PREFACE

The present volume is a collection of survey papers of the talks presented at the very successful fourth China-Japan Seminar on number theory. The seminar was held in Shandong University Academic Center, in Weihai, Shandong, People's Republic of China, during August 30 – September 3, 2006, under the support of Shandong University, the Japan Society for the Promotion of Science (JSPS) and the National Natural Science Foundation of China (NSFC). The organizers Shigeru Kanemitsu and Jianya Liu would like to express their hearty thanks to Shandong University for their generosity and support. The title of the seminar reads "Sailing on the sea of number theory" which suggests that we are supposed to sail freely both on the sea of number theory and the sea of Weihai. The talks were given in a wide conference hall looking over the sea all round and the atmosphere was superb, and all the participants really got the feeling of sailing on the sea of number theory and the sea of Weihai.

Weihai Sea

It should be mentioned that Ms. Nan Luo, together with other staff members of Shandong University, helped out the organizers throughout in preparing and conducting the seminar very successfully. The organizers also would like to thank two of their Chinese colleagues, Professors T. -X. Cai and Z. -W. Sun for their attractions. The former read his own poem which has a passage referring to Pythagoras and the latter made a spot announcement of some recent results in a live fashion.

Shandong University Academic Center (Weihai)

Traditionally, we have been publishing the collection of papers presented at the seminars, but from this volume onwards, we are going to publish survey papers which give good perspectives of recent progress of research in number theory and related fields. We would like to thank the authors of these papers for their kind cooperation regarding the preparation of excellent manuscript and the subsequent unification process of style, which is to make the volume of a readable book quality.

In this volume we assemble the following papers:

1. S. Egami and K. Matsumoto, "Convolutions of the von Mangoldt Function and Related Dirichlet Series" has the theme of analytic continuation of multiple Dirichlet series of various types and their possible natural boundaries. Analytic continuation is furnished largely by means of the Mellin-Barnes integrals which is a form of the definition of the beta-function. The authors consider Φ_2 with the coefficients of the form of the Abel convolution G_2 of the von Mangoldt function as an example of a multiple Dirichlet series which may have a natural boundary. They also consider the Riesz sum of G_2 and obtain an asymptotic formula.

2. K.-Q. Feng and Y. Xue, "Constructing New Non-congruent Numbers by Graph Theory" is regarding the application of graph theory (combined with the results on elliptic curves) to finding non-congruent numbers with arbitrarily many prime factors. The main tools they use from elliptic curve theory is that if the rank of the rational points of a certain elliptic curve (with n in the coefficients) is 0, then n is a non-congruent number and this condition in turn is realized if the corresponding Selmer groups have

the minimal sizes. This last condition is then checked on the basis of the oddness of the suitable graphs. The reader can learn these kinds of various facts about elliptic curves and graph theory.

3. Y. Kitaoka, "Distribution of Units of an Algebraic Number Field Modulo an Ideal" is a massive work expounding the author's new investigation on the distribution of units modulo an ideal. Here the author combines algebraic structures with an analytic output, i.e. the density etc. and should give rise to a new fertile uncultivated land for the coming younger generation. Uncultivated because, compared with extensive study on other ingredients of an algebraic number field like class numbers, the units have not been paid much attention. Not only the field is promising but the problem setting will be very beneficial to younger scientists: Setting a problem in algebraic aspects and incorporate analytic tools to arrive at some statistical data. In the paper the reader can learn really all notions and tools in algebraic number theory, the Frobenius automorphism, Hilbert's ramification theory, Galois action, the Čebotarëv density theorem, and the Artin conjecture.

4. W. Kohnen, "Sign Changes of Fourier Coefficients and Eigenvalues of Cusp Forms". In the paper the recent results are summarized on the sign change problem of the Fourier coefficients $a(n)$ of cusp forms f (Siegel modular forms). The Theorem in §1 about infinitely many sign changes is proved using the Hecke L-series for f and the Rankin-Selberg zeta-function, which motivates the study of the first occurrence of the sign change. For a normalized Hecke eigenform that is a new form of even integral weight k and level N, sharp bounds for the occurrence of the first sign change is obtained, with or without the symmetric square L-function and the Hecke relations, (sub-)convexity bounds, prime number theorem, etc. The same ideas give results including sign changes in short intervals. Sign changes for Siegel modular forms of genus 2 are also summarized. Infinitely many sign changes may not occur in general, yet there are theorems which have similar flavor as those for the elliptic modular case. The proof uses the spinor zeta-function in place of Hecke L-series.

5. Y. -K. Lau, J. -Y. Liu and Y. -B. Ye, "Shifted Convolution Sums of Fourier Coefficients of Cusp Forms". Let α denote the infimum of the exponent of t in the estimation of the Riemann zeta-function on the critical line $\sigma = 1/2$. The GLH (Generalized Lindelöf Hypothesis) which follows from the GRH (Generalized Riemann Hypothesis) implies $\alpha = 0$. The convexity bound which is obtained with the aid of the Phragmén-Lindelöf convexity principle is $\alpha = 1/4$ and the authors call any improvement on the convex-

ity bound a subconvexity bound, while the bound $\alpha = 1/6$ which is 2/3-rd power of convexity and is due to Weyl, is called a Weyl-like bound. The paper is concerned with summarizing the research made hitherto toward these subconvexity and Weyl-like bounds for automorphic L-functions. The authors refer to the L-functions of degree n according to the degree of the generic polynomial factor in p^{-s} in their Euler product and after mentioning the degree 2 case, they present their newest Weyl-like bound for the Rankin-Selberg L-function $L(1/2+it, f \times g)$ formed from f, a holomorphic Hecke eigenform for $\Gamma_0(N)$ of weight k (or a Maass Hecke eigenform for $\Gamma_0(N)$ with Laplace eigenvalue $1/4+k^2$) and g a fixed holomorphic or Maass cusp form for $\Gamma_0(N)$, or for $\Gamma_0(N')$ with $(N, N') = 1$. The Weyl-like bound obtained is 2/3 in the weight aspect, which is expounded in the authors' most recent article of 78 pages. The method hinges on the meromorphic continuation of the shifted convolution sum formed from the Fourier coefficients. In the paper, the intermediate developments toward the above bound are explained in some detail and the reader can learn a quite rich mixture of many new methods which have their origin in analytic number theory and spectral theory.

6. K. Miyake, "Two Expositions on Arithmetic of Cubics". The paper consists of two parts. Part I is devoted to the study of cubic generic polynomials $R = R(t; x)$ and $Q = Q(s; x)$ with parameters t and s for the symmetric group of degree 3 and the cyclic group of order 3, respectively. As an application, the divisibility of the class numbers of quadratic fields by 3 is obtained. For the algebraic tools used, the reader is referred to the recent books by G. Gras or J. Neukirch. Thus Part I motivates the study of cubic and more generally, non-abelian fields. In Part II, two types of families of elliptic curves are considered whose sets of rational points over $\mathbb{Q}$ are described by certain subsets of cubic fields. One type is given by $u^3 = u^3 + au^2 + bu + c$ whose short forms are Mordell curves, while the other family consists of the twists of Hessian family of elliptic curves over the splitting field of the cubic polynomial R and also over the quadratic field contained in the splitting field. Part II therefore introduces the reader to the world of arithmetic on elliptic curves. Basic knowledge is to be found in the textbooks of J. Silverman and of Silverman and Tate.

7. I. Shparlinski, "Distribution of Points on Modular Hyperbolas". For a positive integer m and an arbitrary integer a with $\gcd(a, m) = 1$, the author defines the modular hyperbola as $xy \equiv a (\text{mod } m)$ and designates all the points (x, y) on it by $\mathcal{H}_{a,m}$. The author considers the distribution and geometric properties of points of $\mathcal{H}_{a,m}$, denoted $\mathcal{H}_{a,m}(\mathcal{X}, \mathcal{Y})$ whose coordinates

x and y lie in prescribed sets of integers $\mathcal{X}$ and $\mathcal{Y}$, respectively. Acquiring precise asymptotic formulas and establishing positivity of $\#\mathcal{H}_{a,m}(\mathcal{X}, \mathcal{Y})$ for various interesting sets have been the central theme in this area. However, as the author writes, "there is a large number of papers which rather routinely study various problems related to $\mathcal{H}_{a,m}$ on the case by case basis. Here, we explain some standard principles which can be used to derive these and many other results of similar spirit about the points on $\mathcal{H}_{a,m}$ as simple corollaries of just one general result about the uniformity of distribution of point on $\mathcal{H}_{a,m}$ in certain domains. In §3.1, this result, Theorem 10, is derived in a very straightforward fashion from (1.1)—the known bound of Kloosterman sums—using some standard arguments", and we learn one important lesson that if one keeps doing routine work, eventually all the result obtained will be relinquished into history's dustbox and will not be referred to as illustrating examples in the subsequent publications. §§2.1–2.3 give a nice survey on a diversity of methods used to study the point on $\mathcal{H}_{a,m}$, and especially, it is remarked that multiplicative character sums can sometimes give better results than Kloosterman sums.

Various applications are described as well. In particular in §5, one can find some interesting results, especially an unexpected result on torsion of elliptic curves in §5.4.

8. Z. -W. Sun, "A Survey of Problems and Results on Restricted Sumsets". The paper gives a useful survey on recent results in additive number theory, now being exciting, related to combinatorics. A restricted sumset has the form $\{a_1 + \cdots + a_n : \ a_1 \in A_1, \ldots, a_n \in A_n, \ P(a_1, ..., a_n) \neq 0\}$, where $A_1, \ldots, A_n$ are finite subsets of $\mathbb{Z}$, or a field or an abelian group, and P is a suitable polynomial. The book "Additive Combinatorics" (2006) by T. Tao and V. Vu, mainly summarizes important results on sumsets without restrictions, obtained by various different tools.

The author is concerned with the lower bounds for cardinalities of various sumsets obtained by algebraic methods. §1 gives rather enlightening discussion on the powerful "polynomial method" via Alon's "Combinatorial Nullstellensatz" the proof of which is given and which implies the Alon-Nathanson-Ruzsa theorem, generalizing Dias da Silva and Hamidoune's extension of the Erdős-Heilbronn conjecture. In the remaining sections one can find recent developments by the author and his school. The survey also contains some open conjectures.

9. H. Tsukada, "A General Modular Relation in Analytic Number Theory" presents equivalent forms of the functional equation with multiple gamma factors satisfied by two sets of Dirichlet series, in the form of mod-

ular relation between the corresponding two sets of H-function series, where H-function means the Fox H-function. As far as we have checked, there is no formula which does not come under this big umbral theorem. In the paper, there are some illustrating examples with multiple gamma factors. Hopefully, the theorem covers also those zeta-functions studied in the papers of Kohnen and Lau-Liu-Ye.

10. D. -Q. Wan, "L-functions of Function Fields". Let $\mathbb{F}_q$ denote a finite field of q elements with characteristic $p > 0$. As a generalization of the case of $C=$ the projective plane, $U=$ projective plane with the origin and infinity removed and $K = \mathbb{F}_q(t)$, the function field, the following general situation is considered: Let C denote a smooth projective geometrically connected curve defined over $\mathbb{F}_q$ with function field K, with its absolute Galois group denoted by G_K and let $j : U \to C$ be a Zariski open dense subset of C.

Let F_ℓ be a finite extension of $\mathbb{Q}_\ell$, ℓ being a prime number. With V a finite dimensional vector space over $\mathbb{F}_\ell$, let $\rho : G_K \to GL(V)$ be a continuous representation of G_K unramified on U. There are two L-functions introduced, the L-function $L(U, \rho)$ and the complete L-function of ρ on C, $L(C, \rho)$ which differ by a finite number of Euler factors. The paper gives a concise survey of recent results on analyticity of these L-functions, especially, for those representations arising from geometry. It is stated that in the ℓ-adic case, with $\ell \neq p$, all ℓ-adic representations are essentially geometric from L-function point of view including ℓ-adic function field analogue of Artin's entireness conjecture. In the more complicated $\ell = p$ case, the location of zeros and poles on the compact unit disc is determined and the author's result says that if the p-adic representation is geometric, the L-function $L(U, \rho)$ is p-adic meromorphic everywhere. The interested reader can go on by reading the references given.

All in all, the interested reader can really get benefited by going through this volume and we hope we will keep doing this work of putting together the recent developments in handy volumes.

The editors would like to express their hearty thanks to Professor Haruo Tsukada, Dr. Jing Ma and Ms. Nan Luo for their devoted help toward the completion of this volume. Dr. Jing Ma, especially, spent an immense amount of time to edit the final versions of all the papers; without her help, the volume could not have been completed so well and in time. Professor Haruo Tsukada kindly checked the final version of the manuscript and made essential improvement therein to whom the editors would like to express their hearty gratitude. Last but not least, thanks are due to the editor of

the World Scientific, Mrs. Ji Zhang, for her kind and timely help throughout the preparation of the proceedings.

As usual, we end up with recording a poem describing our feeling, and this time it is the following.

威海連天碧
宇內盡光華
把酒歌聚散
一躬向天涯

The fifth China-Japan Seminar will be held in Osaka, at Kinki University and we do hope we'll meet there again. Wo men xi wang zai jian dao ni men.

Jin Gunagzi and Liu Jianya—the editors-organizers