

Contents

| | |
|---|-----|
| Preface | vii |
| Introduction | 1 |
| 1. Vectors and Matrices | 7 |
| 1.1 Introduction | 7 |
| 1.2 Vector Inner Product | 9 |
| 1.3 Vector Cross Products and Skew Symmetric Matrix Algebra | 10 |
| 2. Coordinate Transformation between Orthonormal Frames | 17 |
| 2.1 Introduction | 17 |
| 2.2 Direction Cosine Matrices | 18 |
| 2.3 The Direction Cosine Matrix is a Unitary Matrix | 20 |
| 2.4 The Direction Cosine Matrix is a Transformation Matrix | 21 |
| 2.5 DCM Fixed Axis | 24 |
| 2.6 The Rotation Matrix | 26 |
| 2.7 Inner and Outer Transformation Matrices | 29 |
| 2.8 The Quaternion | 32 |
| 3. Forms of the Transformation Matrix | 35 |
| 3.1 Introduction | 35 |
| 3.2 Simple Frame Rotations | 36 |
| 3.3 Euler Angles | 37 |
| 3.4 Rotation Vector | 38 |
| 3.5 Quaternion | 39 |
| 3.6 Simple Quaternions | 43 |
| 3.7 Conversion between Forms | 45 |
| 3.7.1 Conversion between DCM and Euler | 45 |
| 3.7.2 Conversion between DCM and Quaternion | 45 |
| 3.7.3 Conversion between Euler Angles and Quaternion | 47 |

| | | |
|-------|---|-----|
| 3.8 | Dynamics of the Transformation Matrix | 47 |
| 3.8.1 | DCM Differential Equation | 48 |
| 3.8.2 | Quaternion Differential Equation | 50 |
| 3.8.3 | Rotation Vector Differential Equation | 52 |
| 3.8.4 | Euler Angles Differential Equation | 55 |
| 4. | Earth and Navigation | 58 |
| 4.1 | Introduction | 58 |
| 4.2 | Earth, Geoid and Ellipsoid | 59 |
| 4.3 | Radii of Curvature | 63 |
| 4.4 | Earth, Inertial and Navigation Frames | 65 |
| 4.5 | Earth Rate | 67 |
| 4.6 | The Craft Rate ω_{en}^n | 67 |
| 4.7 | Solution of the DCM C_e^n | 70 |
| 4.8 | Gravitational and Gravity Fields | 70 |
| 5. | The Inertial Navigation System Equations | 75 |
| 5.1 | Introduction | 75 |
| 5.2 | Body Frame of Reference | 76 |
| 5.3 | Inertial Sensors | 77 |
| 5.3.1 | The Accelerometer | 77 |
| 5.3.2 | The Rate Gyro | 78 |
| 5.4 | The Attitude Equation | 78 |
| 5.5 | The Navigation Equation | 80 |
| 5.6 | Navigation Equations Computational Flow Diagram | 83 |
| 5.7 | The Navigation Equation in Earth Frame | 84 |
| 6. | Implementation | 86 |
| 6.1 | Introduction | 86 |
| 6.2 | The Rotation Vector Differential Equation | 87 |
| 6.3 | The Attitude Equation | 92 |
| 6.4 | The Craft Velocity Equation | 95 |
| 6.5 | The Craft Position Equation | 99 |
| 6.6 | The Vertical Channel | 101 |
| 7. | Air Data Computer | 104 |
| 7.1 | Introduction | 104 |
| 7.2 | US Standard Atmosphere 1976 | 105 |
| 7.3 | Pressure Altitude | 107 |
| 7.4 | Vertical Channel Parameter Estimation Using Inertial and Air Data | 111 |
| 7.5 | Density Altitude | 116 |

| | | |
|--------|--|-----|
| 7.6 | Altitude (Descend /Climb) Rate | 117 |
| 7.7 | Air Speed | 117 |
| 7.8 | Indicated Air Speed (IAS) | 119 |
| 8. | Polar Navigation | 121 |
| 8.1 | Introduction | 121 |
| 8.2 | The Wander Azimuth Navigation | 123 |
| 8.3 | Prospective of the Wander Azimuth Approach | 126 |
| 8.4 | Polar Circle Navigation Algorithm | 128 |
| 8.5 | Alternative Polar Circle Navigation Frame | 132 |
| 9. | Alignment | 136 |
| 9.1 | Introduction | 136 |
| 9.2 | IMU Alignment | 137 |
| 9.3 | Alternative Algorithm for C_n^b | 144 |
| 9.4 | Estimation of the Accelerometer and Gyro Biases | 149 |
| 9.5 | Effects of Biases on Estimate of C_n^b | 150 |
| 10. | Attitude and Heading Reference System | 152 |
| 10.1 | Introduction | 152 |
| 10.2 | Attitude Initialization | 152 |
| 10.3 | Heading Initialization | 155 |
| 10.4 | Gyro Drift Compensation | 159 |
| 10.5 | G Slaving | 160 |
| 10.5.1 | X-Gyro Bias | 160 |
| 10.5.2 | Y-Gyro Bias | 162 |
| 10.5.3 | Z-Gyro Bias | 163 |
| 10.6 | Alternative Approach for Gyro Drift Compensation | 163 |
| 10.7 | Maneuver Detector | 165 |
| 10.7.1 | Rate Gyro Threshold Selection | 165 |
| 11. | GPS Aided Inertial System | 167 |
| 11.1 | Introduction | 167 |
| 11.2 | Navigation Frame Error Equation | 168 |
| 11.2.1 | Craft Rate Error $\delta\omega_{en}^n$ | 169 |
| 11.2.2 | Earth Rate Error $\delta\omega_{ie}^n$ | 170 |
| 11.2.3 | Position Errors | 171 |
| 11.2.4 | Attitude Error | 173 |
| 11.2.5 | Gravity Error | 176 |
| 11.2.6 | Velocity Error | 177 |
| 11.2.7 | Navigation Frame Error State Equation | 179 |
| 11.2.8 | Error Block Diagram | 179 |

| | |
|--|-----|
| 11.3 Earth Frame Error Equations | 180 |
| 11.3.1 Attitude Error | 181 |
| 11.3.2 Velocity Error | 182 |
| 11.3.3 Position Error | 183 |
| 11.3.4 Earth Frame Error State Equation | 183 |
| 11.4 Inertial Sensors Error Models | 183 |
| 11.5 The Global Positioning System | 187 |
| 11.6 Mechanization of the INS/GPS Equations | 191 |
| | |
| Appendix A. The Vector Dot and Cross Products | 194 |
| | |
| Appendix B. Introduction to Quaternion Algebra | 197 |
| | |
| Appendix C. Simulink [®] Models | 202 |
| | |
| Appendix D. Ellipse Geometry | 206 |
| | |
| Appendix E. Vector Dynamics | 213 |
| | |
| Appendix F. Derivation of Air Speed Equations | 219 |
| | |
| Appendix G. DCM Error Algebra | 222 |
| | |
| Appendix H. Kalman Filtering | 226 |
| | |
| Index | 237 |