} v_{\mathbf{k}} u_{\mathbf{k}'} v_{\mathbf{k}'}, \quad (12.6-10)$$

$$\delta W = \delta \left\{ 2 \sum_{\mathbf{k}} [\varepsilon(\mathbf{k}) - \mu] v_{\mathbf{k}}^2 + \sum_{\mathbf{k}, \mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}} v_{\mathbf{k}} u_{\mathbf{k}'} v_{\mathbf{k}'} \right\} = 0. \quad (12.6-10')$$

When carrying out the variation we can eliminate  $\delta u$  since  $u \delta u + v \delta v = 0$ ; hence,

457, §1, 2ff. The clarity of these paragraphs is compromised by the *over-usage* of the term quasi-particles. We propose some minor changes by denoting the constituents of the quantum fluid as *pseudo-particles*; then – following Landau – the excitations above the ground state will be referred to as quasi-particles, whether massless or hole-particle pairs. An *optional* new version of p. 457-459 follows.

Due to the exclusion principle and its concomitant need for antisymmetrized wave functions, the theory of Fermi liquids is in many respects more complicated than the Bose-type systems we considered previously. There are a number of readable surveys from which the student may profit. We mention the older treatment of Pines and Nozières (1966<sup>47</sup>), modern reviews by Baym and Pethick (1978<sup>48</sup>) or Leggett (1975<sup>49</sup>) and a useful section in Mahan's book (2000<sup>50</sup>). Detailed considerations need a diagrammatic treatment, as discussed in later sections. Here we just mention that the imaginary part of the self-energy,  $\text{Im}\Sigma$ , is found to be small, so that the spectral function  $A(\mathbf{p}, \omega)$  is peaked and resembles a delta function<sup>51</sup>; we can thus for given  $\hbar\omega$  associate a momentum  $\mathbf{p}$  with the excitations.

The constituents of the ground-state quantum liquid will in this text be denoted as *pseudo-particles*. Basically, we may think of these as 'dressed' <sup>3</sup>He atoms, thereby acquiring an effective mass  $m^*$  (Leggett<sup>49</sup>). Thus, a pseudo-particle in the Fermi-type spectrum can, in a sense, be regarded as an atom in the self-consistent field of the surrounding atoms. Experimental data suggest that  $m^* = 2.76m$ , where  $m$  is the bare mass of the atom. We can also surmise this as follows. The energy  $\varepsilon_{\mathbf{p}}$  should be zero for  $\mathbf{p} = 0$ . A formal expansion about  $\mathbf{p} = 0$  will not have any odd terms – in contrast to a Bose liquid – because of particle-hole symmetry; the first important term is then  $\varepsilon_{\mathbf{p}} \propto p^2$ , which defines the effective mass with  $\varepsilon_{\mathbf{p}} = p^2/2m^*$ . In accordance with Landau's viewpoint we shall also introduce a self-consistent energy  $\tilde{\varepsilon}_{\mathbf{p}}$  which is more than  $\varepsilon_{\mathbf{p}}$ , see the developments below.

Landau's theory, as for the case of <sup>4</sup>He, was guided by experimental results on the specific heat and the compressibility, required for the velocity of sound; it is therefore to be regarded as a phenomenological theory. The following basic assumptions will be made and are upheld by comparison of the theory with the experimental data. First of all, it must be assumed that the interactions can in some way be 'adiabatically turned on', so that the number of pseudo-particles remains the same as the number of separate atoms in the gaseous state. Secondly and as a consequence of this supposition, the pseudo-particles of the ground state obey Fermi–Dirac statistics. Fermi liquid theory now aims primarily at describing low-lying excited states, which naturally involve particle-hole pairs; these excitations or quasi-particles are fermions.<sup>52</sup> In addition, there are two other types of excitations. Density fluctuations or zero-sound waves can be viewed as collective resonances of the

<sup>47</sup> D. Pines and P. Nozières, "The Theory of Quantum Liquids", Benjamin, NY and Amsterdam 1966.

<sup>48</sup> G. Baym and C. Pethick in "The Physics of Liquid and Solid Helium," Part 2 (J.B. Ketterson and K. Benneman, Eds.), J. Wiley, NY 1971, Chapter 1.

<sup>49</sup> A.J. Leggett, Rev. Modern Physics **47**, 331-414 (1975).

<sup>50</sup> G.D. Mahan, Op. Cit., 3<sup>rd</sup> Ed., Kluwer Acad./Plenum, 2000, pp.713-742.

<sup>51</sup> W. Jones and N.H. March, "Theoretical Solid State Physics" Vol. I, Wiley, London 1973, p. 140 ff.

<sup>52</sup> Following Landau, the denotations 'quasi-particles' and 'excitations' will be taken to be synonymous.

primary particle-hole fluid and are carried by phonons.<sup>52a</sup> Further there are spin waves carried by paramagnons; they represent spin fluctuations involving pairs of opposite spin. They are inadequately accounted for by the original Fermi liquid theory, but a number of modern approaches, using quantum-field Hamiltonians of the types we studied before, have been developed. And, last but not least, we must mention that at very low temperatures pairing can take place, resulting in a boson-like superfluid. Contrary to Cooper pairs in BCS theory, these pairs usually have the triplet state, with  $S = 1$ .<sup>52b</sup> Superfluidity in  $^3\text{He}$  was discovered in 1972 by Osheroff, Richardson and Lee.<sup>53</sup> They identified two phases in the liquid, designated  $A$  and  $B$ ; a Nobel prize followed. Curiously, the theory for this phenomenon did exist already, since Balian and Werthamer<sup>54</sup> developed the triplet pairing theory as an alternative for the BSC theory in 1963, in order to (possibly) explain some anomalous results in superconducting materials like Sn, Hg and others. All these developments will be briefly discussed in the next few sections.

We now return to Fermi liquid theory, which is our basic quest in this section. The ground state is a ‘quasi-vacuum’ – as for an electron gas – and will be designated as  $|O\rangle$ , its energy being  $\mathcal{E}_0$ . At  $T=0$  the state is *grosso modo* filled up to the Fermi radius  $p_F$ , although there is some fuzziness for the distribution in systems with interactions, cf. Ref. 51, loc. cit. Fig. 2.5; the distribution will be denoted by  $n_{\mathbf{p}}^0$ . Mainly, this is a fictitious concept, except near the Fermi radius. The ground-state energy will be written as  $\mathcal{E}_0 = \text{const} + \sum_{\mathbf{p},\sigma} n_{\mathbf{p}}^0 \varepsilon_{\mathbf{p}}$ , where  $\sigma$  denotes the spin; the ‘constant’ needs no discussion. At non-zero temperatures the exclusion principle forces many excitations in states outside  $p_F$ ; we shall denote their occupancy by  $n_{\mathbf{p}}$ . We will assume that the average occupancy (Heaviside-like behaviour or step-down function) can still be represented by the ordinary F–D distribution,

$$n_{\mathbf{p}}^0 = n_F(\varepsilon_{\mathbf{p}} - \zeta) = \lim_{T \rightarrow 0} \left\{ 1 / [e^{\beta(\varepsilon_{\mathbf{p}} - \zeta)} + 1] \right\}; \quad (12.7-1)$$

note that the Fermi-Dirac function will be denoted by the middle member for the present considerations. More important is the ‘difference distribution’  $\delta n_{\mathbf{p}} = n_{\mathbf{p}} - n_{\mathbf{p}}^0$ ; for the energy with respect to the ground state we have,

$$\mathcal{E} = \mathcal{E}_0 + \sum_{\mathbf{p},\sigma} \varepsilon_{\mathbf{p}} \delta n_{\mathbf{p}} = \mathcal{E}_0 + \int \Delta_{p\sigma}^3 \varepsilon_{\mathbf{p}} \delta n_{\mathbf{p}}. \quad (12.7-2)$$

where  $\Delta_{p\sigma}^3$  is a shortcut notation for integration over  $p$ -space including the density of states  $1/8\pi^3 \hbar^3$  [we set  $V_0 = \text{unity}$ ] and summation over the spin  $\sigma = \pm 1$ .

It must now be born in mind that the total number of pseudo-particles in the

<sup>52a</sup> The sound-wave phonons in  $^3\text{He}$  are the equivalent of plasmons in an electron gas, cf. Chapter XVI.

<sup>52b</sup> It is customary to use the symbols  $S, s$  and  $\sigma$  (instead of  $I$ ) for the nuclear spin.

<sup>53</sup> D.D. Osheroff, R.C. Richardson and D.M. Lee, Phys. Rev. Lett. **28**, 885 (1972).

<sup>54</sup> R. Balian and N.R. Werthamer, Phys. Rev. **131**, 1553 (1963).

ground state  $N_0$  is not a constant of motion, nor is the number of quasi-particles (or excitations)  $\Sigma \delta n_{\mathbf{p}}$  above the ground state. Due to their fluctuations we need an ensemble with specified chemical potential, such as we used in previous sections. Fortunately, for the Fermi gas  $\zeta$  is hardly affected by  $\Sigma \delta n_{\mathbf{p}}$  and we can use the ground state value. Instead of  $\mathcal{E}$  we should consider the grand-ensemble Legendre transform

$$\mathcal{E} - \zeta N \equiv \hat{\mathcal{E}}, \quad N = N_0 + \Sigma \delta n_{\mathbf{p}}, \quad (12.7-3)$$

where we shall call  $\hat{\mathcal{E}}$  the grand energy. Denoting further  $\mathcal{E}_0 - \zeta N_0$  by  $\hat{\mathcal{E}}_0$ , we find with trivial algebra,

$$\hat{\mathcal{E}} = \hat{\mathcal{E}}_0 + \int \Delta_{p\sigma}^3 (\varepsilon_{\mathbf{p}} - \zeta) \delta n_{\mathbf{p}}. \quad (12.7-4)$$

To be noted here is that  $|\mathbf{p}|$  is always very near the Fermi value  $p_F$  so that  $\varepsilon_{\mathbf{p}}$  is close to  $\zeta$ ; hence, the integrand is actually of second order smallness, i.e.,  $\propto (\delta n_{\mathbf{p}})^2$ .

Landau now recognised that, after all, Eq. (12.7-4) cannot be the full story, since in the above each quasi-particle contributes independently to the grand energy. Therefore, he added a binary interaction term, which redefines the character of each excitation. Let  $\varphi_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}$  be a binary interaction energy; the grand energy up to (but not including) terms  $\mathcal{O}(\delta n_{\mathbf{p}})^3$  should then be modified from (12.7-4) to read:

$$\hat{\mathcal{E}} = \hat{\mathcal{E}}_0 + \int \Delta_{p\sigma}^3 (\varepsilon_{\mathbf{p}} - \zeta) \delta n_{\mathbf{p}\sigma} + \frac{1}{2} \iint \Delta_{p\sigma}^3 \Delta_{p'\sigma'}^3 \varphi_{\mathbf{p}\sigma, \mathbf{p}'\sigma'} \delta n_{\mathbf{p}\sigma} \delta n_{\mathbf{p}'\sigma'}. \quad (12.7-5)$$

The interaction terms are spin-dependent, but they do not represent ordinary dipole-dipole coupling, which is negligibly small. Rather, the exchange hole around each quasi-particle causes an exchange energy which is spin-dependent analogous to the spin-spin coupling in a Heisenberg Hamiltonian. A full discussion is found in Leggett, Op. Cit. Generally,  $\varphi_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}$  contains a spin-symmetric and asymmetric contribution and we have

$$\varphi_{\mathbf{p}\sigma, \mathbf{p}'\sigma'} = \varphi_{\mathbf{p}\mathbf{p}'}^s + (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') \varphi_{\mathbf{p}\mathbf{p}'}^a, \quad (12.7-6)$$

where  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  refers to the Pauli spin matrices. As to the two momenta of each term, we note that both have magnitudes which for all practical purposes are equal to  $p_F$ ; therefore only their mutual angle  $\gamma$  is relevant. Consequently, an expansion in Legendre polynomials is a natural representation. It is customary to remove a factor  $\rho_F$ , being the density of states at the Fermi surface, from the factors  $\varphi$ ; we have  $\rho_F = m^* p_F / \pi^2 \hbar^3$ . Thus we employ the expansions

$$\varphi_{\mathbf{p}\mathbf{p}'}^{s,a}(\gamma) = (1/\rho_F) \sum_{\ell=0}^{\infty} F_{\ell}^{s,a} P_{\ell}(\cos \gamma). \quad (12.7-7)$$

We note that the coefficients  $F_{\ell}^s$  and  $F_{\ell}^a$  are also called  $F_{\ell}$  and  $Z_{\ell}$  in many papers. Generally only the coefficients for  $\ell = 0, 1$  are important. These, then, are the four

474, Eq. (12.9-3): altered as follows,

$$\begin{aligned} U_I(t, t_0) &\equiv U_I(t - t_0) \equiv U^{(0)\dagger}(t - t_0)U(t - t_0) \\ &= e^{iK^0(t-t_0)/\hbar} e^{-i(K^0 + \lambda V)(t-t_0)/\hbar}. \end{aligned} \quad (12.9-3)$$

§2, 1: We review some basic results, which have been further elaborated

480, §1, 8: Using the rules (12.9-36)

Eq. (12.9-45): remove the tildes from the time-ordering operators  $\mathcal{T}$  (2×)

483, (iv), 2: from  $\tau_j$  to  $\tau_i$ ,

493, Fig. 12-24: Little contour  $C'$  should be small  $c$ .

497, Eq. (12.11-20), last part,  $\Psi_{\mu}^{\dagger}(\mathbf{r}', \tau') = \sum_{\mathbf{k}'} \varphi_{\mathbf{k}'}^*(\mathbf{r}') \hat{e}_{\mathbf{k}', \mu}^{\dagger}(\tau')$ ,

499, §1, 1:  $K = \mathcal{H} - \zeta \sum_{\mathbf{k}} \mathbf{n}_{\mathbf{k}}$

506, §1, 2: Eqs. (12.11-5), (12.11-13) and (12.11-16).

508, Eq. (12.13-6), first line,  $\lim_{\delta \rightarrow 0} e^{i\omega_n \delta}$

513, §2, 7: Chapter XIII, Eqs. (13.4-11, 12)

519, §1, 1-2: replace by: Although the numerator and the denominator are zero for  $\mathbf{q} = 0$ , the ratio is finite, as we will see below. We write as always...

Eq. (12.13-49): remove the  $\mathcal{P}$ 's and in the second line write  $\int_0^{\infty} k dk n_{\mathbf{k}}^0$ .

§1, 4: where <sup>97a</sup>

footnote <sup>97a</sup> added:

<sup>97a</sup> The integral is rewritten as  $\varphi(x) = \frac{1}{\pi x} \lim_{u \rightarrow 0^+} \int_u^{\infty} \xi d\xi e^{-\xi^2/4\pi} \ln \left[ 1 + \frac{2x}{2\xi - x} \right]$ . For  $x \ll \xi$  with  $\xi > u > 0$ ,

upon expansion of the logarithm we find,  $\varphi(0) = \frac{1}{\pi} \lim_{u \rightarrow 0^+} \int_u^{\infty} d\xi e^{-\xi^2/4\pi} = 1$ .

## Chapter XIII

539, footnote <sup>9</sup>: See Reference 13 of Chapter VIII.

540, §2, 1: Let there be  $\hat{N} \Delta^3 r$  scatterers <sup>9a</sup>

footnote <sup>9a</sup> added:

<sup>9a</sup> E.g.  $\hat{N}$  is the *density* of ionized donors in electron-impurity scattering, cf. subsection 13.4.2 example (b) and Problem 13.5.

548, Fig. 13-5(a): the obtuse angle  $\theta$  should be labelled  $\Theta$

551, §1, 7:  $\sum_{\mathbf{k}} F(1 + \varepsilon_2 F) \rightarrow \hat{N} \Delta^3 r$ .

557, §2, 11: Comparing with Eq. (1.3-8)

## Chapter XIV

567, §2, 2:  $u = \sum_i (1/\Delta^3 r) \int \Delta^3 k_i \varepsilon_{\mathbf{k}^i} f^i$ .

568, §2, 1: we set  $\Psi$  equal (cap  $\Psi$ )

- 573, Eq. (14.2-7): last factor,  $f_1^i(\mathbf{r}, \mathbf{v}^i, t)$  ;  
 575, §1, 4: and  $\varphi$  the corresponding azimuthal angle,  
 Fig. 14-1: the ordinate should be labelled  $z$ ,  $\alpha$ ,  $\mathbf{g}_1$   
 576, Eq. (14.2-22): superscript on  $m$  is 'i', not '1'  
 578, §1, 2: if  $u \rightarrow u^i$ ,  $\zeta \rightarrow \zeta^i$  and  $\tau \rightarrow \tau^i$ .  
 579, Eq. (14.2-35), last line, last term:  $-L^{SS}(\nabla T)/T$ .  
 587, footnote 20, second line:  
 where the summation convention is implied. So in order that  $L^{iT} : \mathbf{PQ} = L^{Ti} : \mathbf{QP}$  we need that

## Chapter XV

- 591, Eqs.(15.1-12) and (15.1-14): the diffusion coefficient here is  $D_n'$  (add the prime)  
 593, §1, 2 and 3: Section 15.3.  
 §2, 12ff. Replace by new paragraph:  
 At low temperatures at which the electron gas is fully degenerate, it is faster to use (15.2-7) as is; we can set  $\partial f_0 / \partial \varepsilon = -\delta(\varepsilon - \varepsilon_F)$  and pull the relaxation time out of the integrand as  $\tau_F$ . For spherical energy surfaces, noting  $\mathbf{v}_\mathbf{k} \mathbf{v}_\mathbf{k} \rightarrow \frac{1}{3} v_k^2$  and  $n(\mathbf{r}) \approx n_0 = k_F^3 / 3\pi^2$  [cf. (8.4-11)], one readily confirms the elementary result  
 594, §1, 12: this into (15.2-20) gives  
 595, Eq. (15.2-24):  $\mathbf{S}_n \cdot \nabla T$   
 600, Eq. (15.3-5):  $\mathbf{J}_e^i(\mathbf{r}, t) = -e\mathbf{J}_p^i(\mathbf{r}, t)$  (minus sign added)  
 Eqs. (15.3-8) and (15.3-9): lhs, bold  $\mathbf{r}$ ; also, Eq. (15.3-10):  $T(\mathbf{r}, t)$   
 601, Eqs. (15.4-1) and (15.4-2): bold  $\mathbf{r}$  in  $T(\mathbf{r})$   
 §1, 14:  $\gamma = (\zeta - \varepsilon_C) / k_B T$ .  
 602, §1, 5: cap  $\Psi$   
 612, Fig. 15-2a, 2b: the arrowheads on the vectors  $\mathbf{k}$  and  $\mathbf{k}'$  should be restored  
 617, §3, in Fig. 14-1  $\chi'$  be the angle of  $\mathbf{v}'$  with the polar axis (add the prime to chi)  
 619, §2, 6: where we chose the direction of  $\mathbf{g}_1$  such that the vector...  
 621, §3, 3: form as in (15.2-32).  
 622, §1, 1: This is substituted into (15.8-1) together with (15.7-10) and the terms  
 624, §1, 2: comparable to (15.6-15)  
 626, §2, 1 and 2: this should read  
 Rests to evaluate the collision integral in first order (*c.i.f.o.*). From (15.10-3) and (15.9-1) we have, since  $\bar{\mathcal{E}}_1$  in the absence of a  $\mathbf{B}$ -field was taken along  $\alpha$ ,  
 627, Fig. 15-4: replace  $\theta$  by  $\Theta$   
 628, §2, 2: Thus from (15.10-16) and (15.9-6):  
 §2, 7: ... both results (15.10-18) and (15.10-20) are substituted into (15.9-5)  
 631, §1, 1: Previously we established [cf. (15.10-27)]  
 §1, 3: With  $A_p^i$  given by (15.10-28), this yields  
 632, Problem 15.2: in Eq. (15.2-23).  
 Problem 15.4, first line: was given in (15.2-20). Last line:

(b) From (15.2-20), obtain Landsberg's result, (15.2-21).

## Chapter XVI

641, Eq. (6.2-14) and (6.2-15): we regret that the dotted symbols of mathtype are routinely too mall, in particular when occurring as subscripts; they have all been redone, e.g.,

$$\langle \Delta \dot{B}(t) \rangle = \int_0^t dt' \phi_{BA}^{\dot{}}(t-t') F(t'), \quad (16.2-14)$$

$$\phi_{BA}^{\dot{}}(t) = (1/\hbar i) \text{Tr}\{[A, \dot{B}(t)] \rho_{eq}\}. \quad (16.2-15)$$

666, Eqs. (16.6-12)-(16.6-15): there are factors two to be inserted,

$$\chi(\mathbf{q}, i\omega) = \lim_{\eta \rightarrow 0} \frac{1}{2\pi\hbar V_0} \int_{-\infty}^{\infty} d\omega'' S_d(\mathbf{q}, \omega'') \left\{ \frac{1}{\omega + \omega'' - i\eta} - \frac{1}{\omega - \omega'' - i\eta} \right\}. \quad (16.6-12)$$

In the quantum limit (very large  $\omega$ ) we find that  $\chi$  becomes real; combining the two parts in the curly brackets of (16.6-12), we find

$$\begin{aligned} \chi(\mathbf{q}, i\omega) &= -\lim_{\eta \rightarrow 0} \frac{1}{\pi\hbar V_0} \int_{-\infty}^{\infty} \omega'' d\omega'' S_d(\mathbf{q}, \omega'') / [(\omega - i\eta)^2 - \omega''^2] \\ &= \frac{2}{\pi} \lim_{\eta \rightarrow 0} \int_{-\infty}^{\infty} \omega'' d\omega'' \text{Im} \chi(\mathbf{q}, \omega'') / [(\omega - i\eta)^2 - \omega''^2] \sim \frac{2q^2 \langle n \rangle}{m\omega^2}, \end{aligned} \quad (16.6-13)$$

where we used (16.5-22) and the  $f$ -sum rule (16.5-35). Returning to the general form (16.6-12) and using (16.3-11), we find for the real and imaginary parts,

$$\chi'(\mathbf{q}, \omega) = -\frac{1}{\pi\hbar V_0} \int_{-\infty}^{\infty} d\omega'' S_d(\mathbf{q}, \omega'') \mathcal{P} \frac{\omega''}{(\omega^2 - \omega''^2)}, \quad (16.6-14)$$

$$\chi''(\mathbf{q}, \omega) = \frac{1}{2\hbar V_0} \{S_d(\mathbf{q}, -\omega) - S_d(\mathbf{q}, \omega)\}. \quad (16.6-15)$$

669, Eq. (16.6-33):  $d^3r$  replace by  $d^3k$

§2, 9 and 10: for the  $k$ -integral we note that the Heaviside function sets  $k \leq k_F$  and we use  $\langle n \rangle = k_F^3 / 3\pi^2$ , cf. (8.4-11).

$$674, \sigma_{\mu\nu} = \frac{\beta}{4} \lim_{T \rightarrow \infty} V_0 \int_{-T}^T dt \sum_{k\ell} \text{Tr} \left\{ \rho_{gcan} \mathbf{n}_k \mathbf{n}_\ell \left[ \langle k | j_\nu | k \rangle \langle \ell | j_\mu | \ell \rangle + \text{transp} \right] \right\}, \quad (16.7-3)$$

§2, 11: *etc.*; *transp* means the transpose,  $\nu \leftrightarrow \mu$ .

675, §1, 5: of the electron gas (omit 'free').

681, §2, 7: which reads, writing  $U_\gamma^0(t-t') = \exp[-i\mathcal{C}_\gamma^\circ(t-t')/\hbar]$  and noting that the perturbations  $\lambda^\nu$  are time-independent,

last equation, second line:

$$\times U_\gamma^0(t-t_n) \langle \gamma | \mathcal{V}^\nu(t_n) | \gamma_{n-1} \rangle U_{\gamma_{n-1}}^0(t_n - t_{n-1}) \langle \gamma_{n-1} | \mathcal{V}^\nu(t_{n-1}) | \gamma_{n-2} \rangle \dots$$

682, Equation top line:  $\times U_{\gamma_1}^0(t_2 - t_1) \langle \gamma_1 | \mathcal{U}^{\mathcal{L}}(t_1) | \gamma_0 \rangle U^0(t_1)$ . (16.9-15)

684, Eq. (16.9-32), second line:  $|\langle \gamma'' | \mathcal{U} | \gamma \rangle|^2$  (script  $\mathcal{U}$ )

690, §1, 3 and 4: (even if not, this part goes with  $\lambda$  and does not give the diagonal singularity). Thus, ...

697, Eq. (16.10-62) should be changed to read:

$$\dot{A}_d^R(-i\hbar\tau) = e^{\tau\mathcal{L}^0} \dot{A}_d^R(0) e^{-\tau\mathcal{L}^0} = \dot{A}_d^R(0) \equiv (\dot{A}^R)_d. \quad (16.10-62)$$

698, §1, 1: The dots have been inadvertently omitted:  $\dot{A}_{nd}^R(0) \equiv (\dot{A}^R)_{nd} = \dot{A}_{nd}$ ,

701, §3, 5:

(differentiation to the upper limit  $t$  of the integral gives a contribution of order  $\lambda$ , which will be dismissed – as we did in the Heisenberg case). We thus find

715, Eq. (16.15-25) should read:

$$\lim_{\lambda, t} \lim_{\text{th}} e^{\pm i\mathcal{L}^0 t} = \exp \left[ - \left( (\Lambda_d \mp i\mathcal{L}^0) - \frac{qE_z}{m} \sum_i \frac{\partial}{\partial v_{iz}} \right) t \right]. \quad (16.15-25)$$

Problem 16.1, last line: show that there are *no poles* in the lower half plane.

716, Problem 16.4: Eq. (6) should read  $\ln \coth(\dots)$ .

Problem 16.5, part (a): Employing a convergence factor  $\exp(-\eta t)$ , show...

717, Problem 16.7, Eq. (10):  $\frac{2}{\pi} \int_0^\infty \text{Re} \sigma_{\mu\nu}^s(i\omega) d\omega = \frac{q^2 n}{m} \delta_{\mu\nu}$ , (10)

## Chapter XVII

722, Eq. (17.1-10), second line:

$$= \sum_{\eta' \eta''} \langle N_{\eta''} \rangle_{eq} (1 + \langle N_{\eta'} \rangle_{eq}) Q(\zeta'' \eta''; \zeta' \eta'), \quad (17.1-10)$$

725, Eq. (17.2-1), in the lhs, uses script symbol  $\mathcal{B}_\zeta$ ; similarly in Eq. (17.2-2).

726, §2, 11: function  $\Psi_{\zeta\zeta'}$ .

728, Eq. (17.39), add:  $= 0$ .

729, Eq. (17.3-12)

$$\text{first term} = \frac{1}{8\pi^3} \frac{\partial}{\partial t} \int d^3\bar{r} \int d^3u e^{i\mathbf{u} \cdot (\mathbf{r} - \bar{\mathbf{r}})} [h^3 \rho_1(\mathbf{k}, \bar{\mathbf{r}}, t)] = \frac{\partial f}{\partial t}, \quad (17.3-12)$$

§2. 3: Wigner occupation function (16.10-56) and

730, §1, 2 and 3: Also we set  $\sum_{\bar{\mathbf{k}}} \rightarrow (V_0 / 8\pi^3) \int d^3\bar{k}$ . Then the last part of (17.3-10) inside the curly brackets becomes:

739, Eq. (17.7-6): add  $(q / 2V_0)$  in front of the sum sign

740, Eq. (17.7-8): add  $(\beta q / 2V_0)$  in front of the sum sign

742, Page has been *corrected and improved* as follows:

boson bath states, we have by (17.1-15) and (17.1-16)

$$\begin{aligned} \left\langle e^{-\Lambda_d t} \right\rangle_b &= \sum_{\gamma} |\gamma\rangle \langle \gamma| \sum_{k=0}^{\infty} \langle (-M_{\gamma} t)^k \rangle_b / k! \\ &= \sum_{\{n_{\zeta}\}} |\{n_{\zeta}\}\rangle \langle \{n_{\zeta}\}| \sum_{k=0}^{\infty} (-\mathcal{B}_{\zeta} t)^k / k! = \sum_{\{n_{\zeta}\}} |\{n_{\zeta}\}\rangle \langle \{n_{\zeta}\}| e^{-\mathcal{B}_{\zeta} t}, \end{aligned} \quad (17.8-4)$$

which is formally correct, but of no value unless the operator on the rhs can be computed. Although  $M$  is a linear operator,  $\mathcal{B}_{\zeta}$  is not. Therefore, we will limit ourselves to the case that  $\mathcal{B}$  is linear or linearized and allows the definition of a relaxation time, i.e.,  $\mathcal{B} \rightarrow [1/\tau(\varepsilon)]$ . Substituting these results in (17.8-2) yields

$$\left\langle \frac{1}{i\omega + \Lambda_d} \right\rangle_b = \sum_{\{n_{\zeta}\}} |\{n_{\zeta}\}\rangle \langle \{n_{\zeta}\}| \frac{1}{i\omega + \tau^{-1}(\varepsilon_{\zeta})}. \quad (17.8-5)$$

The diagonal many-particle velocity operators are as usual given by

$$\hat{v}_{\mu,d} = \sum_{\zeta} \mathbf{n}_{\zeta} \cdot (\zeta' | v_{\mu} | \zeta'), \quad \hat{v}_{\nu,d} = \sum_{\zeta} \mathbf{n}_{\zeta} \cdot (\zeta'' | v_{\nu} | \zeta''). \quad (17.8-6)$$

Returning to (17.8-1) and going over to the *grand-canonical* ensemble, we find

$$\begin{aligned} \sigma_{\mu\nu}^d(i\omega) &= \frac{\beta q^2}{V_0} \sum_{\zeta', \zeta''} \sum_{\{n_{\zeta}\}} \text{Tr} \left\{ \rho_{gcan} |\{n_{\zeta}\}\rangle \langle \{n_{\zeta}\}| \mathbf{n}_{\zeta'} \cdot \frac{\tau(\varepsilon_{\zeta''})}{1 + i\omega\tau(\varepsilon_{\zeta''})} \mathbf{n}_{\zeta''} \right\} \\ &\quad \times (\zeta' | v_{\mu} | \zeta') (\zeta'' | v_{\nu} | \zeta''). \end{aligned} \quad (17.8-7)$$

In evaluating the trace we need the expression – with  $X_{\zeta}$  denoting the frequency term,

$$\begin{aligned} &\sum_{\zeta', \zeta''} \langle n_{\zeta'} n_{\zeta''} \rangle_{eq} (\zeta' | v_{\mu} | \zeta') (\zeta'' | v_{\nu} | \zeta'') X_{\zeta''}(i\omega) \\ &= \sum_{\zeta' \neq \zeta'', \zeta''} \langle n_{\zeta'} \rangle_{eq} \langle n_{\zeta''} \rangle_{eq} (\zeta' | v_{\mu} | \zeta') (\zeta'' | v_{\nu} | \zeta'') X_{\zeta''}(i\omega) \\ &+ \sum_{\zeta'} \langle n_{\zeta'}^2 \rangle_{eq} (\zeta' | v_{\mu} | \zeta') (\zeta' | v_{\nu} | \zeta') X_{\zeta'}(i\omega) \\ &= \sum_{\zeta'} \langle n_{\zeta'} \rangle_{eq} (\zeta' | v_{\mu} | \zeta') \sum_{\zeta''} \langle n_{\zeta''} \rangle_{eq} (\zeta'' | v_{\nu} | \zeta'') X_{\zeta''}(i\omega) \\ &+ \sum_{\zeta'} [\langle n_{\zeta'}^2 \rangle_{eq} - \langle n_{\zeta'} \rangle_{eq}^2] (\zeta' | v_{\mu} | \zeta') (\zeta' | v_{\nu} | \zeta') X_{\zeta'}(i\omega). \end{aligned} \quad (17.8-8)$$

The next to last line of (17.8-8) is zero since the equilibrium current  $\sum_{\zeta'} \dots$  vanishes. The last line involves the variance,

$$\langle n_{\zeta}^2 \rangle_{eq} - \langle n_{\zeta} \rangle_{eq}^2 = \langle \Delta n_{\zeta}^2 \rangle_{eq} = \langle n_{\zeta} \rangle_{eq} (1 - \langle n_{\zeta} \rangle_{eq}) = -\beta^{-1} \frac{\partial \langle n_{\zeta} \rangle_{eq}}{\partial \varepsilon_{\zeta}}. \quad (17.8-9)$$

The final result now becomes

$$\sigma_{\mu\nu}^d(i\omega) = -\frac{q^2}{V_0} \sum_{\zeta} \frac{\partial \langle n_{\zeta} \rangle_{eq}}{\partial \varepsilon_{\zeta}} \frac{\tau(\varepsilon_{\zeta})}{1 + i\omega\tau(\varepsilon_{\zeta})} (\zeta | v_{\mu} | \zeta) (\zeta | v_{\nu} | \zeta). \quad (17.8-10)$$

This result is exactly what we obtained earlier from the linearized QBE, cf. (17.6-6).

749, Fig. 17-4. Lettering ordinate and abscissa to be made larger.

753, Eq. (17.10-18): last member is  $(1/\ell^2) \int du$ .

754, Eq. (17.10-21), first line should read:

$$\sigma_{xx} = \frac{\beta q^2}{2\pi\hbar} \frac{A_0}{2\pi\ell^2} \sum_{N, N', n, n'} f_{Nn}^0 (1 - f_{N'n'}^0) \int u du \int dq_z |\mathcal{F}(\mathbf{q})|^2 |J_{NN'}(u)|^2 |F_m(\pm q_z)|^2$$

Eq. (17.10-24), first line should be

$$\sigma_{xx}(\text{opt}) = \frac{3\beta q^2 D'}{4\pi\hbar\ell^2 L_z} \sum_{NMn} \left\{ f_{Nn}^0 (1 - f_{N+M,n}^0) \mathcal{N}_0(2N + M + 1) \right.$$

755, Eq. (17.10-25) has as first factor:  $(3\beta q^2 D' / 4\pi\hbar\ell^2 L_z^2)$

Eq. (17.10-30) has as first factor:  $(3\beta q^2 D' / 4\pi\hbar^2 \omega_c \ell^2 L_z^2)$

756, Eq. (17.10-31): remove  $\kappa_0$  in prefactor

768, §2, 5: Since  $n \neq n'$ , only the ...

772, §1,7: (16.10-41) and ...

## Chapter XVIII

791, Eq. (18.5-3):

$$\begin{aligned} \frac{\partial}{\partial t} \langle e^{-\mathbf{s}\cdot\mathbf{a}(t)} \mathbf{a}\mathbf{a} \rangle_{\mathbf{a}} &= \langle e^{-\mathbf{s}\cdot\mathbf{a}(t)} \mathbf{a}\mathbf{a} [-\mathbf{s}\cdot\mathbf{A}(\mathbf{a}) + \frac{1}{2}\mathbf{s}\cdot\mathbf{B}(\mathbf{a})\cdot\mathbf{s} + \dots] \rangle_{\mathbf{a}}, \\ &+ \langle e^{-\mathbf{s}\cdot\mathbf{a}(t)} \mathbf{a} [\mathbf{A}(\mathbf{a}) - \frac{1}{2}\mathbf{B}(\mathbf{a})\cdot\mathbf{s} - \frac{1}{2}\mathbf{s}\cdot\mathbf{B}(\mathbf{a}) + \dots] \rangle_{\mathbf{a}}, \\ &+ \langle e^{-\mathbf{s}\cdot\mathbf{a}(t)} [\mathbf{A}(\mathbf{a}) - \frac{1}{2}\mathbf{B}(\mathbf{a})\cdot\mathbf{s} - \frac{1}{2}\mathbf{s}\cdot\mathbf{B}(\mathbf{a}) + \dots] \mathbf{a} \rangle_{\mathbf{a}}, \\ &+ \langle e^{-\mathbf{s}\cdot\mathbf{a}(t)} [\mathbf{B}(\mathbf{a}) + \dots] \rangle_{\mathbf{a}}. \end{aligned} \quad (18.5-3)$$

801, Fig. 18-3(a): trap level should be close to the valance band

803, Eq. (18.5-72):

$$i_0 = \frac{-n_0(\kappa + \delta)}{2\kappa} + \frac{1}{2\kappa} [(\kappa + \delta)^2 n_0^2 + 4\delta\kappa n_0 I]^{1/2}. \quad (18.5-72)$$

807, §1, 1-3 should be changed as follows,

Averaging over an ensemble with initial values fixed to  $\mathbf{a}(0) = \mathbf{a}'$ , we have  $\langle \mathbf{a}(\Delta t) - \mathbf{a}' \rangle_{\mathbf{a}'} = -\mathbf{M} \langle \Delta \mathbf{a} \rangle_{\mathbf{a}'} \Delta t$ . Now, using the definition of the first-order F-P moment (18.5-17) and dividing by  $\Delta t$ , we also find  $\mathbf{A}(\mathbf{a}') = -\mathbf{M} \langle \Delta \mathbf{a} \rangle_{\mathbf{a}'}$ , in accord with (18.5-4). Next, we multiply (18.6-8) by its transpose and average again with  $\mathbf{a}(0) = \mathbf{a}'$ ; thus,

815, Eq. (18.7-31), middle line needs a + sign in the last bracket:

$$= \int \left[ \nabla_{\mathbf{v}} f \cdot d\mathbf{v} - \frac{1}{\beta} (\nabla_{\mathbf{r}} f) \cdot \frac{d\mathbf{v}}{d\mathbf{r}} d\mathbf{r} \right] = \int \left[ \frac{\partial f}{\partial \mathbf{v}} \cdot d\mathbf{v} + \frac{\partial f}{\partial \mathbf{r}} \cdot d\mathbf{r} \right]$$

816, Eq. (18.7-32), second line:

$$= \frac{k_B T}{m\beta} \nabla_{\mathbf{r}'} \cdot \int \left\{ \nabla_{\mathbf{r}'} P(\mathbf{r}' - \beta^{-1} \mathbf{v}, \mathbf{v}, t | \mathbf{r}_0, \mathbf{v}_0) - \frac{m\mathbf{K}}{k_B T} P(\mathbf{r}' - \beta^{-1} \mathbf{v}, \mathbf{v}, t | \mathbf{r}_0, \mathbf{v}_0) \right\} d\mathbf{v}.$$

817, Eq. (18.7-39): [ ] of denominator last exponent:  $[1 - e^{-2(\omega^2/\beta)t}]$

820, §1, 6: result (15.2-17)!

822, Eq. (18.9-2b): replace  $\mathbb{S}_{\alpha\alpha}$  by  $\mathbb{G}_{\alpha\alpha}$

826, Eq. (18.10-25):  $\overline{M_\theta} = \sum_M M W(M, \theta) = \left\langle \lambda \int_t^{t+\theta} N(t') dt' \right\rangle = \lambda \theta \langle N \rangle$ , (18.10-25)

827, §2, 4: Let the light intensity on a photocathode of unit efficiency be ...

Eq. (18.10-34), lhs:  $W(M, \mathcal{T}) =$

§2, 11 and 12: (18.10-24).<sup>41</sup> For the counting variance in  $(t, t + \mathcal{T})$ ,  $\langle \Delta M_{\mathcal{T}}^2 \rangle$ , we obtain a result analogous to (18.10-27). With the usual change of variables and integrating separately over the two regions of Fig 18-9, we then obtain

832, §2, 8: the Fourier transform is  $e$  and the ...

838, Eq. (18.12-22), last line: denominator should be  $\omega^{p-2}$

840, Eq. (18.12-30): – sign in front of the rhs

846, Fig. 18-14: in caption: interchange  $p_{12}$  and  $p_{21}$ ; also,  $p_{31} = \delta n_1 i$

848, Fig. 18-15(a): draw a horizontal line at ‘1’ on the ordinate; this needs the designation ‘thermal noise’

## Chapter XIX

858, first equation, in lhs:  $P(X_k)$

859, §1, 13: cf. (18.10-38).

Eq. (19.1-18): The products run from  $k = 0$  to  $N$

860, (iii), 1: Bayes’ rule (19.1-3).

962, Eq. (19.2-11): the sum runs from  $\ell = 0$  to  $\infty$

863, §1, 2: Hence, we have the *boson distribution*

after Eq. (19.2-17') insert :

The binomial theorem now takes the form of the well-known expansion

$$\sum_{m=0}^{\infty} \frac{(-n)!}{m!(-n-m)!} (-q)^m = 1 + \sum_{m=1}^{\infty} \frac{(-n)(-n-1)\dots(-n-m+1)}{m!} (-q)^m = (1-q)^{-n}. \quad (19.2-18)$$

For the generating function of this distribution, employing (19.2-12), we find

$$G(z) = \sum_{m=0}^{\infty} z^m P(m|n) = [1 + M(1-z)]^{-n}. \quad (19.2-18')$$

The factor  $[1 + M(1-z)]^{-1}$  is clearly  $\phi(z)$ . Thus, the generating function ...

§2, 6:  $\omega(z) = \lambda(z-1) + 1$ .

§2, 7: is  $X = 1 + p_1$ . Hence, by the addition theorem,

$$\phi(z) = z\omega(z) = \lambda z(z-1) + z. \quad (19.2-19)$$

863, footnote<sup>6</sup> has been changed to

<sup>6</sup> The negative factorials in these formulae are singular [cf. Problem 4.14], but their ratio is finite.

869, §2,5:  $e|E_i| \ell_i \sim \varepsilon_G$ , where ...

872, Eq. (19.5-15): put an *equal sign* = after  $\phi'_{N,0}$

880, §2, 4: by  $\tilde{G}^*(\mathbf{r}, \mathbf{r}', i\omega) = \tilde{G}(\mathbf{r}, \mathbf{r}', -i\omega)$  [add tildes]

§2, 5:  $a(\mathbf{r}, t)$  replace by  $\hat{a}(\mathbf{r}, \omega)$

889, Fig. 19-8(b): lettering insert II:  $\tau = \tau_a$

892, §1, 13: The noise spectra are found from (19.6-26), first line.

in last line of bottom formula: replace  $p_s(\mathbf{r})$  by  $p_s(\mathbf{r}')$

893, Eq. (19.8-17): replace  $p_s(\mathbf{r})$  by  $p_s(\mathbf{r}')$

894, §2, 3: extension of (19.8-3)

## Chapter XX

906, Eqs. (20.3-5) and (20.3-6):

$$\mathbf{E}^+(\mathbf{r}, t) |\{\alpha_q\}\rangle = \sum_q i(\hbar\omega_q/2V_0)^{1/2} \alpha_q \mathbf{u}_q(\mathbf{r}) e^{-i\omega_q t} |\{\alpha_q\}\rangle \equiv \mathcal{E}^+(\mathbf{r}, t) |\{\alpha_q\}\rangle. \quad (20.3-5)$$

Accordingly, the ‘pseudo-classical field’ takes the form

$$\mathcal{E}^+(\mathbf{r}, t) = \sum_q i(\hbar\omega_q/2V_0)^{1/2} \alpha_q \mathbf{u}_q(\mathbf{r}) e^{-i\omega_q t}. \quad (20.3-6)$$

915, §2, 3: change names to Hanbury–Brown and Twiss.

## Appendix A

923, first line. Section **A.1** should have been named: **The Schrödinger Picture**

926, Eq. (A.3-3) should read:  $dU^0(t, t_0)/dt = (1/\hbar i)\mathcal{H}^0 U^0(t, t_0)$ .

927, Eq. (A.3-9) should be improved in clarity:

$$\begin{aligned} \langle \gamma | \bar{U}^{(n)}(t, t_0) | \gamma_0 \rangle &= \left( \frac{\lambda}{\hbar i} \right)^n \sum_{\gamma_1 \dots \gamma_{n-1}} \int_{t_0}^t dt_n \int_{t_0}^{t_n} dt_{n-1} \dots \int_{t_0}^{t_2} dt_1 \\ &\times e^{-i\mathcal{E}_\gamma(t-t_n)/\hbar} \langle \gamma | \mathcal{V}^\lambda(t_n) | \gamma_{n-1} \rangle e^{-i\mathcal{E}_{\gamma_{n-1}}(t_n-t_{n-1})/\hbar} \langle \gamma_{n-1} | \mathcal{V}^\lambda(t_{n-1}) | \gamma_{n-2} \rangle \dots \\ &\times e^{-i\mathcal{E}_{\gamma_1}(t_2-t_1)/\hbar} \langle \gamma_1 | \mathcal{V}^\lambda(t_1) | \gamma_0 \rangle e^{-i\mathcal{E}_{\gamma_0}(t_1-t_0)/\hbar}. \end{aligned} \quad (A.3-9)$$

928, Eq. (A.3-12) should read:  $a_{k,H}^\dagger(t) = U_I^\dagger(t) a_{k,I}^\dagger U_I(t)$ ,  $a_{k,H}(t) = U_I^\dagger(t) a_{k,I} U_I(t)$ .

## Appendix B

935, and following pages. The dots on the symbols are too small and the dotted indices are barely discernible. All formulas with such symbols have been redone and will as such appear in a second, revised printing.