Preface and Acknowledgments

In the view of the authors, and as we hope to convince the reader, Lie theory, broadly understood, lies at the center of modern mathematics. It is linked to algebra, analysis, algebraic and differential geometry, topology and even number theory, and applications of some of these other subjects are crucial to many of the arguments we shall present here. This also holds true in the opposite direction: Lie theory can be used to clarify or derive results in these other areas. In a philosophical sense Lie groups are pervasive within much of mathematics, for whenever one has some system, the "automorphisms" of it will frequently be a Lie group. This even occurs in the oldest deductive system in mathematics, namely Euclidean geometry. Here the key issue is congruent figures, particularly triangles. Two such planar triangles are congruent if and only if they differ by an element of the Lie group $E(2) = \mathrm{O}(2, \mathbb{R}) \ltimes \mathbb{R}^2$, the group of rigid motions of the Euclidean plane. The reader will find in these many interrelations a vast panorama well worth studying.

This book is the result of courses taught by one of the authors over many years on various aspects of Lie theory at the City University of New York Graduate Center. The primary reader to which it is addressed is a graduate student in mathematics, or perhaps physics, or a researcher in one of these subjects who wants a comprehensive reference work in Lie theory. However, by a judicious selection of topics, some of this material could also be used to give an introduction to the subject to well-grounded advanced undergraduate mathematics majors.

For example, Chapters 3 and most of 7 could form a semester's course in Lie algebras. Similarly, Chapters 0, 2 and 5 (respectively 0, 2 and 8) could be a semester's course in integration in topological groups and their homogeneous spaces (respectively lattices and their applications). For the reader's convenience we have included a diagram of the interdependence of the chapters. We have also tried to make the text as self-contained as possible even at the cost of increased length. We shall assume the reader has some knowledge of basic group theory, topology, and linear algebra, and a general acquaintance with the grammar of mathematics. While reading this book one may wish to consult some of the other books on the subject for clarification, or to see another viewpoint or treatment; especially useful books are listed in the bibliography. We have not attempted to detail the historical development of our subject, nor to systematically give credit to the individual researchers who discovered these results.

The book is organized as follows:

Chapter 0 introduces the players; topological and Lie groups, coverings, group actions, homogeneous spaces, and Lie algebras.

Chapter 1 deals with the correspondence between Lie groups and their Lie algebras, subalgebras and ideals, the functorial relationship determined by the exponential map, the topology of the classical groups, the Iwasawa decomposition in certain key cases, and the Baker Campbell Hausdorff theorem, and local Lie groups.

Chapter 2 concerns Haar measure both on a group and on cocompact and finite volume homogeneous spaces together with a number of applications.

Chapter 3 gives the elements of Lie algebra theory in some considerable detail (except for the detailed structure of complex semisimple Lie algebras, which we defer until Chapter 7).

Chapter 4 deals with the structure of a compact connected Lie group in terms of a maximal torus and the Weyl group.

Chapter 5 contains the representation theory of compact groups.

Chapter 6 concerns symmetric spaces of non-compact type.

Chapter 7 presents the detailed structure of complex semisimple Lie algebras.