

# THREE NUCLEON FORCES AND NUCLEAR STRUCTURE

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The author's earlier research with Professor Bruce McKellar, some 30 years ago, led to the Tucson-Melbourne three-nucleon interaction, which is still relevant today. The new significance of three-nucleon forces in determining the structure of atomic nuclei will be discussed. This has led to increased efforts to learn more about the nature of these three-nucleon interactions, both experimentally and theoretically. The recently developed no-core shell model (NCSM) has the ability to test different theoretical models for three-nucleon forces by making direct comparisons of results produced by these forces with experimental data.

*Keywords:* nuclear structure, Tucson-Melbourne three-nucleon interaction, no-core shell model.

## 1. Introduction

Congratulations to Bruce McKellar and Girish Joshi on their retirements from the University of Melbourne! It is indeed a great pleasure for me to take part in this retirement Festschrift in their honor, because I have known Bruce McKellar for 40 years and have had a connection with the University of Melbourne since 1973.

My most memorable and significant connection is related to the topic of three-nucleon (NNN) interactions. In the early 1970s I taught a graduate level course on nuclear many-body theory at the University of Arizona, which was being audited by Dr. Sidney Coon, who was a research associate in our group at that time. When I got to the part regarding the theory of the contribution of NNN interactions to the binding energy of nuclear matter, Dr. Coon suggested that we take up this problem with Dr. Michael Scadron, also in our department. The basic idea was to include the pion-nucleon scattering amplitude into the standard two-pion exchange NNN interaction of the Fujita-Miyazawa<sup>1</sup> structure. The results of this investigation<sup>2</sup> were published in 1975.

Dr. Scadron was also a collaborator of Bruce McKellar and, hence, took up this problem with him, which produced a number of improvements and refinements, leading to what is known today as the Tucson-Melbourne three-nucleon (TM NNN) interaction, which is one of the most-recognized NNN interactions worldwide. Our paper<sup>3</sup> on the TM NNN interaction in 1979 has become a widely cited publication,

still being referenced today, because of the now-recognized importance of NNN interactions to the binding energies and other properties of finite atomic nuclei as well as of nuclear matter. The latest version of the TM NNN interaction has been developed by Coon and collaborators,<sup>4</sup> with input from Chiral Effective Field Theory<sup>5</sup> ( $\chi$ EFT).

## 2. The Significance of Three-Nucleon Interactions

The importance of NNN interactions to the theory of nuclear structure has become more apparent within the last ten years with tremendously improved procedures for solving the nuclear many-body problem, such as, the Green Function Monte Carlo<sup>6,7</sup> (GFMC) and No-Core Shell Model<sup>8,9</sup> (NCSM) approaches. In a groundbreaking benchmark investigation<sup>10</sup> in 2001, seven nuclear-theory groups (totally 17 theorists!) calculated the properties of the  $A = 4$  system using the same nucleon-nucleon (NN) potential, *i.e.*, the Argonne V8' (AV8') potential.<sup>6</sup> Within the limits of error of the seven different approaches, all the groups obtained the same result for the binding energy of  ${}^4\text{He}$ . The most significant result of this investigation is that nuclear-structure theory has now progressed to the stage, where truly accurate and meaningful calculations can be performed for the properties of light nuclei, given your choice for the NN interaction. The bad news is that the mean result obtained for the binding energy underbinds  ${}^4\text{He}$  by about 2.3 MeV. However, the positive conclusion to be drawn from this negative result is that the discrepancy between theory and experiment has real physical significance, because the theoretical results are accurate and meaningful. Studies<sup>6-9,11</sup> for other light nuclei from  $A = 5$  to  $A = 12$  show a consistent underbinding of nuclei, compared with experiment, when only local NN interactions are utilized. Consequently, something is actually missing in the theory of nuclear structure, when only local NN interactions are employed. The logical choice for the missing ingredient is NNN interactions, since it is well-known that such forces naturally arise theoretically, when the interaction among nucleons is derived from  $\chi$ EFT.<sup>5,12-14</sup> This has led to a great interest, both theoretically and experimentally, in obtaining as much information as possible about the nature of these NNN interactions.

## 3. The No-Core Shell Model (NCSM)

In the last 15 years a new *ab initio* microscopic approach has been formulated for calculating the properties of atomic nuclei, known as the No-Core Shell Model<sup>8,9</sup> (NCSM). In the NCSM approach, there is no inert, closed core of nucleons, surrounded by a few valence nucleons;<sup>15,16</sup> but all  $A$  nucleons are taken as being active. This new procedure has a number of advantages over the older, standard shell model,<sup>16</sup> such as avoiding the intruder-state effect; yielding an exact, converged result for the effective interaction in a given model space; providing energies relative to the vacuum; and allowing for an exact treatment of the spurious center-of-mass (CM) motion problem.

The NCSM formalism has been given in earlier publications<sup>8,9</sup> and will not be repeated here. The reader is referred to the literature for a detailed description of this approach, which can be summarized as follows:

- (1) Start with the  $A$ -nucleon Hamiltonian in relative coordinates and your choice for the NN interaction (or NN + NNN interactions).
- (2) Bind the CM of the nucleus in an harmonic-oscillator (HO) potential well,<sup>17</sup> *i.e.*, add the CM HO potential to the Hamiltonian in part (1). This does not change any of the intrinsic energies but provides a single-particle HO basis for the performing of numerical calculations. It also confines the nucleons in a HO potential well, which aids in the convergence of the final numerical results.
- (3) In order to solve the nuclear Schrödinger equation for the  $A$ -nucleon Hamiltonian, truncate to a finite model space. This produces a tractable numerical calculation but converts the initial NN (or NN + NNN) interactions into effective  $A$ -nucleon interactions.
- (4) Because one cannot, in general, solve the  $A$ -nucleon problem with  $A$ -nucleon interactions (*i.e.*, for  $A > 4$ ), it is necessary to restrict the calculations to effective NN (or effective NNN) interactions and operators, known as the two-body cluster (or three-body cluster) approximation.
- (5) Pick a given cluster level, *e.g.*, the two-body cluster, and solve the Schrödinger equation at this level, *i.e.*, for the eigenenergies and eigenfunctions of two nucleons in an HO potential well interacting through an NN interaction.
- (6) Pick the size of the model space, in which you want to perform your calculations (*i.e.*, in terms of the eigenstates up to a given energy above the ground-state energy), and then use the solutions in part (5) to calculate exactly the two-body effective interaction in this model space, as well as other two-body effective operators, using the unitary transformation method.<sup>18–22</sup>
- (7) Finally, by diagonalization, solve the Schrödinger equation in the truncated model space for all  $A$  nucleons active, using the effective interaction in part (6). Repeat the calculation for increasing size of the model space to check the convergence of the numerical results. There is also a pseudo-dependence on the HO energy,  $\hbar\Omega$ , due to the truncation of the Hilbert space, but this dependence goes away as the size of the model space approaches infinity.

#### 4. Testing Different Theoretical NNN Interaction Models

How all of this becomes relevant to the earlier discussion of NNN interactions is the fact that the above procedure can be repeated at the three-body cluster level, including NNN interactions. Our present knowledge regarding the physical nature of NNN interactions is based purely on theoretical models, such as the TM,<sup>4</sup> the Urbana-IX (UIX) and the Illinois-2 (IL2).<sup>7,23</sup> One of the main problems is that most numerical calculations involve NNN interactions, which are *not* theoretically self-consistent with the accompanying NN interactions, such as AV18 + UIX or AV8'

+ TM. In these cases the unknown parameters in the NNN-interaction models are adjusted to fit some data for light nuclei, such as, the binding energies of  $^3\text{H}$  and  $^4\text{He}$ , calculated with the NN + NNN interactions. These NN + fitted-NNN interactions are then used to calculate the properties of heavier nuclei.

In practice, one wants to utilize interactions among the nucleons based on the QCD Lagrangian. This is now possible in terms of  $\chi\text{EFT}$ , in which NNN interactions consistent with the accompanying NN interactions are generated as one calculates higher-order terms in the  $\chi\text{EFT}$  expansion.<sup>5,13</sup> Figure 1 shows the first three terms, which occur for the NNN interaction in this expansion at next-to-next-to-lowest order ( $\text{N}^2\text{LO}$ ). The first term is related to the Fujita-Miyazawa two-pion exchange

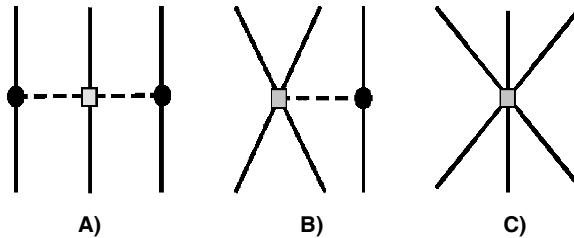


Fig. 1. The first three terms in the NNN interaction, occurring in  $\text{N}^2\text{LO}$  of the  $\chi\text{EFT}$  expansion.

contribution<sup>1,5</sup> to the NNN interaction. The second and third processes contain what are known as contact terms, which involve low-energy constants (LECs). At some future time these LECs will need to be determined from Lattice Gauge calculations, but for now they must be obtained by fitting to experimental data. In the case of the Idaho  $\chi\text{EFT}$  interaction,<sup>24</sup> there are two choices for these LECs, which describe the experimental data equally well.<sup>25</sup> Consequently, a test is needed for deciding between these two choices for the LECs, or, in general, among any set of different theoretical models for NNN interactions.

Such a test exists in the form of the NCSM, which allows nuclear-structure calculations at the three-body cluster level, including NNN forces. Because of the sensitive dependence of theoretical NNN interactions on spin and isospin, such forces can have a dramatic effect not only on the binding energies of nuclei but also on their spectra. For example, the calculated ground-state ( $I^\pi, T$ ) for  $^{10}\text{B}$  is  $(1^+, 0)$  using only local NN interactions,<sup>26</sup> while it is the correct value of  $(3^+, 0)$ , when NNN interactions are included in the calculations.<sup>7,9</sup>

Thus, by performing NCSM calculations at the three-body cluster level with different theoretical models for the NNN interactions and comparing the results with experiment, one has a test for these theoretical models. Preliminary calculations<sup>25,27</sup> of this kind have been performed for  $^6\text{Li}$  and  $^7\text{Li}$  and are shown in Figs. 2 and 3, respectively. The 3NF-A and 3NF-B results in these two figures refer to the two equally good choices for the LECs in Fig. 1, *i.e.*, terms B) and C). At this time, the purpose is *not* to make an argument as to whether choice 3NF-B is better than

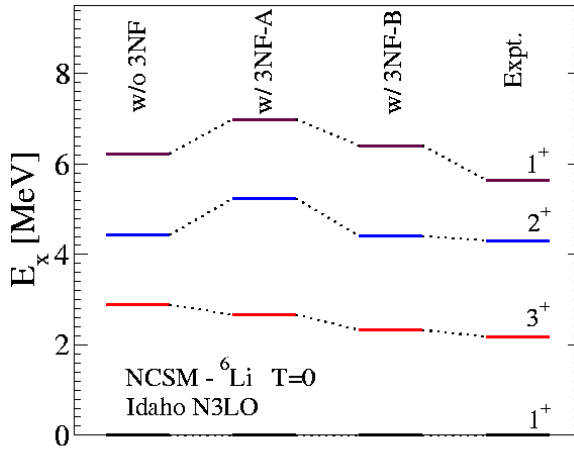


Fig. 2. Dependence of the excitation energies of the lowest states of  ${}^6\text{Li}$  on the interaction.<sup>27</sup> Results with the NN interaction only and with the 3NF-A and 3NF-B included (see text for details) are compared with the experimental values.

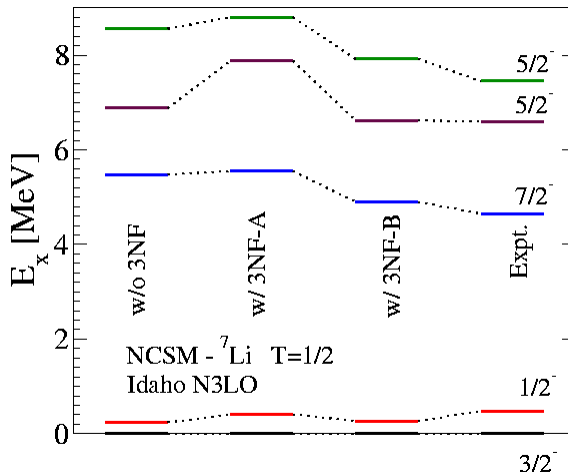


Fig. 3. Dependence of the excitation energies of the lowest states of  ${}^7\text{Li}$  on the interaction.<sup>25</sup> Results with the NN interaction only and with the 3NF-A and 3NF-B included (see text for details) are compared with the experimental values.

choice 3NF-A, based on these limited calculations for  ${}^6\text{Li}$  and  ${}^7\text{Li}$ , but to illustrate how the NCSM can be employed to make such evaluations. Clearly, more extensive calculations for many more nuclei are necessary before one can meaningfully evaluate and rate different NNN interactions.

## 5. Conclusions

Because NNN interactions are important for understanding the details of nuclear structure and because all current models of such forces are purely theoretical, there is a real need for a procedure for evaluating the accuracy of these different theoretical models. The NCSM provides such a procedure, because of its ability to perform nuclear-structure calculations at the three-body cluster level, including NNN interactions.

In conclusion, NNN interactions are as interesting and timely an area of research in nuclear-structure theory today, as they were some 30 years ago, when Bruce McKellar and I first collaborated, leading to a long, enjoyable and productive scientific relationship as well as friendship.

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