# Preface

This year will be my 60th birthday, which naturally makes one look back to see what one has accomplished and failed to accomplish. This is also an age at which the technical details begin to seem less important and the big, philosophical ideas stand out.

For four decades I have been using the idea of complexity to try to understand the significance of Gödel's famous incompleteness theorem. Most logicians and mathematicians think that this theorem can be dismissed. I, on the contrary, believe that mathematics must be conceived of and carried out differently because of Gödel.

The fundamental question: What is a proof? Why is it convincing? What is mathematics? In fact, Gödel's proof is a *reductio ad absurdum* of the idea of a formal axiomatic math theory. Gödel, Turing and myself, what we do each in our own unique way, is to assert increasingly emphatically that math is not a formal theory, it is **not** mechanical. What then **is** math? Where do new truths come from?

Instead of saying what math isn't, how about a new, optimistic metamathematics that says what math is, and how creativity, imagination and inspiration make it progress? For like any living organism, math must progress or it withers and dies. It cannot be static, it must be dynamic, it must

constantly evolve. And there are other pressing questions, particularly for those of us who feel so attracted, so obsessed by mathematics. We sometimes wonder: Why is it so beautiful? What precisely attracts us? Why do we feel so attracted? Is there no end to such beauty?

This collection contains twenty-three papers that I still feel to be stimulating and which discuss these issues at a philosophical rather than at a technical level. They are presented in chronological order, in order of publication, and taken together I hope they begin to make a case for a new way of looking at mathematics, for a new idea of what math is all about, more in line with how physicists work, perhaps.

This is, I hope, a case in which the whole is greater than the sum of its parts. Many interesting topics are discussed, including Cantor's diagonal method, Gödel's 1931 proof, Turing's halting problem, program-size complexity, algorithmic irreducibility and randomness, as well as important ideas on complexity and the limits of mathematics of Leibniz and Emile Borel that I feel are insufficiently appreciated.

By going through this collection in chronological order one can also appreciate how difficult it is for new ideas to emerge, as one slowly gropes in the dark for understanding. One keeps turning things over and over in one's mind. Each tiny step forward takes years. One only perseveres because one must and because one believes that things ultimately make sense, even if one will never fully understand why.

I also hope that this collection, showing as it does how a few simple ideas, mere seedlings at first, gradually developed into a large tree, will encourage others to believe in the power of ideas, in math as an art, and in the search for deep understanding and fundamental ideas.

Just look at those magic creative moments in human history, like ancient Greece, the Italian renaissance, France before the revolution, Vienna between the wars, when there is enough anarchy for new ideas to flourish, and individuals can affect events. Can we understand the sociodynamics of this magic? Can we encourage and promote such moments?

The basic lesson that I have learned from Gödel is that mathematics is not a machine. Creativity is essential. And it is also mysterious. Just look at the ease with which rivers of beautiful mathematics flowed from Euler's pen. Look at Ramanujan's remark that a goddess brought him ideas while he slept, and that no equation is worthwhile unless it expresses one of God's

thoughts.[1] Or, for that matter, look at Gödel's faith that mathematicians can overcome the incompleteness theorem by intuiting new concepts and principles whenever this is needed for mathematics to advance. So it is high time for us to give up on static, formal mathematics and instead begin to study creativity and how ideas evolve.

Finally, I want to say that I am very grateful to my friends in Chile, Eric Goles, Oscar Orellana, and Ricardo Espinoza, for encouraging me to put together this collection and helping me to find an appropriate title.

Gregory Chaitin, Viña del Mar, January 2007

[1] In this connection, see Ira Hauptman's play *Partition* about Hardy, Ramanujan, and Ramanujan's goddess Namagiri. Available from `http://www.playscripts.com`.