Chapter 1

Introduction

The present scientific knowledge is very often successful in the so-called border areas when a few different and sometimes distant scientific theories and methods interlace and gather round new phenomena thus forming a new configuration of scientific models and approaches. Later on many of such configurations might transform into independent scientific branches.

The first steps in the early stage formation of new directions are always the same, trying to collect physical objects, models, analytical methods, computer modeling, etc…, from different areas of science.

In this book, the object of computer modeling are the new structured materials (micro- and nanocomposite materials), the object of analytical study is the new solitary waves in such materials. Both are studied with methods of wave analysis, but the main results obtained by a computer simulation are based on wavelet analysis.

The first goal of this book is to give a full set of models and techniques, which is the essential tool for modeling wave propagation, in modern structured materials, by using wavelet and wave analysis.

The second goal is to discuss each one of the main components of modeling:

- the analysis of solitary waves in structured materials,
- the analysis of mechanical behaviour of structured materials of micro- and nanolevel of the structure,
- the wavelet analysis as a new tool in the modeling of materials.

Each component has enough resources for being developed as an individual self-consistent scientific branch. However, by developing wavelet analysis techniques as applied to composite nanomaterials (both in engineering, and non-engineering applications) and, in perspective, to nanomaterials and nano-formations of biological nature seems to be the most interesting task.

The main goal of this book is to unify three new scientific directions

- the analysis of new materials,
- the analysis of solitary waves in materials,
- the wavelet analysis

and to develop the investigations of waves in new structured materials, which need the knowledge of the models and methods coming from all three directions.

The first part of the book contains three chapters, on wavelet analysis (Chapter 2), theory of materials with internal structure (Chapter 3), and theory of waves in materials (Chapter 4). These chapters are self-sufficient, they don't require any background on the specific subject, and they give all necessary information to understand the remaining two chapters, where all three directions are united for modeling the wave evolution in structured materials.

The wavelet analysis can be roughly understood as a part of mathematics like mathematical analysis, functional analysis, harmonic analysis, or fractal analysis. It is widely accepted that wavelet analysis is one of the best achievements of mathematics in the twentieth century. However, wavelet analysis was developed and recognised mainly for its applications in applied mathematics, physics, engineering sciences, etc. Nowadays it is considered as an independent scientific branch placed on the border among mathematics, scientific computer modeling and applied theory of signals and images. Despite their hard mathematical tools, wavelets spread in many fundamental sciences other than mathematics, as medicine, biology, geophysics, physics, mechanics, economy etc. A special development of the wavelet analysis is achieved in the theory of information, coding theory, and the theory of signals and images.

Wavelet analysis is the topic of Chapter 2 which consists of 11 sections.

There exists a close link between Fourier (harmonic) analysis and wavelet analysis. Therefore very often wavelets are approached through Fourier analysis. Let us remember that the Fourier series of function $f(x) \in L^2(-\pi,\pi)$ by the orthonormal system of functions $\{e_n(x)\} \in L^2(-\pi,\pi), n \in \mathbb{Z}$ is defined as the series

$$\sum_{k\in\mathbb{Z}} c_k\, e_k \equiv \sum_{k\in\mathbb{Z}} \langle f, e_k \rangle e_k\ ,$$

where $\langle f, g \rangle = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(x)\overline{g(x)}dx$ is the inner product. So, the given function $f(x)$ which take an infinite number of values over some interval is replaced by a series which is characterized by given functions $e_k(x)\,(k \in \mathbb{Z})$ and infinite number of coefficients $c_k = \langle f, e_k \rangle$. For a fixed k we can say that the continuous function is discretized, since it is replaced by a discrete set of coefficients of Fourier series.

We will see, in discrete wavelet analysis, that the following representation of function $f(t)$

$$f(t) = \sum_{k\in\mathbb{Z}} c_{j_o,k}\varphi_{j_o,k}(t) + \sum_{j=j_o}^{\infty}\sum_{k\in\mathbb{Z}} d_{j,k}\psi_{j,k}(t)\ ,$$

holds, where $\varphi_{k,m}(t) = 2^{k/2}\varphi\left(2^k t - m\right)$, $\psi_{k,m}(t) = 2^{k/2}\psi\left(2^k t - m\right)$ are a family of functions based on the mother $\varphi(t)$ and father $\psi(t)$ wavelet functions. Each term of the family is obtained by a simple operation of dyadic scaling and translation. They might form sets of different kind (including orthogonal ones and frames). The sets of wavelet transform coefficients $\{c_{j,k}\}_{j,k\in\mathbb{Z}}, \{d_{j,k}\}_{j,k\in\mathbb{Z}}$ are treated as system of numbers, which in the definite way is set in accordance to the signal $f(t)$ (this signal is discretized) and which is called the discrete wavelet transform.

The first sum in wavelet representation gives the coarse approximation and the second sum gives the details of the signal in any necessary scale. Such, more detailed, representation into two separate parts of the signal seems to be impossible with Fourier integral transform and this explains why the wavelet transform is sometimes called the mathematical microscope.

Wavelet is a special function, which vanishes everywhere except a “small” interval, where its profile looks like a piece of some wave-like function. There exist many different families of wavelets, each one reflects some special features, as the Mexican or French hats, or its discoverer like Daubechies or Shannon wavelets.

Let us now consider the integral Fourier transform. If the complex-valued function $f(x)$ is given over the real axis and it is absolutely integrable over this axis, that is, $\int_{-\infty}^{+\infty} |f(x)|\,dx < \infty$, then the Fourier integral transform of function $f(x)$ is the integral

$$\mathsf{F}_f(\omega) = \int_{-\infty}^{+\infty} f(x)e^{-i\omega x}dx \,.$$

Wavelet analysis is based on a similar integral transform. The basic idea, in wavelet analysis, is the decomposition of the function (signal) into two families of functions constructed by means of the scaling function (mother wavelet) $\varphi(x) \in L^2(\mathbb{R})$. The first family is built by translation and scaling of the scaling function. The second one is produced by translation and scaling also, of the wavelet function (father wavelet) $\psi(x) \in L^2(\mathbb{R})$. The wavelet family (set) is

$$\left\{\sqrt{s}\,\psi\left(s(x-\tilde{x})\right)\right\}_{s,\tilde{x}\in\mathbb{R}^2} .$$

The wavelet transform of a function $f(x) \in L^2(\mathbb{R})$ is conventionally defined as

$$\mathsf{W}_{f}(s,\tilde{x})=\int_{-\infty}^{+\infty} f(x)\sqrt{s}\,\psi\big(s(x-\tilde{x})\big)\,dx\,.$$

Among the many similarities of Fourier and wavelet representations, the common representation property of isometry or the property of measure invariance is very important. This property is often commented as the existence of adequateness in the description of the signal and its Fourier or wavelet transforms.

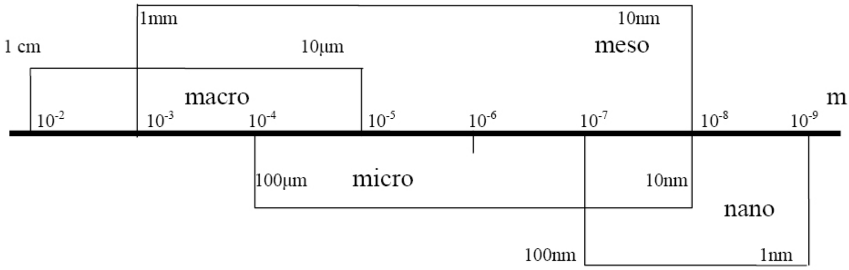
So, two facts are shown as having the deciding role in the explanation of the wavelet theory, the link of wavelet analysis with the classical Fourier analysis and the introduction of the new notion of frames into the wavelet analysis, which can be understood as some development of classical functional analysis. Therefore Chapter 2, especially the first 6 sections, should be considered as having a theoretical character also, which seems to be quite natural owing to the mathematical origins of wavelet analysis.

In section 7, the multiresolution analysis (MRA) is described as the discrete variant of wavelet analysis. In section 10 the most known wavelet families are shown and section 11 is devoted to the integral wavelet transforms.

Chapter 3 describe the theory of materials with internal structure. Eight sections contain the primary information about materials from the notion of the internal structure of macro-, meso-, micro-, and nanolevels till the useful computer modeling data on real micro- and nanocomposites.

A special attention is drawn to composite materials as the main representatives of materials having an internal structure. The basic models (one structural model of the first order and different structural models of the second order) utilized in the modern studies of composite materials are shown and commented on.

The new classification of internal structure of materials by the attribute of admissible size of particles of materials including nanomechanics is shown schematically below and commented on in chapter 3.



The last section seems important for the numerical modeling. It contains different data on granular, fibrous, and layered micro- and nanocomposites, which are the necessary component of modeling the waves in materials.

Chapter 4 complete the representation of the introductory information on the three basic directions and is devoted to the wave analysis in materials. It includes five sections, which cover this topic in full. The main purpose here is to give the sufficiently transparent and simple representation of linear and nonlinear (free) elastic wave propagation in materials.

First we give two basic theoretical models for linear elastic waves: the plane and cylindrical waves. Then the modern nonlinear theory of elastic materials is explained in the most comprehensible form. Finally the nonlinear wave equations and basic approaches to their solution (including two most often utilized procedures: the method of successive approximations and the method of slowly varying amplitudes – van der Pol method) are discussed.

The object of united efforts of three mentioned above theories is described in Chapter 5: the solitary waves in structured elastic materials.

The solitary wave is defined as the wave with initial profile equal to zero practically everywhere besides some finite interval where one or more humps form the wave profile. So, the shape of solitary wave is very similar to the wavelet shape. A typical solitary wave is the classical bell-shaped signal. We study, in particular, two types of solitary elastic waves in materials: the waves with initial profiles in the form of one special function as Chebyshev-Hermite, Mathieu, Whittaker functions and the waves with profiles, which are observed in some experiments on wave propagation in materials.

A new hierarchy of elastic waves in materials is considered:

1. Periodic and nonperiodic waves with the constant phase velocity: classical non-dispersive waves.
2. Periodic waves with the phase velocity depending on frequency: classical dispersive waves.
3. Nonperiodic waves with the phase velocity depending on the actual phase: new type of waves.
4. Nonperiodic waves with the phase velocity depending on the actual amplitude: classical Riemann waves.

The third type of waves is studied in Chapters 5 and 6.

From Chapter 6 we start immediately with applications and form the skeleton of a new field in the analysis of material where

– the object is defined as solitary waves,
– the model is taken from wave analysis and mechanics of materials,
– the tools of analysis combine techniques from wave analysis and elastic wavelet analysis.

Chapter 6 consists of six sections representing different stages of computer modeling and starts with Kaiser physical (optical and acoustic) wavelets, Newland harmonic wavelets, and elastic wavelets proposed by the authors.

Optical wavelets own this name because they satisfy the linear wave equations of optics in the simplified form of Maxwell electromagnetic equations.

The acoustic wavelets were proposed as those wavelets satisfying the linear wave equations in acoustics. These wavelets have a very special shape, which correspond to the so-called *chirp*-signals, very common in acoustic antennas. These signals have the explicit analytical representation $x^{\alpha}\sin\left(1/x^{\beta}\right)$ and, in accordance with the name, they represent those sounds similar to the sounds emitted by dolphins or bats with very characteristic oscillations.

Harmonic wavelets were suggested by Newland. The Newland harmonic wavelets can be referred to physical family of wavelets for many reasons, but mainly because they are especially proposed for the analysis of physical problems on oscillations. Although they have a slow decay in space variable they are very well localized in frequency domain. The base for this kind of physical wavelets family is formed by the Shannon wavelets:

$$\psi_{T,k}(x)=\frac{\sin\frac{\pi}{T}\left(2x-k\right)}{\frac{\pi}{T}\left(2x-k\right)}\quad .$$

The mother harmonic wavelet is defined as follows

$$\varphi(x)=\frac{e^{i2\pi x}}{i2\pi x},$$

$$\varphi_{j,k}(x)=\varphi(2^{j}x-k)=\frac{e^{i2\pi(2^{j}x-k)}-1}{i2\pi(2^{j}x-k)}\quad .$$

The corresponding family of wavelets has the form

$$w\left(2^{j}x-k\right)=\frac{e^{i4\pi\left(2^{j}x-k\right)}-e^{i2\pi\left(2^{j}x-k\right)}}{i2\pi\left(2^{j}x-k\right)}$$

with Fourier transform in the form of trapezoidal box

$$W(\omega)=\begin{cases}(1/2\pi)(1/2^j)e^{-i\omega k/2^j}, & \omega\in\left[2\pi 2^j,4\pi 2^j\right],\\ 0, & \omega\notin\left[2\pi 2^j,4\pi 2^j\right].\end{cases}$$

We propose a new family of wavelets: elastic wavelets on the base of:

1. Kaiser's idea of constructing the physical wavelets as (admissible) solutions of wave equations;
2. the theory of solitary waves (with profiles of the Chebyshev-Hermite functions) propagation in elastic dispersive media;
3. the theory and practice of using the Mexican hat wavelet family, the mother and father wavelets (and their Fourier transforms), which are analytically represented as the Chebyshev-Hermite functions of different indexes.

The elastic wavelet is the solution of the wave equations for the linear elastic dispersive medium. In particular, we consider the two-phase elastic medium. The base system of plane wave equations describe the propagation of solitary waves, where the profiles of solitary waves are chosen as Chebyshev-Hermite functions.

The Chebyshev-Hermite functions can be associated with the Mexican Hat (MH) wavelets. This is the main reason why the MH wavelets were appointed as the first candidate for the elastic wavelets. MH wavelet family is an example of continuous wavelets with infinite support (that is, it is defined over the whole real axis), whereas the most famous wavelet families are defined on a finite support (that is, a finite interval of the real axis). The MH wavelets form a frame, whereas many families of wavelets form an orthonormal basis.

Further we study the regularities of propagation of solitary waves with different initial profiles. On this stage, the elastic wavelet technique is the basic tool for investigation.

We show the ability of elastic wavelet technique to describe adequately the evolution of the wave initial profile as the fundamental phenomenon accompanying the solitary wave propagation.

As application of the elastic wavelets we describe in details the numerical modeling of solitary elastic waves with MH initial profile, the assumptions on weak nonlinearity of the material, the choice of the frame limit, the determination of resolution exactness and the choice of the limit value of scale order in the fine resolution.

The propagation of solitary waves has revealed that there are at least two fundamental questions:

- How can we establish correctly the relationship between the parameters of the wave (its trough as the most typical parameter) and of the material (the characteristic microstructural dimension as the most important parameter in any microstructural theory)?
- At what distance from the starting point of motion or at what time from the beginning of motion are data on the change of the initial wave profile already incorrect?

A solitary wave is defined by two parameters: profile and amplitude. Usually, the wave profile has the so-called trough (bottom), i.e., an interval of the abscissa axis within which the corresponding ordinates on the profile are considerably different from zero. The word "considerably" in the definition of trough implies that its length can be calculated differently. The trough of a solitary wave may be regarded as the length of a harmonic wave, i.e., it is possible to try to compare the trough with the characteristic structural dimension of medium (material) and to find out how, if at all, to account for the internal structure in the model of the medium where the wave propagates.

For a solitary wave, it is obvious that a trough length that exceeds the characteristic structural dimension of material by an order of magnitude should also be considered a threshold value. This minimum trough length, corresponding, possibly, to the most intensive manifestation of profile evolution, is used in computer simulation of wave profile evolution. The maximum trough length, at which the evolution is still somewhat noticeable, can be found from computer simulation.

The utilized elastic wavelet technique permits to describe many wave effects, considered in the last three sections of Chapter 6. For example, the effect of transition from one mode to two modes (breaking-up the primary wave into two modes) can be observed and in the process of evolution of the solitary wave three characteristic stages can be separated.

On the first stage the profile moves in a finite time interval without significant distortions. The second stage consists in the separation (breaking-up) of the wave into two waves which propagate with different phase velocities. The profile distorts on the leading edge of an impulse and then the separation of the second impulse starts. The third final stage shows the separate propagation of two waves with distinguishing phase velocities and amplitudes.

We tried to generalize the procedures of Chapter 6 in a such way, to be used also for studying other evolution problems arising in many different processes of applied science.