PREFACE

"Order, Disorder, and Criticality" - a book with this title appeared in the World Scientific almost three years ago.[a] The aim of the volume was to review a number of problems related to phase transitions and critical phenomena that have attracted considerable attention due to essentially new contributions. More broadly, the book also aimed to demonstrate that the phase transition theory, which experienced its 'golden age' during the 1970s and 1980s, was far from over and there was still a good deal of work to be done, both on the fundamental level and in respect of applications. It is, of course, impossible to cover all recent developments on pages of a single book; moreover, new directions in research continue to appear. For this reason, the World Scientific Publishing Co. has decided to establish a series of review volumes serving this purpose.

This, the second volume in this series, consists of five chapters. The first chapter, written by Bertand Delamotte, serves as an introduction to the non-perturbative renormalization group. In a traditional survey, an application of a renormalization group (RG) formalism in the quantitative theory of critical phenomena originates from two seminal papers by Kenneth Wilson[b] relying on Kadanoff-Wilson coarse graining ideas which do not assume *per se* any perturbation expansion. However, in 1970-80s – the most advanced period for the application of the method – the term 'renormalization group' was used nearly synonymously with the perturbative field theoretical RG. The last, indeed, served as a powerful tool for giving an accurate quantitative description of criticality. As rare exceptions we can mention papers by G. R. Golner, E. K. Riedel, K. E. Newman, C. Bagnuls, C. Bervillier, G. Zumbach et al., where the non-perturbative approach was further developed and exploited. Also worthy of mention in this context is the work

[a] *Order, Disorder and Criticality. Advanced Problems of Phase Transition Theory*, edited by Yu. Holovatch (World Scientific, Singapore, 2004).
[b] K. G. Wilson, Phys. Rev. B **4**, 3174 (1971); *ibid.* **4**, 3184 (1971).

of I. R. Yukhnovskii and his group,[c] where momentum-shell integration was used in combination with the collective variables method. We are now witenessing a certain return to the original ideas of Wilson, resulting in an increasing interest in the non-perturbative RG. In particular, the method serves as an alternative tool for tackling problems where controversial results are obtained by standard approaches based on weak coupling perturbative expansions. At the beginning of the chapter, the non-perturbative RG a la Wilson is introduced in the context of statistical field theory and compared with the perturtbative RG. Basic notions of renormalization are introduced taking the 2d Ising model on the triangular lattice as an example. Thereafter two 'modern' non-perturbative formalisms are described: the Wilson-Polchinski and the effective average action approaches. Explaining in greater detail how the second one works, the author, at the same time, reviews main results obtained with the help of the non-perturbative RG for the Ising and $O(N)$ models.

A second chapter, written by Reinhard Folk, is devoted to critical dynamics. At the beginning of the chapter, an overview is given of the experimental situation in dynamics, with emphasis on liquids (transport coefficients, light and sound scattering), ferromagnets (neutron scattering) and the lambda transition in liquid He-4 (thermal conductivity, second sound); the van Hove theory and the concept of dynamic scaling theory are also discussed. Thereafter, the main steps of the dynamic renormalization group are explained. It is technically much more difficult to obtain dynamic RG perturbative expansions than static ones. Typically, one works within the two-loop order in dynamics (cf. with the fifth or sixth order in statics). Recently, it was recognized[d] that dynamical vertex functions have a structure, allowing the singling out of genuine dynamic and static parts. This result has led to considerable progress in the quantitative description of dynamic criticality. In particular, it has allowed for the simplification of perturbation theory expansions and has made tractable the two-loop critical dynamics of the superfluid transition in He-3/He-4 mixtures, critical dynamics of model C. The rest of the review is devoted to the comparison of theoretical results with experiments. The main topic in this comparison is an explanation of observed crossover phenomena from the hydrodynamic region to the critical one.

[c] I.R. Yukhnovskii. *Phase Transitions of the Second Order. Collective Variables Method* (World Scientific, Singapore, 1987).
[d] R. Folk and G. Moser, Phys. Rev. Lett. **91**, 030601 (2003); *ibid* **89**, 125301 (2002); Erratum: *ibid* **93**, 229902 (2004).

The first two chapters of this book make use of the field-theoretical formalism to analyze different features of criticality. The third chapter, written by Wolfhard Janke and Adriaan Schakel, makes use of field-theoretical ideas in a rather unexpected and, as the reader will discover, very fruitful way – namely, in a similar fashion as the Feynman path integral formalism provides another view on quantum field theory, the spacetime approach, discussed in this chapter, provides one more – geometric – way to describe critical phenomena. The fact that a geometric description is inherent to criticality has already been demonstrated by the celebrated 'polymer' $N \to 0$ limit[e] of the $O(N)$ spin model: on the one hand, it has allowed for the introduction of powerful methods, elaborated to analyze magnetic systems, in polymer physics; on the other hand, it serves as a link between criticality of thermodynamic phase transitions and purely geometrical scaling of self-avoiding walks. In the $O(N = 0)$ model, the statistics of self-avoiding walks naturally arise if one analyzes high-temperature expansion. In a more general context, the high-temperature expansion provides a geometrical description of phase transitions in spin models, as detailed in this chapter. Information about critical exponents governing temperature scaling at second-order phase transition can be recovered by an analysis of the fractal (geometrical) structure of appropriate high-temperature graphs. In particular, it is shown that the fractal dimensions of the relevant geometrical objects (Peierls domain walls in the Ising model, worldlines in Bose-Einstein condensates, vortex loops in superfluid He-4) encode the critical exponents.

In standard critical phenomena, there is a control parameter that can be varied to obtain a radical change in behaviour. Self-organized critical phenomena, by contrast, are exhibited by driven systems that reach a critical state by means of their intrinsic dynamics, independently of the value of any control parameter. Self-organized criticality offers considerable insight into various phenomena ranging from an archetypal sandpile to earthquakes and traffic jams and is the subject of the chapter by Alexander Olemskoi and Dmytro Kharchenko. After giving a short overview of different approaches to the description of self-organized critical behaviour, the authors provide new results obtained in this field. Initial models for describing self-organized critical phenomena were introduced by means of computer-aided procedures[f]. Since then, computer simulations have been playing an impor-

[e] P. G. de Gennes, Phys. Lett. A **38**, 339 (1972).
[f] P. Bak, C. Tang, and K. Wiesenfeld, *Phys. Rev. Lett.* **59**, 381 (1987); C. Tang and P. Bak, *Phys. Rev. Lett.* **60**, 2347 (1988).

tant, if not a leading, role in this field. However, a particular feature of the review is that it is focused on an analytic perspective in the description of self-organized criticality. The authors consider self-organized criticality as an example of complexity in a system's behaviour and follow its inherent features (fractal geometry, anomalous time evolution, power-law distribution of catastrophic events – avalanches) throughout the review. Using the framework of Langevin dynamics, combined with generalized statistics, a unified picture is offered that allows for the description of both a single avalanche formation and the behaviour of an avalanche ensemble.

The last chapter, written by Ihor Stasyuk, deals with typical descriptions of phase transition problems in solid-state physics. Here, the question of principal interest is a change in the phase diagram of strongly-correlated electron systems caused by (correlation-induced) changes in their energy spectrum. A comprehensive review of a phase behaviour of a so-called pseudospin–electron model is given. The last is a generalization of a Hubbard model to account for the vibrational degrees of freedom, described as pseudospins. Recent applications of the model include high-T_c superconduction crystals, molecular and crystalline hydrogen-bonded systems, intercalated layered structures, etc. An analysis is performed in two complementary ways: the non-perturbative dynamical mean field theory approach and a generalized random phase approximation. The last relies on an elaborated diagram technique for Hubbard operators,[g] which is a generalization of a corresponding technique for the spin operators. Reviewed results for thermodynamics and the energy spectrum bring about rich phase behaviour with a variety of uniform and modulated (commensurate or incommensurate) phases.

There is one more reason behind the choice of the topics for this review series. Since 1997, with the assistance of the Institute for Condensed Matter Physics of the National Academy of Sciences of Ukraine and Ivan Franko National University of Lviv (Ukraine), we have been organizing annual workshops called "The Ising Lectures". The aim of these workshops is to bring together students, as well as young and mature scientists, interested in phase transitions and critical phenomena. These workshops provide a possibility for participants to attend review lecture courses, to take part in informal discussions and to obtain information about new results directly from the principal investigators. This is the context in which the review

[g] P. M. Slobodjan, I. V. Stasyuk, Teor. Mat. Fiz. **19**, 423 (1974) [Theor. Math. Phys. USSR **19**, 616 (1974).]

chapters of this volume were initiated.

I am deeply indebted to my colleagues, the authors of this volume, for coming to Lviv and for their devoted work, both in connection with the solving of various puzzles posed by nature and with preparing future generations for solving such puzzles. Special thanks are due to the World Scientific for their interest in getting this book published.

Yurij Holovatch
ICMP, National Academy of Sciences of Ukraine,
Lviv, Ukraine
& Johannes Kepler Universität Linz, Austria
10 August 2006