PREFACE

The primary interest in Gabor and wavelet analyses and their theories stems from and is propelled by the explosion of data arising from rapid advances in communication, sensing and computational power. The usefulness of these data for human knowledge is determined by their accessibility and portability. Pertinent research efforts in mathematics focus on developing new theories, technologies and algorithms for the representation, processing, analysis and interpretation of such scientific data sets. The Gabor and wavelet representations are among the most successful mathematical tools to this end, and were found widespread in applications on signal analysis, image processing and many other information-related areas. Both methodologies deliver representations that are simultaneously local in time and in frequency. The Gabor representation is obtained by windowing the signal through a fixed-size window, which implies that it tiles uniformly the frequency domain. The wavelet representation employs windows that are arbitrarily small, hence is especially suitable for the analysis of transient events.

In response to the recent exciting developments in the mathematical research on image, signal and information processing, the program "Mathematics and Computation in Imaging Science and Information Processing" was held in Singapore at the Institute for Mathematical Sciences, National University of Singapore, from July to December 2003 and in August 2004. A major goal of the program was to promote multidisciplinary research in the area. Among the core topics, one finds time-frequency analysis and applications, wavelet theory and its applications in image and signal processing, and numerical methods in image and information processing. Altogether, three conferences, six workshops, eleven tutorials and three public lectures were set. More than 340 participants took part in these activities including 130 international attendees. We take this opportunity to acknowledge with thanks the essential contribution of the Institute for Mathematical Sciences, which provided us with ample funds and efficient administrative support. We would also like to express our appreciation to the authors for their contributions towards this volume.

The tutorials of the program, each comprising a series of lectures, were conducted by international experts, and they covered a wide spectrum of topics in the field of mathematical image, signal and information processing. The compiled volume includes exposition articles by the tutorial speakers on the foundations of Gabor analysis, subband filters and wavelet algorithms, and operator-theoretic interpolation of wavelets and frames. The volume also presents research papers on Gabor analysis. The accompanying volume focuses more on applications and contains survey articles by the tutorial speakers on subdivision in geometric modeling and computer graphics, high order numerical methods for time dependent Hamilton-Jacobi equations, variational methods in mathematical image processing, data hiding and image steganography, and the apriori algorithm in data mining. The two volumes collectively provide graduate students and researchers new to the field a comprehensive introduction to a number of important topics in mathematical image, signal and information processing. The chapters in each volume were written by specialists in their respective areas. In what follows, we outline the organization of this volume and briefly discuss the topics that are presented.

The first three chapters focus on Gabor frames and its general theory. The first chapter by H. G. Feichtinger, F. Luef and T. Werther is a panoramic introduction to Gabor analysis, from basic principles of linear algebra up to the foundations of advanced time-frequency concepts. It provides the motivation and technical background on some of the key topics in Gabor analysis, namely, the Gabor frame operator, the notion of (dual) Gabor frames, the Janssen representation of the Gabor frame operator, and the spreading function as a tool to describe operators. The first part of the article is devoted to finite dimensional signal spaces, and allows one to get familiar with the general concepts in a concrete linear algebraic setup. The second part deals with Gabor analysis of functions on Euclidean spaces. In this context, the Banach Gelfand triple built upon the Segal algebra appears as a universal tool for a treatment of questions of time-frequency analysis, and the associated modulation spaces.

The second chapter by A. J. E. M. Janssen analyzes several iterative algorithms for computing canonical tight windows and dual windows for Gabor frames by using the calculus of frame operators, the spectral mapping theorem, and Kantorovich's inequality as tools. For the computation of canonical tight windows, both algorithms that require inversion of intermediate frame operators and algorithms that do not need inversions are considered. For the computation of dual windows, algorithms requiring no frame operator inversions are presented. Analysis of the convergence orders of the algorithms is given. The optimality of the convergence orders and the stability of the algorithms are discussed as well.

The third chapter by F. Luef connects Gabor analysis with noncommutative tori and Feichtinger's algebra. In particular, the strong Morita equivalence of noncommutative tori appears as underlying setting for Gabor analysis, since the construction of equivalence bimodules for noncommutative tori has a natural formulation in the notions of Gabor analysis. This leads to the conclusion that Feichtinger's algebra is such an equivalence bimodule. In addition, based on results about Morita equivalence, the biorthogonality relation of Wexler-Raz on the existence of dual atoms of a Gabor frame operator is discussed.

The remaining two chapters are on general wavelet theory. The fourth chapter by P. E. T. Jorgensen expands on some interconnections between mathematical aspects of wavelets and other areas, both within and outside mathematics, such as operator theory, quantum theory, and especially signal processing. Particular attention is given to the concepts of high-pass and low-pass filters: these are central notions in wavelet representation, that played a significant role in signal processing long before wavelets were formally introduced. The chapter gives a detailed account on subband filtering techniques and the efficient algorithms generated by them for practical applications. It also provides connections between these techniques in wavelet analysis and elements of operator theory. It further indicates a number of developments in operator theory which arise from wavelet problems, but are of independent interest in mathematics.

The last chapter by D. R. Larson is an exposition on an operator-theoretic interpolation of wavelets and frames that was developed by the author in collaboration with others. A wavelet is a special case of a vector in a separable Hilbert space that generates a basis under the action of a system of unitary operators. A detailed description of the operator-interpolation approach to wavelet theory using the local commutant of a system is given. This leads to a comprehensive study on applying the theory of operator algebras to wavelet theory. The concrete applications of this method include results obtained using specially constructed families of wavelet sets. The concept of frames is introduced as a sequence of vectors in a Hilbert space which is a compression of a basis for a larger space. Due to this compression relationship between frames and bases, the unitary system approach to wavelets (and more generally, wandering vectors) is perfectly adaptable to frame theory.

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