

# Preface

The goal of this text is to concisely present the mathematical blocks needed for implementing the main body of a strapped down Inertial Navigation System (INS) in a manner that provides a mental image of the contribution of each block and their interrelation. The text describes the makeup of each block and provides the derivation of its equations. Towards this objective, when the need for clarifying or justifying a certain idea arises, it is presented in an appendix so as not to interfere with the flow of the main ideas.

This treatment should benefit both the novice as well as a practitioner in the field. For a journeyman in the area of navigation, this book can be used to pinpoint the equations that are the basis of such a system, how they are developed and how they are implemented. Those with more experience may use this book as a quick reference guide.

What is navigation anyway? It is the ability to set the course of a ship to move between two desired locations. To do that the navigator must be able to know his location and set the velocity vector towards the desired destination. Thus the prime function of a navigation system is determining the craft's position and velocity.

We will be primarily concerned herein with a special type of navigation: inertial navigation. And why inertial navigation in particular? Inertial systems are self-contained: they are independent of weather conditions and are operable anywhere in seas, underwater, lands, tunnels, or in air. Short of a reliable source of power, they can work almost indefinitely.

If the Earth were flat, inertial navigation algorithms would have been a lot easier. Because navigation usually is on or close to Earth, a spherical body that rotates about itself, we will soon find ourselves entangled in discussing two different elements at the same time: developing the mathematical algorithms and describing the pertinent physics of the Earth.

We have devoted the first three chapters to introduce the mathematical foundation for developing the algorithms. These algorithms rely heavily on vector and matrix notations, and for that, vector and matrix properties are introduced in Chapter 1. On developing the equations, we will discover that our variables of interest are represented in different coordinate systems. Obviously this creates the need of moving from one coordinate system to another and thus this concept is discussed in Chapter 2. For further clarity, Chapter 3 introduces the most common approaches used in performing coordinate transformations. There we discuss these approaches and their relationships amongst one another.

We discuss the physical properties of the Earth in Chapter 4. At this point, armed with the mathematical tools and the geometrical properties of Earth, we develop the inertial navigation equations from first principles in Chapter 5. This yields a set of continuous time differential equations that should be solved to yield the navigation solution. Mechanizing and implementing these equations on a digital computer is introduced in Chapter 6. We discover one of the drawbacks of inertial navigation systems: unreliable vertical channel. This means that they cannot be relied on to provide altitude or vertical velocity.

Typically, the navigation system for aircraft is usually complemented with some sort of altimeter. Integrating the altimeter measurements with the INS is discussed therein. Our reliance on air data for aiding the navigation system does not end at this point: the INS cannot estimate wind speed. Chapter 7 is devoted to the discussion of air data computations and their use for computing rate of climb/descent and relative airspeed.

Using the legendary lines of longitude and latitude to locate a craft location on the surface of the Earth introduces a peculiar navigational

phenomenon: all longitude lines meet at the two polar points of the Earth. Solving navigation equations near these two points can prove to be mathematically cumbersome. Despite the rarity of these events, considerable attention has been given to avert the consequences of such an occurrence. The wander azimuth angle is one such classical technique and it is critiqued in Chapter 8. An innovative simple algorithm for navigating in the polar circle is introduced shortly thereafter.

Two problems remain to be solved. The first addresses the alignment problem. Simply stated, it is determining the initial conditions of the differential equations that were developed in Chapter 5. The second deals with the real life factors: no matter how expensive the inertial sensors are they have errors that must be estimated. Solutions to these problems depends on the inertial sensor level accuracy onboard the specific craft.

To elaborate, we may have noticed that implementing the INS equations not only provide the craft location and speed, but also its attitude and heading. Some applications use navigation grade sensors to estimate all the above parameters. But others utilize low-grade inexpensive sensors focused on estimating only the attitude and heading. In so doing, these applications – called Attitude and Heading Reference Systems (AHRS) – forgo estimating the location and velocity.

Chapter 9 addresses the alignment problem for navigation systems. In Chapter 10, we discuss the AHRS systems, introduce their pertinent alignment algorithms, and estimate sensor errors. Often, an AHRS complements its inertial system with magnetic detectors and utilizes mathematical algorithms to estimate the relatively large errors introduced by the inertial systems.

A system that utilizes navigation grade sensors could enhance its performance by using aids such as a Global Positioning System (GPS). Like the INS, the GPS provides position and velocity. These two, INS and GPS, when mathematically fused with a Kalman filter, can be used to estimate the inaccuracies due to the sensors of the former system. These equations are developed in Chapter 11.

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