Preface

This book is based on material obtained in my Habilitation at the RWTH Aachen. It presents a rigorous mathematical approach to the theory of amplitude equations for stochastic partial differential equations (SPDEs). This is a theory which is on a formal level well established and a valuable tool for physicists and applied mathematicians.

The goal is the approximation of SPDEs near a change of stability. It aims at providing details about the reduction of the dynamics of SPDEs to more simpler equations via amplitude or modulation equations. The key idea is the natural separation of time-scales present near a change of stability.

Main topics of this book are rigorous error estimates for the approximation by amplitude equations. These are well known for deterministic PDEs, and there is a large body of literature over the past about two decades. However there seems to be a lack of literature for stochastic equations, although without reliable error estimates at hand the theory in the stochastic case is successfully used. One celebrated examples is, for instance, the convective instability in Rayleigh-Bénard convection. This book is a first step in closing this gap.

The aims of the book are twofold. On one hand it presents a more elementary introduction to the subject highlighting the new tools necessary for stochastic equations. In a large part of the book not much expertise in stochastic partial differential equations is assumed. On the other hand it should provide a guideline to current research, by summarising results of recent research articles.

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