

Preface in Two Acts with a Prelude, Interlude, and Postlude

Prelude

Thirty-seven years have elapsed between the first version and the present version of this monograph. We begin with the first author's slightly edited preface from his first version. We then provide a lengthier second preface composed by the second author.

The Original Preface

These notes are part of a course on modular forms and applications to analytic number theory given by the first author at the University of Illinois at Urbana-Champaign in the spring of 1970. The existing accounts [47], [48], [87] of Hecke's theory of modular forms and Dirichlet series are somewhat concise. Therefore, it has been our intention to present a more detailed account of a major portion of this material for those who are unfamiliar with this beautiful theory. Readers already familiar with Hecke's theory will find little that is new here.

The first author is especially grateful to Ronald J. Evans for providing a new proof of a fundamental region for Hecke's modular groups, which we present here. We express our thanks also to Elmer Hayashi for a detailed reading of the manuscript and to Harold Diamond for several suggestions.

Bruce Berndt, May, 1970 & May, 2007

Interlude

The first author mailed a copy of his notes on Hecke's theory of modular forms and Dirichlet series to Dr. Jürgen Elstrodt, who at that time was at Universität München. He responded with about a dozen pages of detailed

comments, which, after an undeservedly quick reading, were deposited in the first author's file cabinet for approximately thirty-five years, until they were dusted off and sent to the second author for incorporation in the new version. We hope that it is not too late to thank Elstrodt for his kind suggestions and patience.

The Second Preface

In the spring of 1971, I received the following letter, dated June 17. Since it is brief, I quote it in full.

Under separate cover, I am sending you a copy of some lecture notes, "Hecke's theory of modular forms and Dirichlet series." I would appreciate any comments, corrections, criticisms, or suggestions that you may have. Thank you very much.

Most sincerely, (signed) Bruce

To establish the context of this letter, I recall that in the spring of 1938 Erich Hecke gave an important series of lectures at the Institute for Advanced Study, Princeton, on his correspondence theory published in 1936. The notes from these lectures, taken by Hyman Serbin and produced in planographed form by Edwards Brothers of Ann Arbor, received only limited circulation. To my knowledge there are only a few copies extant in mathematics libraries (for example, the University of Illinois at Urbana-Champaign) and private collections of professional mathematicians.

In 1970 Berndt produced a set of lecture notes based upon Hecke's notes, but with the addition of many details omitted from Hecke's original notes. The more extensive notes, too, had only limited circulation.

For the past thirty-five years I have employed both sets of notes to introduce graduate students to the Hecke theory and the broader theory of modular/automorphic forms. During this time my Ph.D. students and others frequently asked why Berndt's notes had never been published. Because we are convinced that the reactions of these students reflect a genuine usefulness of these notes to the mathematical community, we have undertaken the task of publishing this book based upon them, corrected and modified where necessary, and expanded to include some of the many new developments in the theory during the past decades, as well as relevant earlier work not previously included. We stress that the Hecke correspondence theory has remained an active feature of research in number theory since the 1930s

and, in fact, its importance is perhaps better understood today than it was in 1936.

The first six chapters of this book follow the organization of Berndt's original notes, hence that of the first part of Hecke's notes as well. Beyond this, we have added two completely new chapters based upon work done since 1970 and upon earlier work not originally understood to lie within the circle of ideas surrounding Hecke's correspondence theorem.

Chapter 7 features Bochner's important generalization of Hecke's correspondence theorem and some closely related results. Chapter 8 is devoted to the great variety of identities related to the Hecke correspondence theory (but not explicitly present in that theory) that have been developed over the years. Among others, these identities are due to S. Ramanujan, N. S. Koshliakov, G. N. Watson, A. P. Guinand, K. Chandrasekharan, R. Narasimhan, and Berndt. Some antedate Hecke's work, while others are more recent.

Marvin Knopp, April, 2007

Postlude

We are grateful for the comments made by our students over the past several decades. More recently, Shigeru Kanemitsu and Yoshio Tanigawa offered several additional remarks and references. We thank Hilda Britt for expertly typing most of our manuscript and Tim Huber for his graphical expertise.