

Contents

1	Introduction	1
2	Groups	7
2.1	Definitions and Examples	7
2.2	Computer Algebra Applications	17
2.3	Exercises	20
3	Lie Groups	21
3.1	Definitions	21
3.2	Concepts for Lie Groups	24
3.3	Haar Measure	28
3.4	Computer Algebra Applications	32
3.5	Exercises	33
4	Lie Transformation Groups	35
4.1	Introduction	35
4.2	Classical Groups	36
4.2.1	Affine Groups	45
4.2.2	Group Actions and Group Representations	48
4.3	Computer Algebra Applications	53
4.4	Exercises	54
5	Infinitesimal Transformations	57
5.1	Introduction	57
5.2	Infinitesimal Generators	61
5.3	Multi-Parameter Lie Groups of Transformations	66
5.4	Computer Algebra Applications	70
5.5	Exercises	73

6	Lie Algebras	75
6.1	Definition and Examples	75
6.2	Lie Algebras and Bose Operators	78
6.3	Lie Algebras and Fermi Operators	78
6.4	Lie Algebras and Lie Groups	79
6.5	Classical Lie Algebras	84
6.6	Important Concepts	88
6.7	Adjoint Representation	93
6.8	Casimir Operators	96
6.9	Cartan-Weyl Basis	97
6.10	Computer Algebra Applications	99
6.11	Exercises	101
7	Introductory Examples	103
7.1	One-Dimensional Linear Wave Equation	103
7.2	One-Dimensional Diffusion Equation	107
7.3	Stationary Flow	108
7.4	Gauge Transformation	111
7.5	Computer Algebra Applications	118
7.6	Exercises	119
8	Differential Forms and Tensor Fields	121
8.1	Vector Fields and Tangent Bundles	121
8.2	Differential Forms and Cotangent Bundles	123
8.3	Exterior Derivative	127
8.4	Pull-Back Maps	129
8.5	Interior Product or Contraction	130
8.6	Riemannian Manifolds	131
8.7	Hodge Star Operator	134
8.8	Pseudospherical Surfaces	138
8.9	Computer Algebra Applications	139
8.10	Exercises	142
9	Lie Derivative and Invariance	145
9.1	Introduction	145
9.2	Definitions	145
9.3	Invariance and Conformal Invariance	149
9.4	Computer Algebra Applications	159
9.5	Exercises	160

10 Invariance of Differential Equations	163
10.1 Prolongation of Vector Fields	163
10.1.1 Introductory Example	163
10.1.2 Local Jet Bundle Formalism	166
10.1.3 Prolongation of Vector Fields	170
10.1.4 Partial Differential Equations on Jet Bundles	172
10.2 Invariance of Differential Equations	175
10.3 Similarity Solutions	184
10.4 Transforming Solutions to Solutions	193
10.5 Direct Method	195
10.6 Computer Algebra Application	197
10.7 Exercises	199
11 Lie-Bäcklund Vector Fields	201
11.1 Definitions and Examples	201
11.2 Invariant Solutions	210
11.3 Computer Algebra Applications	213
11.4 Exercises	215
12 Differential Equation for a Given Lie Algebra	217
12.1 Lie Point Symmetry Vector Fields	217
12.2 Lie-Bäcklund Vector Fields	222
12.3 Computer Algebra Applications	224
12.4 Exercises	224
13 A List of Lie Symmetry Vector Fields	225
13.1 Introduction	225
13.2 Listing	225
13.3 Exercises	244
14 Recursion Operators	245
14.1 Gateaux Derivative	245
14.2 Definition and Examples	253
14.3 Computer Algebra Applications	257
14.4 Exercises	258
15 Bäcklund Transformations	259
15.1 Definitions	259
15.2 Examples	261
15.3 Computer Algebra Applications	265
15.4 Exercises	266

16 Lax Representations	267
16.1 Definitions	267
16.2 Examples	269
16.3 Sato's Theory	273
16.4 Computer Algebra Applications	276
16.5 Exercises	277
17 Conservation Laws	279
17.1 Basic Concepts	279
17.2 Exterior Differential Systems	285
17.3 Cartan Fundamental Form	291
17.4 String Theory and Invariants	304
17.5 Computer Algebra Applications	307
17.6 Exercises	310
18 Symmetries and Painlevé Test	311
18.1 Introduction	311
18.2 Ordinary Differential Equations	312
18.3 Invertible Point Transformation	317
18.4 Partial Differential Equations	322
18.5 Symmetries by Truncated Expansions	325
18.6 Painlevé Test and Recursion Operators	328
18.7 Singular Manifold and Similarity Variables	330
18.8 Hirota Technique and Painlevé Test	334
18.8.1 Hirota Technique	334
18.8.2 Connection with Painlevé Test	340
18.9 Integrability Concepts	349
18.10 Computer Algebra Applications	350
18.11 Exercises	351
19 Ziglin's Theorem and Integrability	353
19.1 Introduction	353
19.2 Ziglin's Theorems	354
19.3 Applications	358
19.4 Computer Algebra Application	364
19.5 Exercises	365
20 Lie Algebra Valued Differential Forms	367
20.1 Introduction	367
20.2 Covariant Exterior Derivative	370
20.3 Yang-Mills Equations	371

20.4	Maurer-Cartan Equation	375
20.5	Application	376
20.6	Yang-Mills Equation and Chaos	378
20.7	Self-Dual Yang-Mills Equations	380
20.8	Anti-Self Dual Yang-Mills Equation	384
20.9	Geometry of $SU(n)$	386
20.10	Computer Algebra Applications	389
20.11	Exercises	391
21	Bose Operators and Lie Algebras	393
21.1	Embedding and Bose Operators	393
21.2	Examples	396
21.3	Embedding and Bose Field Operators	398
21.4	Bose Operators and Maps	401
21.5	Computer Algebra Applications	404
21.6	Exercises	405
22	Maps and Invariants	407
22.1	Discrete Dynamical Systems	407
22.2	Logistic Map	409
22.3	Discrete Painlevé Equations	412
22.4	Computer Algebra Applications	413
22.5	Exercises	414
23	Computer Algebra	415
23.1	Computer Algebra Packages	415
23.2	Programs for Lie Symmetries	416
23.3	SymbolicC++ and the Symbolic Class	420
23.3.1	Expression Tree	420
23.3.2	Examples	423
23.4	Examples	428
A	Differentiable Manifolds	433
A.1	Definitions	433
A.2	Examples of Differentiable Manifolds	435
A.3	Exercises	441
	Bibliography	443
	Index	453